Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.2-e-x-^m-a+b-x^n-^p-cosh

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [111]. This is test number [166].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (111)	% 0. (0)
Mathematica	% 100. (111)	% 0. (0)
Maple	% 100. (111)	% 0. (0)
Maxima	% 57.66 (64)	% 42.34 (47)
Fricas	% 100. (111)	% 0. (0)
Sympy	% 23.42 (26)	% 76.58 (85)
Giac	% 63.96 (71)	% 36.04 (40)

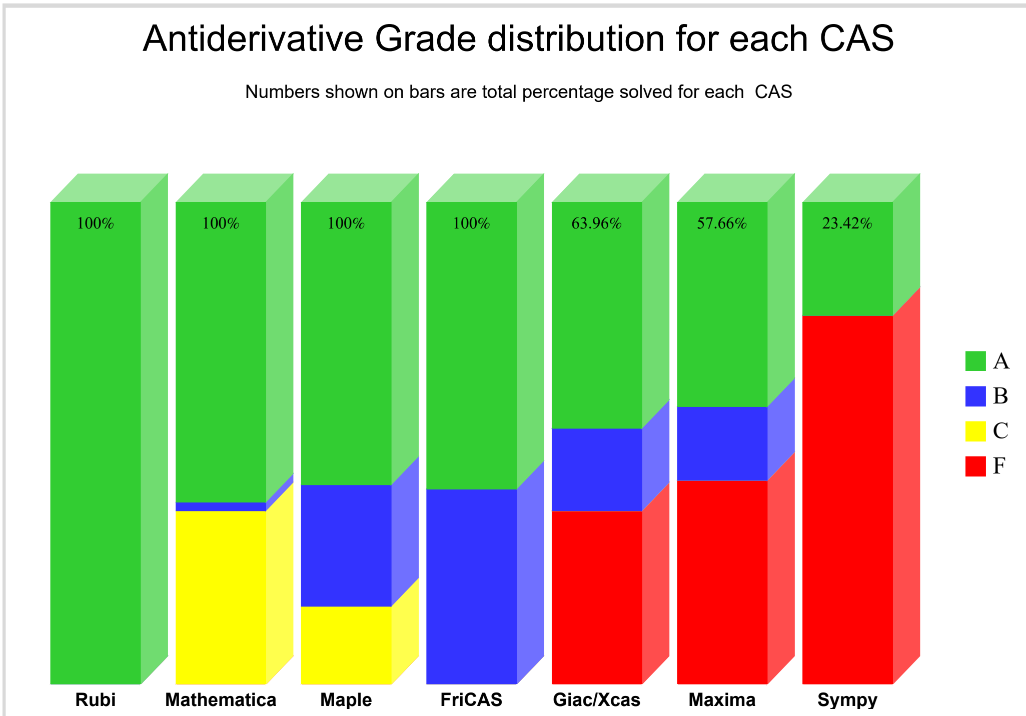
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

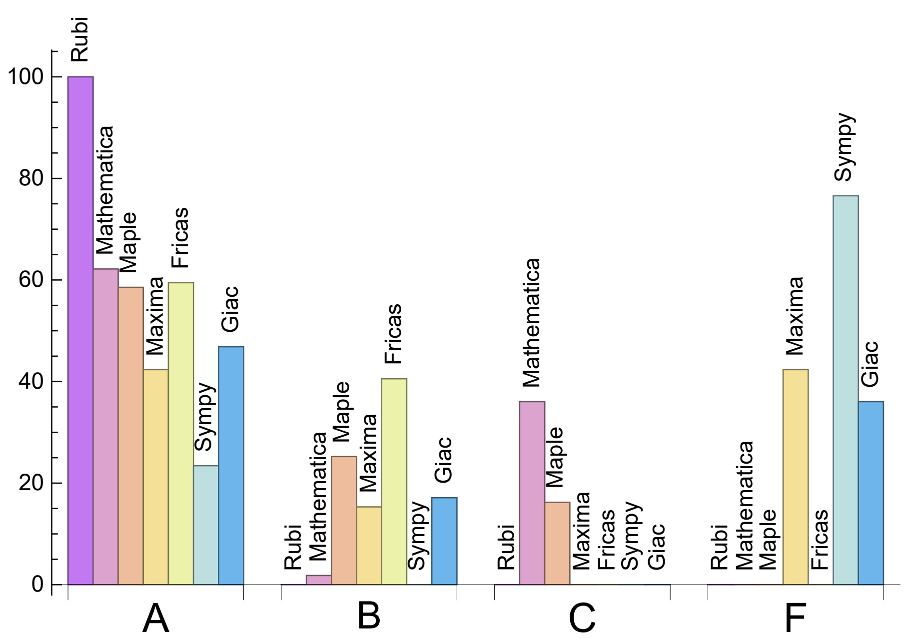
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	62.16	1.8	36.04	0.
Maple	58.56	25.23	16.22	0.
Maxima	42.34	15.32	0.	42.34
Fricas	59.46	40.54	0.	0.
Sympy	23.42	0.	0.	76.58
Giac	46.85	17.12	0.	36.04

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.55	268.55	1.	178.	1.
Mathematica	0.69	289.74	0.97	143.	0.86
Maple	0.06	412.83	1.76	288.	1.69
Maxima	1.24	247.95	2.25	229.5	2.3
Fricas	2.12	1453.68	4.07	439.	3.78
Sympy	4.78	138.96	1.27	127.5	1.23
Giac	1.21	322.94	2.28	244.	2.16

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 78, 104, 106}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

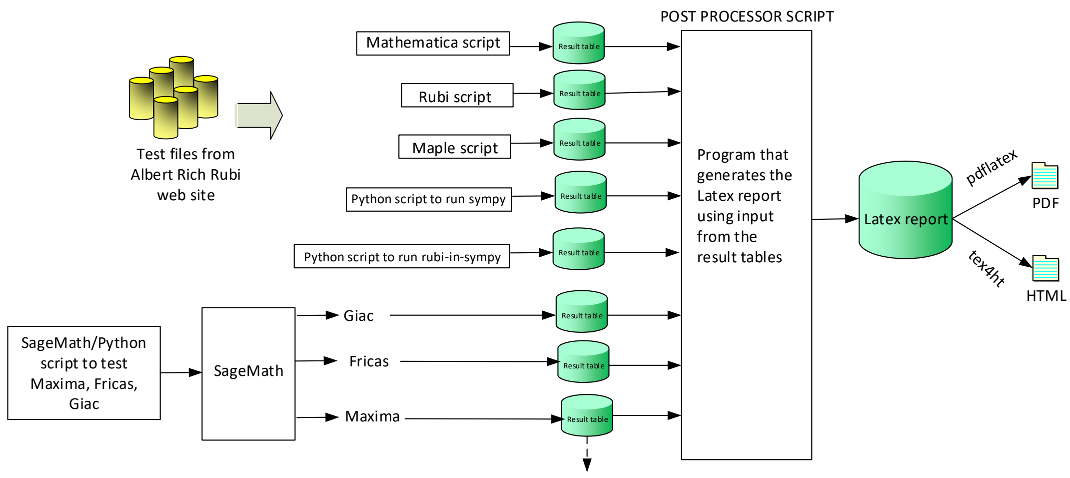
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 37, 38 }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

2.1.3 Maple

A grade: { 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 83, 84, 85, 86, 91, 92, 93 }

B grade: { 1, 2, 10, 11, 12, 33, 34, 35, 36, 38, 39, 40, 41, 42, 49, 50, 51, 52, 72, 78, 79, 80, 81, 82, 87, 88, 89, 90 }

C grade: { 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 22, 24, 25, 26, 27, 28, 29, 30, 31, 36, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93

B grade: { 2, 3, 4, 5, 11, 12, 13, 19, 20, 21, 23, 42, 44, 52, 80, 81, 83 }

C grade: { }

F grade: { 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 62, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 30, 33, 35, 36, 37, 38, 39, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade: { }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 79, 80, 81, 82, 83, 85, 86, 87, 88, 92 }

B grade: { 12, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 84, 89, 90, 91, 93 }

C grade: { }

F grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	82	356	313	193	151	205
normalized size	1	1.	0.66	2.87	2.52	1.56	1.22	1.65
time (sec)	N/A	0.321	0.14	0.011	1.149	1.942	3.639	1.213

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	65	224	265	151	117	157
normalized size	1	1.	0.69	2.38	2.82	1.61	1.24	1.67
time (sec)	N/A	0.217	0.117	0.007	1.124	1.923	2.242	1.164

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	45	122	216	111	82	107
normalized size	1	1.	0.7	1.91	3.38	1.73	1.28	1.67
time (sec)	N/A	0.114	0.088	0.008	1.13	1.919	1.95	1.224

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	92	73	46	62
normalized size	1	1.	0.96	1.89	3.29	2.61	1.64	2.21
time (sec)	N/A	0.021	0.051	0.009	1.139	1.983	0.629	1.203

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	52	131	142	34	63
normalized size	1	1.	1.39	1.86	4.68	5.07	1.21	2.25
time (sec)	N/A	0.149	0.028	0.038	1.201	1.967	4.547	1.182

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	59	77	111	186	0	97
normalized size	1	1.	1.26	1.64	2.36	3.96	0.	2.06
time (sec)	N/A	0.228	0.127	0.039	1.397	1.968	0.	1.151

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	78	139	89	278	0	181
normalized size	1	1.	0.89	1.58	1.01	3.16	0.	2.06
time (sec)	N/A	0.281	0.155	0.047	1.417	1.994	0.	1.152

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	205	105	328	0	269
normalized size	1	1.	0.83	1.55	0.8	2.48	0.	2.04
time (sec)	N/A	0.34	0.272	0.053	1.395	1.98	0.	1.148

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	140	271	111	366	0	359
normalized size	1	1.	0.84	1.63	0.67	2.2	0.	2.16
time (sec)	N/A	0.406	0.318	0.062	1.399	2.021	0.	1.265

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	100	463	444	273	228	319
normalized size	1	1.	0.54	2.52	2.41	1.48	1.24	1.73
time (sec)	N/A	0.346	0.242	0.007	1.184	2.019	3.531	1.177

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	87	283	371	203	172	231
normalized size	1	1.	0.65	2.11	2.77	1.51	1.28	1.72
time (sec)	N/A	0.214	0.188	0.01	1.176	2.008	2.114	1.143

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	56	147	182	140	112	151
normalized size	1	1.	1.14	3.	3.71	2.86	2.29	3.08
time (sec)	N/A	0.048	0.142	0.007	1.206	1.949	1.064	1.215

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	121	236	221	73	153
normalized size	1	1.	0.82	1.95	3.81	3.56	1.18	2.47
time (sec)	N/A	0.185	0.235	0.036	1.34	1.971	5.077	1.263

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	118	184	284	0	161
normalized size	1	1.	0.89	1.69	2.63	4.06	0.	2.3
time (sec)	N/A	0.247	0.23	0.049	1.393	1.994	0.	1.239

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	93	181	170	351	0	244
normalized size	1	1.	0.77	1.5	1.4	2.9	0.	2.02
time (sec)	N/A	0.339	0.373	0.052	1.372	1.95	0.	1.178

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	154	287	158	431	0	385
normalized size	1	1.	0.9	1.67	0.92	2.51	0.	2.24
time (sec)	N/A	0.426	0.448	0.06	1.431	2.016	0.	1.152

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	396	173	510	0	533
normalized size	1	1.	0.83	1.6	0.7	2.06	0.	2.15
time (sec)	N/A	0.521	0.524	0.072	1.398	1.973	0.	1.115

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	159	442	590	481	0	84
normalized size	1	1.	0.73	2.02	2.69	2.2	0.	0.38
time (sec)	N/A	0.484	0.631	0.133	1.448	2.093	0.	1.149

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	118	292	443	392	0	84
normalized size	1	1.	0.79	1.95	2.95	2.61	0.	0.56
time (sec)	N/A	0.329	0.43	0.034	1.276	2.094	0.	1.176

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	89	184	315	327	0	84
normalized size	1	1.	0.89	1.84	3.15	3.27	0.	0.84
time (sec)	N/A	0.262	0.283	0.03	1.378	2.156	0.	1.207

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	64	114	211	251	0	78
normalized size	1	1.	0.94	1.68	3.1	3.69	0.	1.15
time (sec)	N/A	0.176	0.136	0.023	1.333	1.997	0.	1.186

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	81	77	193	0	76
normalized size	1	1.	0.96	1.59	1.51	3.78	0.	1.49
time (sec)	N/A	0.08	0.061	0.019	1.316	2.086	0.	1.171

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	108	209	277	0	101
normalized size	1	1.	0.86	1.48	2.86	3.79	0.	1.38
time (sec)	N/A	0.259	0.141	0.031	1.437	2.066	0.	1.159

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	172	259	400	0	174
normalized size	1	1.	0.89	1.52	2.29	3.54	0.	1.54
time (sec)	N/A	0.369	0.381	0.048	1.456	2.047	0.	1.172

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	178	281	327	591	0	335
normalized size	1	1.	0.94	1.48	1.72	3.11	0.	1.76
time (sec)	N/A	0.485	0.487	0.05	1.626	2.085	0.	1.205

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	173	431	548	740	0	393
normalized size	1	1.	0.75	1.87	2.37	3.2	0.	1.7
time (sec)	N/A	0.536	1.201	0.122	1.516	2.08	0.	1.187

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	156	325	421	663	0	393
normalized size	1	1.	0.86	1.79	2.31	3.64	0.	2.16
time (sec)	N/A	0.423	0.914	0.046	1.343	2.033	0.	1.186

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	115	254	319	570	0	387
normalized size	1	1.	0.78	1.73	2.17	3.88	0.	2.63
time (sec)	N/A	0.375	0.692	0.039	1.348	1.992	0.	1.229

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	97	215	240	414	0	373
normalized size	1	1.	0.78	1.72	1.92	3.31	0.	2.98
time (sec)	N/A	0.297	0.414	0.027	1.363	2.034	0.	1.24

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	132	109	316	0	201
normalized size	1	1.	0.92	1.86	1.54	4.45	0.	2.83
time (sec)	N/A	0.11	0.202	0.025	1.209	2.037	0.	1.211

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	241	254	306	579	0	439
normalized size	1	1.	1.61	1.69	2.04	3.86	0.	2.93
time (sec)	N/A	0.4	1.05	0.05	1.432	2.151	0.	1.211

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	183	312	0	802	0	576
normalized size	1	1.	0.98	1.68	0.	4.31	0.	3.1
time (sec)	N/A	0.501	1.414	0.058	0.	2.11	0.	1.286

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	236	571	0	1142	0	1019
normalized size	1	1.	0.89	2.16	0.	4.33	0.	3.86
time (sec)	N/A	0.618	1.	0.117	0.	2.509	0.	1.161

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	527	0	973	0	1000
normalized size	1	1.	0.63	2.19	0.	4.04	0.	4.15
time (sec)	N/A	0.53	0.928	0.047	0.	2.473	0.	1.181

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	158	435	0	770	0	714
normalized size	1	1.	0.89	2.44	0.	4.33	0.	4.01
time (sec)	N/A	0.37	0.578	0.036	0.	2.236	0.	1.234

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	276	128	518	0	402
normalized size	1	1.	0.85	2.65	1.23	4.98	0.	3.87
time (sec)	N/A	0.146	0.467	0.028	1.19	1.956	0.	1.183

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	614	488	0	1243	0	1130
normalized size	1	1.	2.34	1.86	0.	4.74	0.	4.31
time (sec)	N/A	0.561	4.592	0.055	0.	2.175	0.	1.211

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	710	643	0	1563	0	1358
normalized size	1	1.	2.38	2.16	0.	5.24	0.	4.56
time (sec)	N/A	0.69	1.44	0.073	0.	2.2	0.	1.253

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	627	760	0	1845	0	1578
normalized size	1	1.	1.66	2.02	0.	4.89	0.	4.19
time (sec)	N/A	0.821	1.807	0.082	0.	2.182	0.	1.319

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	92	447	338	212	168	235
normalized size	1	1.	0.66	3.22	2.43	1.53	1.21	1.69
time (sec)	N/A	0.255	0.149	0.009	1.071	2.016	5.485	1.204

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	74	298	289	174	134	186
normalized size	1	1.	0.68	2.73	2.65	1.6	1.23	1.71
time (sec)	N/A	0.187	0.129	0.009	1.03	2.026	2.615	1.198

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	183	288	132	99	136
normalized size	1	1.	0.72	2.32	3.65	1.67	1.25	1.72
time (sec)	N/A	0.121	0.1	0.007	1.086	1.984	1.32	1.178

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	97	116	97	65	95
normalized size	1	1.	0.78	1.9	2.27	1.9	1.27	1.86
time (sec)	N/A	0.067	0.067	0.008	1.019	1.995	0.674	1.153

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	81	165	188	49	103
normalized size	1	1.	1.34	1.98	4.02	4.59	1.2	2.51
time (sec)	N/A	0.098	0.11	0.035	1.169	2.063	3.565	1.176

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	81	108	203	0	108
normalized size	1	1.	1.	1.93	2.57	4.83	0.	2.57
time (sec)	N/A	0.107	0.088	0.046	1.188	2.084	0.	1.222

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	80	110	122	254	0	147
normalized size	1	1.	1.08	1.49	1.65	3.43	0.	1.99
time (sec)	N/A	0.178	0.147	0.044	1.191	2.023	0.	1.16

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	95	172	99	298	0	230
normalized size	1	1.	0.9	1.64	0.94	2.84	0.	2.19
time (sec)	N/A	0.234	0.225	0.059	1.213	1.98	0.	1.183

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	127	238	103	352	0	320
normalized size	1	1.	0.85	1.6	0.69	2.36	0.	2.15
time (sec)	N/A	0.29	0.274	0.066	1.213	2.007	0.	1.199

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	138	738	517	336	286	410
normalized size	1	1.	0.59	3.15	2.21	1.44	1.22	1.75
time (sec)	N/A	0.388	0.321	0.009	1.055	2.059	10.16	1.218

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	113	513	477	273	226	323
normalized size	1	1.	0.61	2.79	2.59	1.48	1.23	1.76
time (sec)	N/A	0.277	0.23	0.008	1.078	2.084	5.259	1.172

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	85	332	255	207	172	243
normalized size	1	1.	0.62	2.44	1.88	1.52	1.26	1.79
time (sec)	N/A	0.181	0.179	0.008	1.039	2.181	2.954	1.175

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	226	317	292	121	300
normalized size	1	1.	0.75	2.05	2.88	2.65	1.1	2.73
time (sec)	N/A	0.189	0.361	0.05	1.187	2.477	5.733	1.201

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	190	242	286	0	266
normalized size	1	1.	1.	2.	2.55	3.01	0.	2.8
time (sec)	N/A	0.177	0.263	0.069	1.204	2.014	0.	1.199

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	97	188	223	359	0	278
normalized size	1	1.	0.85	1.65	1.96	3.15	0.	2.44
time (sec)	N/A	0.225	0.348	0.084	1.347	1.993	0.	1.22

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	222	182	381	0	319
normalized size	1	1.	0.86	1.67	1.37	2.86	0.	2.4
time (sec)	N/A	0.259	0.402	0.092	1.237	2.044	0.	1.158

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	124	291	188	439	0	397
normalized size	1	1.	0.71	1.66	1.07	2.51	0.	2.27
time (sec)	N/A	0.36	0.439	0.116	1.277	1.974	0.	1.175

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	274	369	0	1296	0	0
normalized size	1	1.	1.	1.35	0.	4.75	0.	0.
time (sec)	N/A	0.729	0.419	0.189	0.	2.195	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	210	268	0	1100	0	0
normalized size	1	1.	1.	1.28	0.	5.26	0.	0.
time (sec)	N/A	0.358	0.349	0.06	0.	2.103	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	213	259	0	1087	0	0
normalized size	1	1.	0.94	1.15	0.	4.81	0.	0.
time (sec)	N/A	0.376	0.252	0.045	0.	2.169	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	171	200	0	455	0	0
normalized size	1	1.	0.97	1.13	0.	2.57	0.	0.
time (sec)	N/A	0.254	0.175	0.032	0.	2.092	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	624	0	0
normalized size	1	1.	0.85	1.	0.	2.93	0.	0.
time (sec)	N/A	0.259	0.201	0.021	0.	2.157	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	187	227	0	545	0	0
normalized size	1	1.	0.95	1.15	0.	2.77	0.	0.
time (sec)	N/A	0.372	0.31	0.043	0.	2.203	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	243	288	0	1323	0	0
normalized size	1	1.	0.98	1.16	0.	5.31	0.	0.
time (sec)	N/A	0.496	0.357	0.055	0.	2.499	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	257	330	0	1283	0	0
normalized size	1	1.	0.95	1.22	0.	4.75	0.	0.
time (sec)	N/A	0.504	0.53	0.063	0.	2.51	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	621	532	0	2488	0	0
normalized size	1	1.	1.38	1.18	0.	5.54	0.	0.
time (sec)	N/A	0.859	1.507	0.319	0.	2.313	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	582	495	0	2099	0	0
normalized size	1	1.	1.35	1.15	0.	4.87	0.	0.
time (sec)	N/A	0.701	0.74	0.095	0.	2.153	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	364	491	0	2433	0	0
normalized size	1	1.	0.88	1.18	0.	5.85	0.	0.
time (sec)	N/A	0.611	0.992	0.068	0.	2.196	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	239	291	0	1393	0	0
normalized size	1	1.	1.	1.22	0.	5.83	0.	0.
time (sec)	N/A	0.325	0.514	0.044	0.	2.167	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	590	503	0	2431	0	0
normalized size	1	1.	1.24	1.06	0.	5.11	0.	0.
time (sec)	N/A	0.818	0.688	0.029	0.	2.274	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	363	546	0	2253	0	0
normalized size	1	1.	0.83	1.26	0.	5.18	0.	0.
time (sec)	N/A	0.839	4.804	0.074	0.	2.297	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	500	500	675	595	0	2786	0	0
normalized size	1	1.	1.35	1.19	0.	5.57	0.	0.
time (sec)	N/A	1.253	1.036	0.086	0.	2.337	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	648	820	0	3344	0	0
normalized size	1	1.	1.36	1.72	0.	7.03	0.	0.
time (sec)	N/A	1.062	1.824	0.376	0.	2.408	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	746	746	932	1064	0	4169	0	0
normalized size	1	1.	1.25	1.43	0.	5.59	0.	0.
time (sec)	N/A	1.1	2.509	0.108	0.	2.397	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	637	743	0	3306	0	0
normalized size	1	1.	1.24	1.45	0.	6.46	0.	0.
time (sec)	N/A	0.831	1.712	0.069	0.	2.358	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	856	856	933	1064	0	4342	0	0
normalized size	1	1.	1.09	1.24	0.	5.07	0.	0.
time (sec)	N/A	1.255	2.384	0.037	0.	2.333	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	1558	1090	0	4301	0	0
normalized size	1	1.	2.13	1.49	0.	5.89	0.	0.
time (sec)	N/A	1.682	3.262	0.105	0.	2.152	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	874	874	1359	1178	0	4894	0	0
normalized size	1	1.	1.55	1.35	0.	5.6	0.	0.
time (sec)	N/A	2.689	3.378	0.128	0.	2.172	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	791	791	998	1294	0	4891	0	0
normalized size	1	1.	1.26	1.64	0.	6.18	0.	0.
time (sec)	N/A	1.868	3.717	0.152	0.	2.207	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	100	551	362	240	185	259
normalized size	1	1.	0.65	3.58	2.35	1.56	1.2	1.68
time (sec)	N/A	0.297	0.161	0.008	1.046	1.764	9.675	1.14

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	84	389	360	200	151	211
normalized size	1	1.	0.68	3.14	2.9	1.61	1.22	1.7
time (sec)	N/A	0.228	0.133	0.007	1.042	1.733	6.047	1.189

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	66	257	265	155	116	161
normalized size	1	1.	0.7	2.73	2.82	1.65	1.23	1.71
time (sec)	N/A	0.159	0.11	0.007	1.042	1.719	3.262	1.368

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	158	140	119	82	119
normalized size	1	1.	0.74	2.39	2.12	1.8	1.24	1.8
time (sec)	N/A	0.103	0.081	0.007	1.029	1.813	1.408	1.224

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	49	113	189	211	66	147
normalized size	1	1.	0.88	2.02	3.38	3.77	1.18	2.62
time (sec)	N/A	0.128	0.164	0.032	1.231	1.7	4.863	1.273

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	110	138	223	0	150
normalized size	1	1.	1.	2.	2.51	4.05	0.	2.73
time (sec)	N/A	0.129	0.131	0.054	1.159	1.822	0.	1.32

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	114	117	238	0	159
normalized size	1	1.	1.25	1.65	1.7	3.45	0.	2.3
time (sec)	N/A	0.139	0.141	0.056	1.177	1.833	0.	1.256

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	73	143	128	277	0	190
normalized size	1	1.	0.8	1.57	1.41	3.04	0.	2.09
time (sec)	N/A	0.215	0.238	0.066	1.23	1.728	0.	1.272

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	139	818	517	358	284	409
normalized size	1	1.	0.59	3.5	2.21	1.53	1.21	1.75
time (sec)	N/A	0.395	0.291	0.008	1.071	1.759	15.388	1.233

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	111	592	328	285	226	329
normalized size	1	1.	0.6	3.18	1.76	1.53	1.22	1.77
time (sec)	N/A	0.287	0.22	0.009	1.046	1.741	9.363	1.381

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	335	390	365	168	447
normalized size	1	1.	0.68	2.09	2.44	2.28	1.05	2.79
time (sec)	N/A	0.282	0.468	0.086	1.233	1.786	11.71	1.213

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	143	296	317	358	0	416
normalized size	1	1.	1.	2.07	2.22	2.5	0.	2.91
time (sec)	N/A	0.252	0.348	0.102	1.197	1.775	0.	1.261

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	136	265	274	351	0	378
normalized size	1	1.	0.96	1.88	1.94	2.49	0.	2.68
time (sec)	N/A	0.233	0.348	0.115	1.2	1.752	0.	1.336

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	135	261	254	412	0	377
normalized size	1	1.	0.9	1.74	1.69	2.75	0.	2.51
time (sec)	N/A	0.279	0.57	0.128	1.251	1.76	0.	1.335

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	150	292	208	435	0	425
normalized size	1	1.	0.9	1.75	1.25	2.6	0.	2.54
time (sec)	N/A	0.308	0.539	0.174	1.247	1.804	0.	1.333

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	213	925	0	2627	0	0
normalized size	1	1.	0.57	2.48	0.	7.04	0.	0.
time (sec)	N/A	0.937	0.462	0.087	0.	2.136	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	198	671	0	2591	0	0
normalized size	1	1.	0.55	1.87	0.	7.24	0.	0.
time (sec)	N/A	0.648	0.26	0.036	0.	2.203	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	170	423	0	1322	0	0
normalized size	1	1.	0.6	1.49	0.	4.67	0.	0.
time (sec)	N/A	0.455	0.225	0.032	0.	1.952	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	180	280	0	1773	0	0
normalized size	1	1.	0.52	0.81	0.	5.14	0.	0.
time (sec)	N/A	0.412	0.211	0.027	0.	2.138	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	180	143	0	1770	0	0
normalized size	1	1.	0.52	0.41	0.	5.13	0.	0.
time (sec)	N/A	0.401	0.136	0.022	0.	2.042	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	186	138	0	1412	0	0
normalized size	1	1.	0.61	0.46	0.	4.66	0.	0.
time (sec)	N/A	0.523	0.268	0.036	0.	2.044	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	215	187	0	2921	0	0
normalized size	1	1.	0.56	0.49	0.	7.67	0.	0.
time (sec)	N/A	0.6	0.407	0.047	0.	2.189	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	237	240	0	3071	0	0
normalized size	1	1.	0.58	0.59	0.	7.49	0.	0.
time (sec)	N/A	0.639	0.377	0.054	0.	2.13	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	363	877	0	5017	0	0
normalized size	1	1.	0.51	1.22	0.	6.99	0.	0.
time (sec)	N/A	1.081	0.336	0.15	0.	2.408	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	203	594	0	3200	0	0
normalized size	1	1.	0.54	1.59	0.	8.58	0.	0.
time (sec)	N/A	0.601	0.16	0.044	0.	2.22	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	695	695	387	395	0	4891	0	0
normalized size	1	1.	0.56	0.57	0.	7.04	0.	0.
time (sec)	N/A	1.324	0.188	0.037	0.	2.395	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	739	387	226	0	5042	0	0
normalized size	1	1.	0.52	0.31	0.	6.82	0.	0.
time (sec)	N/A	1.323	0.194	0.026	0.	2.35	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	697	697	5530	338	0	4420	0	0
normalized size	1	1.	7.93	0.48	0.	6.34	0.	0.
time (sec)	N/A	1.472	8.611	0.072	0.	2.327	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	784	784	397	2448	0	6886	0	0
normalized size	1	1.	0.51	3.12	0.	8.78	0.	0.
time (sec)	N/A	1.626	0.595	0.52	0.	3.099	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1105	1105	675	1927	0	10106	0	0
normalized size	1	1.	0.61	1.74	0.	9.15	0.	0.
time (sec)	N/A	1.853	0.524	0.143	0.	3.476	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	776	776	429	1456	0	6494	0	0
normalized size	1	1.	0.55	1.88	0.	8.37	0.	0.
time (sec)	N/A	2.659	0.636	0.112	0.	2.639	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	781	781	423	994	0	6869	0	0
normalized size	1	1.	0.54	1.27	0.	8.8	0.	0.
time (sec)	N/A	1.449	0.422	0.086	0.	2.537	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1147	1147	669	810	0	10103	0	0
normalized size	1	1.	0.58	0.71	0.	8.81	0.	0.
time (sec)	N/A	3.198	0.458	0.063	0.	3.085	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [111] had the largest ratio of [0.5294]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.	15	0.267
2	A	9	4	1.	15	0.267
3	A	7	4	1.	13	0.308
4	A	2	2	1.	12	0.167
5	A	6	5	1.	15	0.333
6	A	9	5	1.	15	0.333
7	A	11	5	1.	15	0.333
8	A	13	5	1.	15	0.333
9	A	15	5	1.	15	0.333
10	A	14	4	1.	17	0.235
11	A	11	4	1.	15	0.267
12	A	3	2	1.	14	0.143
13	A	8	7	1.	17	0.412
14	A	10	6	1.	17	0.353
15	A	14	5	1.	17	0.294
16	A	17	5	1.	17	0.294
17	A	20	5	1.	17	0.294
18	A	15	7	1.	17	0.412
19	A	11	7	1.	17	0.412
20	A	8	7	1.	17	0.412
21	A	6	5	1.	15	0.333
22	A	3	3	1.	14	0.214
23	A	8	4	1.	17	0.235
24	A	12	5	1.	17	0.294
25	A	17	5	1.	17	0.294
26	A	15	8	1.	17	0.471
27	A	12	8	1.	17	0.471
28	A	10	6	1.	17	0.353
29	A	9	5	1.	15	0.333
30	A	4	4	1.	14	0.286
31	A	12	5	1.	17	0.294
32	A	16	5	1.	17	0.294
33	A	15	6	1.	17	0.353
34	A	14	5	1.	17	0.294
35	A	11	5	1.	15	0.333
36	A	5	4	1.	14	0.286
37	A	17	5	1.	17	0.294
38	A	21	5	1.	17	0.294
39	A	26	5	1.	17	0.294
40	A	12	3	1.	17	0.176
41	A	10	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	8	3	1.	15	0.2
43	A	6	3	1.	14	0.214
44	A	7	6	1.	17	0.353
45	A	7	6	1.	17	0.353
46	A	10	5	1.	17	0.294
47	A	12	5	1.	17	0.294
48	A	14	5	1.	17	0.294
49	A	17	3	1.	19	0.158
50	A	14	3	1.	17	0.176
51	A	11	3	1.	16	0.188
52	A	11	6	1.	19	0.316
53	A	10	7	1.	19	0.368
54	A	12	7	1.	19	0.368
55	A	13	6	1.	19	0.316
56	A	17	5	1.	19	0.263
57	A	14	7	1.	19	0.368
58	A	12	6	1.	19	0.316
59	A	11	6	1.	19	0.316
60	A	8	4	1.	17	0.235
61	A	8	4	1.	16	0.25
62	A	13	4	1.	19	0.21
63	A	14	6	1.	19	0.316
64	A	18	5	1.	19	0.263
65	A	24	9	1.	19	0.474
66	A	20	8	1.	19	0.421
67	A	17	6	1.	19	0.316
68	A	9	5	1.	17	0.294
69	A	18	5	1.	16	0.312
70	A	22	6	1.	19	0.316
71	A	32	6	1.	19	0.316
72	A	27	8	1.	19	0.421
73	A	28	7	1.	19	0.368
74	A	19	6	1.	17	0.353
75	A	28	5	1.	16	0.312
76	A	41	7	1.	19	0.368
77	A	60	6	1.	19	0.316
78	A	46	7	1.	19	0.368
79	A	13	4	1.	17	0.235
80	A	11	4	1.	17	0.235
81	A	9	4	1.	15	0.267
82	A	7	4	1.	14	0.286
83	A	8	6	1.	17	0.353
84	A	8	7	1.	17	0.412
85	A	8	6	1.	17	0.353

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	5	1.	17	0.294
87	A	17	4	1.	17	0.235
88	A	14	4	1.	16	0.25
89	A	14	7	1.	19	0.368
90	A	13	8	1.	19	0.421
91	A	12	8	1.	19	0.421
92	A	14	7	1.	19	0.368
93	A	15	7	1.	19	0.368
94	A	15	6	1.	19	0.316
95	A	14	6	1.	19	0.316
96	A	11	4	1.	19	0.21
97	A	11	4	1.	17	0.235
98	A	11	4	1.	16	0.25
99	A	16	4	1.	19	0.21
100	A	17	5	1.	19	0.263
101	A	18	6	1.	19	0.316
102	A	23	6	1.	19	0.316
103	A	12	5	1.	19	0.263
104	A	34	7	1.	17	0.412
105	A	36	8	1.	16	0.5
106	A	41	8	1.	19	0.421
107	A	36	8	1.	19	0.421
108	A	47	9	1.	19	0.474
109	A	71	10	1.	19	0.526
110	A	37	9	1.	19	0.474
111	A	89	9	1.	17	0.529

Chapter 3

Listing of integrals

3.1 $\int x^3(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=124

$$-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{ax^3\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+}{d^2}$$

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d
*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*x*
Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d
 + (b*x^4*Sinh[c + d*x])/d
```

Rubi [A] time = 0.320678, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6742, 3296, 2638, 2637}

$$-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{ax^3\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d
*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*x*
Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d
 + (b*x^4*Sinh[c + d*x])/d
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)\cosh(c+dx)\,dx &= \int \left(ax^3\cosh(c+dx)+bx^4\cosh(c+dx)\right)dx\\ &= a\int x^3\cosh(c+dx)\,dx+b\int x^4\cosh(c+dx)\,dx\\ &= \frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}-\frac{(3a)\int x^2\sinh(c+dx)\,dx}{d}-\frac{(4b)\int x^3\sinh(c+dx}{d}\\ &= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}+\frac{(6a)}{}\\ &= -\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{ax}{}\\ &= -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{6ax}{}\\ &= -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b}{}\end{aligned}$$

Mathematica [A] time = 0.139607, size = 82, normalized size = 0.66

$$\frac{\left(ad^2x\left(d^2x^2+6\right)+b\left(d^4x^4+12d^2x^2+24\right)\right)\sinh(c+dx)-d\left(3a\left(d^2x^2+2\right)+4bx\left(d^2x^2+6\right)\right)\cosh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(3*a*(2 + d^2*x^2) + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^2*x*(6
 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [B] time = 0.011, size = 356, normalized size = 2.9

$$\frac{1}{d^4}\left(\frac{b\left((dx+c)^4\sinh(dx+c)-4\,(dx+c)^3\cosh(dx+c)+12\,(dx+c)^2\sinh(dx+c)-24\,(dx+c)\cosh(dx+c)+24\,\mathrm{si}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^4*(b/d*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh
(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-4*b*c/d*((d*x+c)^3*sinh(d*x+
c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+6*b/d*c^2*(
(d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4*b/d*c^3*((d*x+
c)*sinh(d*x+c)-cosh(d*x+c))+b*c^4/d*sinh(d*x+c)+a*((d*x+c)^3*sinh(d*x+c)-3*
(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-3*a*c*((d*x+c)^2
*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+3*a*c^2*((d*x+c)*sinh(d*x
```

```
+c)-cosh(d*x+c))-a*c^3*sinh(d*x+c))
```

Maxima [A] time = 1.14939, size = 313, normalized size = 2.52

$$-\frac{1}{40}d\left(\frac{5\left(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c\right)ae^{(dx)}}{d^5}+\frac{5\left(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24\right)ae^{(-dx-c)}}{d^5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/40*d*(5*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 5*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 4*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*
x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 4*(d^5*x^5 + 5*d^4*x^4 + 2
0*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) + 1/20*(4*b*x^5
 + 5*a*x^4)*cosh(d*x + c)
```

Fricas [A] time = 1.94238, size = 193, normalized size = 1.56

$$-\frac{\left(4\,bd^3x^3+3\,ad^3x^2+24\,bdx+6\,ad\right)\cosh\left(dx+c\right)-\left(bd^4x^4+ad^4x^3+12\,bd^2x^2+6\,ad^2x+24\,b\right)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((4*b*d^3*x^3 + 3*a*d^3*x^2 + 24*b*d*x + 6*a*d)*cosh(d*x + c) - (b*d^4*x^4
 + a*d^4*x^3 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] time = 3.63855, size = 151, normalized size = 1.22

$$\begin{cases}\frac{ax^3\sinh(c+dx)}{d}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{bx^4\sinh(c+dx)}{d}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{24bx\cosh}{d^4}\\ \left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)\cosh(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**4*sinh(c + d*x)/d - 4*b*x**
3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/
d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*cosh(c),
True))
```

Giac [A] time = 1.21328, size = 205, normalized size = 1.65

$$\frac{\left(bd^4x^4+ad^4x^3-4\,bd^3x^3-3\,ad^3x^2+12\,bd^2x^2+6\,ad^2x-24\,bdx-6\,ad+24\,b\right)e^{(dx+c)}}{2\,d^5}-\frac{\left(bd^4x^4+ad^4x^3+4\,bd^3x^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^3 - 4*b*d^3*x^3 - 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a
*d^2*x - 24*b*d*x - 6*a*d + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*
x^3 + 4*b*d^3*x^3 + 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b*d*x + 6*a
*d + 24*b)*e^(-d*x - c)/d^5
```

3.2 $\int x^2(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4}+$$

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*
x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Sinh
[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d
```

Rubi [A] time = 0.216928, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6742, 3296, 2637, 2638}

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*
x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Sinh
[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \int \left(ax^2\cosh(c+dx)+bx^3\cosh(c+dx)\right)dx$$
$$= a\int x^2\cosh(c+dx)\,dx + b\int x^3\cosh(c+dx)\,dx$$
$$= \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{(2a)\int x\sinh(c+dx)\,dx}{d} - \frac{(3b)\int x^2\sinh(c+dx)}{d}$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} + \frac{(2a)\int}{}$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{ax^2\sin}{}$$
$$= -\frac{6b\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{6bx\sin}{}$$

Mathematica [A] time = 0.117328, size = 65, normalized size = 0.69

$$\frac{d\left(a\left(d^2x^2+2\right)+bx\left(d^2x^2+6\right)\right)\sinh(c+dx)-\left(2ad^2x+3b\left(d^2x^2+2\right)\right)\cosh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-((2*a*d^2*x + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a*(2 + d^2*x^2) + b*
x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Maple [B] time = 0.007, size = 224, normalized size = 2.4

$$\frac{1}{d^3}\left(\frac{b\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d}-3\frac{cb\left((dx+c)^2 \mathrm{s}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^3*(b/d*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*
x+c)-6*cosh(d*x+c))-3*b*c/d*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*
sinh(d*x+c))+3*b/d*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-b*c^3/d*sinh(d*x+c
)+a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*a*c*((d*x
+c)*sinh(d*x+c)-cosh(d*x+c))+a*c^2*sinh(d*x+c))
```

Maxima [B] time = 1.1237, size = 265, normalized size = 2.82

$$-\frac{1}{24}d\left(\frac{4\left(d^3x^3e^c-3d^2x^2e^c+6dxe^c-6e^c\right)ae^{(dx)}}{d^4}+\frac{4\left(d^3x^3+3d^2x^2+6dx+6\right)ae^{(-dx-c)}}{d^4}+\frac{3\left(d^4x^4e^c-4d^3x^3e^c+12d^2x^2\right.}{d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(4*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
+ 4*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^4*x^4*e^c -
```

```
4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b*e^(d*x)/d^5 + 3*(d
^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b*e^(-d*x - c)/d^5) + 1/12*(
3*b*x^4 + 4*a*x^3)*cosh(d*x + c)
```

Fricas [A] time = 1.92279, size = 151, normalized size = 1.61

$$-\frac{\left(3\,bd^2x^2+2\,ad^2x+6\,b\right)\cosh\left(dx+c\right)-\left(bd^3x^3+ad^3x^2+6\,bdx+2\,ad\right)\sinh\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b*d^2*x^2 + 2*a*d^2*x + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3*x^2 +
6*b*d*x + 2*a*d)*sinh(d*x + c))/d^4
```

Sympy [A] time = 2.24197, size = 117, normalized size = 1.24

$$\begin{cases} \frac{ax^2\sinh(c+dx)}{d}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{bx^3\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*s
inh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x**4/
4)*cosh(c), True))
```

Giac [A] time = 1.16419, size = 157, normalized size = 1.67

$$\frac{\left(bd^3x^3+ad^3x^2-3\,bd^2x^2-2\,ad^2x+6\,bdx+2\,ad-6\,b\right)e^{(dx+c)}}{2\,d^4}-\frac{\left(bd^3x^3+ad^3x^2+3\,bd^2x^2+2\,ad^2x+6\,bdx+2\,ad+6\right.}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 + a*d^3*x^2 - 3*b*d^2*x^2 - 2*a*d^2*x + 6*b*d*x + 2*a*d - 6*
b)*e^(d*x + c)/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x^2 + 3*b*d^2*x^2 + 2*a*d^2*x +
 6*b*d*x + 2*a*d + 6*b)*e^(-d*x - c)/d^4
```

3.3 $\int x(a+bx)\cosh(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] -((a*Cosh[c + d*x])/d^2) - (2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/
d^3 + (a*x*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d
```

Rubi [A] time = 0.113946, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 3296, 2638, 2637}

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d^2) - (2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/
d^3 + (a*x*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x(a+bx)\cosh(c+dx)\,dx &= \int \left(ax\cosh(c+dx)+bx^2\cosh(c+dx)\right)dx\\
&= a\int x\cosh(c+dx)\,dx+b\int x^2\cosh(c+dx)\,dx\\
&= \frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}-\frac{a\int \sinh(c+dx)\,dx}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}+\frac{(2b)\int \cosh}{d}\\
&= -\frac{a\cosh(c+dx)}{d^2}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+}{d}
\end{aligned}$$

Mathematica [A] time = 0.0879951, size = 45, normalized size = 0.7

$$\frac{\left(ad^2x+b\left(d^2x^2+2\right)\right)\sinh(c+dx)-d(a+2bx)\cosh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(a + 2*b*x)*Cosh[c + d*x]) + (a*d^2*x + b*(2 + d^2*x^2))*Sinh[c + d*x]
)/d^3
```

Maple [A] time = 0.008, size = 122, normalized size = 1.9

$$\frac{1}{d^2}\left(\frac{b\left((dx+c)^2\sinh(dx+c)-2\,(dx+c)\cosh(dx+c)+2\,\sinh(dx+c)\right)}{d}-2\,\frac{cb\left((dx+c)\sinh(dx+c)-\cosh(dx+\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^2*(b/d*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*b*
c/d*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+b
*c^2/d*sinh(d*x+c)-c*a*sinh(d*x+c))
```

Maxima [B] time = 1.13037, size = 216, normalized size = 3.38

$$-\frac{1}{12}\,d\left(\frac{3\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)ae^{(dx)}}{d^3}+\frac{3\left(d^2x^2+2\,dx+2\right)ae^{(-dx-c)}}{d^3}+\frac{2\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)be^{(dx)}}{d^4}+\frac{2\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/12*d*(3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 3*(d^2*x^2 + 2
*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c -
 6*e^c)*b*e^(d*x)/d^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^(-d*x - c)/
d^4) + 1/6*(2*b*x^3 + 3*a*x^2)*cosh(d*x + c)
```

Fricas [A] time = 1.9195, size = 111, normalized size = 1.73

$$-\frac{(2\,bdx+ad)\cosh(dx+c)-\left(bd^2x^2+ad^2x+2\,b\right)\sinh(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((2*b*d*x + a*d)*cosh(d*x + c) - (b*d^2*x^2 + a*d^2*x + 2*b)*sinh(d*x + c)
)/d^3
```

Sympy [A] time = 1.95005, size = 82, normalized size = 1.28

$$\begin{cases} \frac{ax\sinh(c+dx)}{d} - \frac{a\cosh(c+dx)}{d^2} + \frac{bx^2\sinh(c+dx)}{d} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**2*sinh(c + d*x
)/d - 2*b*x*cosh(c + d*x)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x**
2/2 + b*x**3/3)*cosh(c), True))
```

Giac [A] time = 1.22437, size = 107, normalized size = 1.67

$$\frac{\left(bd^2x^2 + ad^2x - 2\,bdx - ad + 2\,b\right)e^{(dx+c)}}{2\,d^3} - \frac{\left(bd^2x^2 + ad^2x + 2\,bdx + ad + 2\,b\right)e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + a*d^2*x - 2*b*d*x - a*d + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^
2*x^2 + a*d^2*x + 2*b*d*x + a*d + 2*b)*e^(-d*x - c)/d^3
```

3.4 $\int (a+bx)\cosh (c+dx)\,dx$

Optimal. Leaf size=28

$$\frac{(a+bx)\sinh (c+dx)}{d}-\frac{b\cosh (c+dx)}{d^2}$$

```
[Out] -((b*Cosh[c + d*x])/d^2) + ((a + b*x)*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0205861, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3296, 2638}

$$\frac{(a+bx)\sinh (c+dx)}{d}-\frac{b\cosh (c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Cosh[c + d*x],x]

[Out] -((b*Cosh[c + d*x])/d^2) + ((a + b*x)*Sinh[c + d*x])/d
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\cosh (c+dx)\,dx &= \frac{(a+bx)\sinh (c+dx)}{d}-\frac{b\int \sinh (c+dx)\,dx}{d}\\ &= -\frac{b\cosh (c+dx)}{d^2}+\frac{(a+bx)\sinh (c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0513288, size = 27, normalized size = 0.96

$$\frac{d(a+bx)\sinh (c+dx)-b\cosh (c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Cosh[c + d*x],x]

[Out] (-(b*Cosh[c + d*x]) + d*(a + b*x)*Sinh[c + d*x])/d^2
```

Maple [A] time = 0.009, size = 53, normalized size = 1.9

$$\frac{1}{d}\left(\frac{b\left((dx+c)\sinh\left(dx+c\right)-\cosh\left(dx+c\right)\right)}{d}-\frac{cb\sinh\left(dx+c\right)}{d}+a\sinh\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c),x)
```

```
[Out] 1/d*(b/d*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-b*c/d*sinh(d*x+c)+a*sinh(d*x+c))
```

Maxima [B] time = 1.13935, size = 92, normalized size = 3.29

$$\frac{ae^{(dx+c)}}{2\,d}+\frac{(dxe^{c}-e^{c})be^{(dx)}}{2\,d^{2}}-\frac{(dx+1)be^{(-dx-c)}}{2\,d^{2}}-\frac{ae^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(d*x + c)/d + 1/2*(d*x*e^c - e^c)*b*e^(d*x)/d^2 - 1/2*(d*x + 1)*b*e
^(-d*x - c)/d^2 - 1/2*a*e^(-d*x - c)/d
```

Fricas [A] time = 1.98283, size = 73, normalized size = 2.61

$$-\frac{b\cosh\left(dx+c\right)-(bdx+ad)\sinh\left(dx+c\right)}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*cosh(d*x + c) - (b*d*x + a*d)*sinh(d*x + c))/d^2
```

Sympy [A] time = 0.629005, size = 46, normalized size = 1.64

$$\begin{cases}\frac{a\sinh\left(c+dx\right)}{d}+\frac{bx\sinh\left(c+dx\right)}{d}-\frac{b\cosh\left(c+dx\right)}{d^{2}} & \text{for}\ d\neq 0\\ \left(ax+\frac{bx^{2}}{2}\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x*sinh(c + d*x)/d - b*cosh(c + d*x)/d**2,
Ne(d, 0)), ((a*x + b*x**2/2)*cosh(c), True))
```

Giac [A] time = 1.20289, size = 62, normalized size = 2.21

$$\frac{(bdx+ad-b)e^{(dx+c)}}{2\,d^{2}}-\frac{(bdx+ad+b)e^{(-dx-c)}}{2\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + a*d - b)*e^(d*x + c)/d^2 - 1/2*(b*d*x + a*d + b)*e^(-d*x - c)/
d^2
```

3.5 $\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx$

Optimal. Leaf size=28

$$a\cosh(c)\mathbf{Chi}(dx) + a\sinh(c)\mathbf{Shi}(dx) + \frac{b\sinh(c+dx)}{d}$$

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[
d*x]
```

Rubi [A] time = 0.148764, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2637, 3303, 3298, 3301}

$$a\cosh(c)\mathbf{Chi}(dx) + a\sinh(c)\mathbf{Shi}(dx) + \frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[
d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x}\,dx + b\int \cosh(c+dx)\,dx\\ &= \frac{b\sinh(c+dx)}{d} + (a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + (a\sinh(c))\int \frac{\sinh(dx)}{x}\,dx\\ &= a\cosh(c)\mathbf{Chi}(dx) + \frac{b\sinh(c+dx)}{d} + a\sinh(c)\mathbf{Shi}(dx)\end{aligned}$$

Mathematica [A] time = 0.0279314, size = 39, normalized size = 1.39

$$a\cosh(c)\mathbf{Chi}(dx) + a\sinh(c)\mathbf{Shi}(dx) + \frac{b\sinh(c)\cosh(dx)}{d} + \frac{b\cosh(c)\sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x
])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.038, size = 52, normalized size = 1.9

$$-\frac{a\mathrm{e}^{-c}\mathrm{Ei}\,(1,dx)}{2} - \frac{b\mathrm{e}^{-dx-c}}{2d} - \frac{a\mathrm{e}^{c}\mathrm{Ei}\,(1,-dx)}{2} + \frac{b\mathrm{e}^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x,x)
```

```
[Out] -1/2*a*exp(-c)*Ei(1,d*x)-1/2*b/d*exp(-d*x-c)-1/2*a*exp(c)*Ei(1,-d*x)+1/2*b/
d*exp(d*x+c)
```

Maxima [B] time = 1.20109, size = 131, normalized size = 4.68

$$-\frac{1}{2}\left(b\left(\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + \frac{2\,a\cosh(dx+c)\log(x)}{d} - \frac{\left(\mathrm{Ei}\,(-dx)\,e^{(-c)} + \mathrm{Ei}\,(dx)\,e^{c}\right)a}{d}\right)d + (bx + a\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + 2*a*co
sh(d*x + c)*log(x)/d - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + (b*x + a*lo
g(x))*cosh(d*x + c)
```

Fricas [A] time = 1.96747, size = 142, normalized size = 5.07

$$\frac{(ad\mathrm{Ei}\,(dx) + ad\mathrm{Ei}\,(-dx))\cosh(c) + 2\,b\sinh(dx+c) + (ad\mathrm{Ei}\,(dx) - ad\mathrm{Ei}\,(-dx))\sinh(c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*((a*d*Ei(d*x) + a*d*Ei(-d*x))*cosh(c) + 2*b*sinh(d*x + c) + (a*d*Ei(d*x
) - a*d*Ei(-d*x))*sinh(c))/d
```

Sympy [A] time = 4.54689, size = 34, normalized size = 1.21

$$a\sinh(c)\operatorname{Shi}(dx) + a\cosh(c)\operatorname{Chi}(dx) + b\left(\begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x\cosh(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((sinh(c + d*x)/d, Ne(
d, 0)), (x*cosh(c), True))
```

Giac [A] time = 1.1822, size = 63, normalized size = 2.25

$$\frac{ad\operatorname{Ei}(-dx)\,e^{(-c)} + ad\operatorname{Ei}(dx)\,e^{c} + be^{(dx+c)} - be^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a*d*Ei(-d*x)*e^(-c) + a*d*Ei(d*x)*e^c + b*e^(d*x + c) - b*e^(-d*x - c)
)/d
```

3.6 $\int \frac{(a+bx)\cosh(c+dx)}{x^2} dx$

Optimal. Leaf size=47

$$ad\sinh(c)\mathbf{Chi}(dx) + ad\cosh(c)\mathbf{Shi}(dx) - \frac{a\cosh(c+dx)}{x} + b\cosh(c)\mathbf{Chi}(dx) + b\sinh(c)\mathbf{Shi}(dx)$$

```
[Out] -((a*Cosh[c + d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] + a*d*CoshIntegral[d*x
]*Sinh[c] + a*d*Cosh[c]*SinhIntegral[d*x] + b*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.227998, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3298, 3301}

$$ad\sinh(c)\mathbf{Chi}(dx) + ad\cosh(c)\mathbf{Shi}(dx) - \frac{a\cosh(c+dx)}{x} + b\cosh(c)\mathbf{Chi}(dx) + b\sinh(c)\mathbf{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] + a*d*CoshIntegral[d*x
]*Sinh[c] + a*d*Cosh[c]*SinhIntegral[d*x] + b*Sinh[c]*SinhIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}dx = \int \left(\frac{a\cosh(c+dx)}{x^2} + \frac{b\cosh(c+dx)}{x}\right)dx$$
$$= a\int \frac{\cosh(c+dx)}{x^2}dx + b\int \frac{\cosh(c+dx)}{x}dx$$
$$= -\frac{a\cosh(c+dx)}{x} + (ad)\int \frac{\sinh(c+dx)}{x}dx + (b\cosh(c))\int \frac{\cosh(dx)}{x}dx + (b\sinh(c))\int \frac{\text{s}}{}$$
$$= -\frac{a\cosh(c+dx)}{x} + b\cosh(c)\mathbf{Chi}(dx) + b\sinh(c)\mathbf{Shi}(dx) + (ad\cosh(c))\int \frac{\sinh(dx)}{x}dx + ($$
$$= -\frac{a\cosh(c+dx)}{x} + b\cosh(c)\mathbf{Chi}(dx) + ad\mathbf{Chi}(dx)\sinh(c) + ad\cosh(c)\mathbf{Shi}(dx) + b\sinh(c$$

Mathematica [A] time = 0.127054, size = 59, normalized size = 1.26

$$ad(\sinh(c)\mathbf{Chi}(dx) + \cosh(c)\mathbf{Shi}(dx)) - \frac{a\sinh(c)\sinh(dx)}{x} - \frac{a\cosh(c)\cosh(dx)}{x} + b\cosh(c)\mathbf{Chi}(dx) + b\sinh(c)\mathbf{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c]*Cosh[d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] - (a*Sinh[c]*Sinh[
d*x])/x + b*Sinh[c]*SinhIntegral[d*x] + a*d*(CoshIntegral[d*x]*Sinh[c] + Co
sh[c]*SinhIntegral[d*x])
```

Maple [A] time = 0.039, size = 77, normalized size = 1.6

$$-\frac{a\mathrm{e}^{-dx-c}}{2x} + \frac{da\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{2} - \frac{b\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{2} - \frac{a\mathrm{e}^{dx+c}}{2x} - \frac{da\mathrm{e}^{c}\mathrm{Ei}(1,-dx)}{2} - \frac{b\mathrm{e}^{c}\mathrm{Ei}(1,-dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^2,x)
```

```
[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2*b*exp(-c)*Ei(1,d*x)-1/2*
a/x*exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)-1/2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.39651, size = 111, normalized size = 2.36

$$-\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)e^{(-c)} - \mathrm{Ei}(dx)e^{c}\right)a + \frac{2b\cosh(dx+c)\log(x)}{d} - \frac{\left(\mathrm{Ei}(-dx)e^{(-c)} + \mathrm{Ei}(dx)e^{c}\right)b}{d}\right)d + \left(b\log(x) - \frac{a}{x}\right)\cosh(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a + 2*b*cosh(d*x + c)*log(x)/d - (Ei(
-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + (b*log(x) - a/x)*cosh(d*x + c)
```

Fricas [A] time = 1.96812, size = 186, normalized size = 3.96

$$-\frac{2\,a\cosh\left(dx+c\right)-\left((ad+b)x\mathrm{Ei}\left(dx\right)-(ad-b)x\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left((ad+b)x\mathrm{Ei}\left(dx\right)+(ad-b)x\mathrm{Ei}\left(-dx\right)\right)\sinh\left(c\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c) - ((a*d + b)*x*Ei(d*x) - (a*d - b)*x*Ei(-d*x))*cosh
(c) - ((a*d + b)*x*Ei(d*x) + (a*d - b)*x*Ei(-d*x))*sinh(c))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)*cosh(c + d*x)/x**2, x)
```

Giac [A] time = 1.15134, size = 97, normalized size = 2.06

$$-\frac{adx\mathrm{Ei}\left(-dx\right)e^{(-c)}-adx\mathrm{Ei}\left(dx\right)e^{c}-bx\mathrm{Ei}\left(-dx\right)e^{(-c)}-bx\mathrm{Ei}\left(dx\right)e^{c}+ae^{(dx+c)}+ae^{(-dx-c)}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c - b*x*Ei(-d*x)*e^(-c) - b*x
*Ei(d*x)*e^c + a*e^(d*x + c) + a*e^(-d*x - c))/x
```

3.7 $\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=88

$$\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\mathrm{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+bd\sinh(c)\mathrm{Chi}(dx)+bd\cosh(c)\mathrm{Shi}(d$$

```
[Out] -(a*Cosh[c + d*x])/(2*x^2) - (b*Cosh[c + d*x])/x + (a*d^2*Cosh[c]*CoshInteg
ral[d*x])/2 + b*d*CoshIntegral[d*x]*Sinh[c] - (a*d*Sinh[c + d*x])/(2*x) + b
*d*Cosh[c]*SinhIntegral[d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] time = 0.28109, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\mathrm{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+bd\sinh(c)\mathrm{Chi}(dx)+bd\cosh(c)\mathrm{Shi}(d$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^3,x]
```


```
[Out] -(a*Cosh[c + d*x])/(2*x^2) - (b*Cosh[c + d*x])/x + (a*d^2*Cosh[c]*CoshInteg
ral[d*x])/2 + b*d*CoshIntegral[d*x]*Sinh[c] - (a*d*Sinh[c + d*x])/(2*x) + b
*d*Cosh[c]*SinhIntegral[d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\cosh(c+dx)}{x^3}\,dx + b\int \frac{\cosh(c+dx)}{x^2}\,dx\\
&= -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx+(bd)\int \frac{\sinh(c+dx)}{x}\,d\\
&= -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x}\,dx+(\\
&= -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}+bd\mathrm{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{2x}+bd\cosh(c)\\
&= -\frac{a\cosh(c+dx)}{2x^2}-\frac{b\cosh(c+dx)}{x}+\frac{1}{2}ad^2\cosh(c)\mathrm{Chi}(dx)+bd\mathrm{Chi}(dx)\sinh(c)-\frac{ad\,\mathrm{si}}{}
\end{aligned}
$$

Mathematica [A] time = 0.155352, size = 78, normalized size = 0.89

$$
\frac{dx^2\mathrm{Chi}(dx)(ad\cosh(c)+2b\sinh(c))+dx^2\mathrm{Shi}(dx)(ad\sinh(c)+2b\cosh(c))-adx\sinh(c+dx)-a\cosh(c+dx)-2b}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

```
[Out] (-(a*Cosh[c + d*x]) - 2*b*x*Cosh[c + d*x] + d*x^2*CoshIntegral[d*x]*(a*d*Co
sh[c] + 2*b*Sinh[c]) - a*d*x*Sinh[c + d*x] + d*x^2*(2*b*Cosh[c] + a*d*Sinh[
c])*SinhIntegral[d*x])/(2*x^2)
```

Maple [A] time = 0.047, size = 139, normalized size = 1.6

$$
\frac{da\mathrm{e}^{-dx-c}}{4x}-\frac{a\mathrm{e}^{-dx-c}}{4x^2}-\frac{d^2a\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{4}-\frac{b\mathrm{e}^{-dx-c}}{2x}+\frac{db\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{2}-\frac{a\mathrm{e}^{dx+c}}{4x^2}-\frac{ad\mathrm{e}^{dx+c}}{4x}-\frac{d^2a\mathrm{e}^{c}\mathrm{Ei}(1,-dx)}{4}-\frac{b\mathrm{e}^{dx+c}}{2x}-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^3,x)
```

```
[Out] 1/4*d*a*exp(-d*x-c)/x-1/4*a*exp(-d*x-c)/x^2-1/4*d^2*a*exp(-c)*Ei(1,d*x)-1/2
*b*exp(-d*x-c)/x+1/2*d*b*exp(-c)*Ei(1,d*x)-1/4*a/x^2*exp(d*x+c)-1/4*d*a/x*e
xp(d*x+c)-1/4*d^2*a*exp(c)*Ei(1,-d*x)-1/2*b/x*exp(d*x+c)-1/2*d*b*exp(c)*Ei(
1,-d*x)
```

Maxima [A] time = 1.417, size = 89, normalized size = 1.01

$$
\frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx)+ade^{c}\Gamma(-1,-dx)-2b\mathrm{Ei}(-dx)e^{(-c)}+2b\mathrm{Ei}(dx)e^{c}\right)d-\frac{(2bx+a)\cosh(dx+c)}{2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*b*Ei(-d*x)*e^(
-c) + 2*b*Ei(d*x)*e^c)*d - 1/2*(2*b*x + a)*cosh(d*x + c)/x^2
```

Fricas [A] time = 1.99403, size = 278, normalized size = 3.16

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\left(2\,bx+a\right)\cosh\left(dx+c\right)-\left(\left(ad^2+2\,bd\right)x^2\mathrm{Ei}\left(dx\right)+\left(ad^2-2\,bd\right)x^2\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(\left(ad^2+2\right.\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*(2*b*x + a)*cosh(d*x + c) - ((a*d^2 + 2*b*d
)*x^2*Ei(d*x) + (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b*d)*x^
2*Ei(d*x) - (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.15187, size = 181, normalized size = 2.06

$$\frac{ad^2x^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+ad^2x^2\mathrm{Ei}\left(dx\right)e^c-2\,bdx^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+2\,bdx^2\mathrm{Ei}\left(dx\right)e^c-adxe^{(dx+c)}+adxe^{(-dx-c)}-2\,bxe^{(dx+c)}-2\,b}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c - 2*b*d*x^2*Ei(-d*x)
*e^(-c) + 2*b*d*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) -
2*b*x*e^(d*x + c) - 2*b*x*e^(-d*x - c) - a*e^(d*x + c) - a*e^(-d*x - c))/x^
2
```

3.8 $\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$

Optimal. Leaf size=132

$$\frac{1}{6}ad^3\sinh(c)\mathbf{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathbf{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{a\cosh(c+dx)}{3x^3}+\frac{1}{2}bd^2\cosh(c)($$

```
[Out] -(a*Cosh[c + d*x])/(3*x^3) - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(6*x) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^3*CoshIntegral[d*x]*
Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) - (b*d*Sinh[c + d*x])/(2*x) + (a*d
^3*Cosh[c]*SinhIntegral[d*x])/6 + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] time = 0.339799, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3\sinh(c)\mathbf{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathbf{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{a\cosh(c+dx)}{3x^3}+\frac{1}{2}bd^2\cosh(c)($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(a*Cosh[c + d*x])/(3*x^3) - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(6*x) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^3*CoshIntegral[d*x]*
Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) - (b*d*Sinh[c + d*x])/(2*x) + (a*d
^3*Cosh[c]*SinhIntegral[d*x])/6 + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^3}\right)dx$$
$$= a\int \frac{\cosh(c+dx)}{x^4}\,dx + b\int \frac{\cosh(c+dx)}{x^3}\,dx$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} + \frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx + \frac{1}{2}(bd)\int \frac{\sinh(c+dx)}{x^2}\,dx$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{6x^2} - \frac{bd\sinh(c+dx)}{2x} + \frac{1}{6}\left(ad^2\right)\int \frac{\cosh}{}$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} - \frac{ad\sinh(c+dx)}{6x^2} - \frac{bd\sinh(c+dx)}{2x}$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} + \frac{1}{2}bd^2\cosh(c)\mathbf{Chi}(dx) - \frac{ad\sinh(}{6x^2}$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{6x} + \frac{1}{2}bd^2\cosh(c)\mathbf{Chi}(dx) + \frac{1}{6}ad^3\mathbf{Chi}$$

Mathematica [A] time = 0.2723, size = 110, normalized size = 0.83

$$-\frac{-d^2x^3\mathbf{Chi}(dx)(ad\sinh(c)+3b\cosh(c)) - d^2x^3\mathbf{Shi}(dx)(ad\cosh(c)+3b\sinh(c)) + ad^2x^2\cosh(c+dx) + adx\sinh(c+d}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(2*a*Cosh[c + d*x] + 3*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d^2*x
^3*CoshIntegral[d*x]*(3*b*Cosh[c] + a*d*Sinh[c]) + a*d*x*Sinh[c + d*x] + 3*
b*d*x^2*Sinh[c + d*x] - d^2*x^3*(a*d*Cosh[c] + 3*b*Sinh[c])*SinhIntegral[d*
x])/(6*x^3)
```

Maple [A] time = 0.053, size = 205, normalized size = 1.6

$$-\frac{ad^2e^{-dx-c}}{12x} + \frac{dae^{-dx-c}}{12x^2} - \frac{ae^{-dx-c}}{6x^3} + \frac{d^3ae^{-c}\mathrm{Ei}(1,dx)}{12} + \frac{bde^{-dx-c}}{4x} - \frac{be^{-dx-c}}{4x^2} - \frac{d^2be^{-c}\mathrm{Ei}(1,dx)}{4} - \frac{ae^{dx+c}}{6x^3} - \frac{ade^{dx+c}}{12x^2} - \frac{ad^2e}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^4,x)
```

```
[Out] -1/12*d^2*a*exp(-d*x-c)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3+1/
12*d^3*a*exp(-c)*Ei(1,d*x)+1/4*d*b*exp(-d*x-c)/x-1/4*b*exp(-d*x-c)/x^2-1/4*
d^2*b*exp(-c)*Ei(1,d*x)-1/6*a/x^3*exp(d*x+c)-1/12*d*a/x^2*exp(d*x+c)-1/12*d
^2*a/x*exp(d*x+c)-1/12*d^3*a*exp(c)*Ei(1,-d*x)-1/4*b/x^2*exp(d*x+c)-1/4*d*b
/x*exp(d*x+c)-1/4*d^2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.39474, size = 105, normalized size = 0.8

$$\frac{1}{12}\left(2\,ad^2e^{(-c)}\Gamma(-2,dx) - 2\,ad^2e^c\Gamma(-2,-dx) + 3\,bde^{(-c)}\Gamma(-1,dx) + 3\,bde^c\Gamma(-1,-dx)\right)d - \frac{(3\,bx+2\,a)\cosh(dx+c)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/12*(2*a*d^2*e^(-c)*gamma(-2, d*x) - 2*a*d^2*e^c*gamma(-2, -d*x) + 3*b*d*e
^(-c)*gamma(-1, d*x) + 3*b*d*e^c*gamma(-1, -d*x))*d - 1/6*(3*b*x + 2*a)*cos
h(d*x + c)/x^3
```

Fricas [A] time = 1.98033, size = 328, normalized size = 2.48

$$-\frac{2\left(ad^2x^2+3\,bx+2\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^3+3\,bd^2\right)x^3\mathrm{Ei}\left(dx\right)-\left(ad^3-3\,bd^2\right)x^3\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)+2\left(3\,bdx^2+adx\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(a*d^2*x^2 + 3*b*x + 2*a)*cosh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*E
i(d*x) - (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(3*b*d*x^2 + a*d*x)*si
nh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) + (a*d^3 - 3*b*d^2)*x^3*Ei(-d*
x))*sinh(c))/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14764, size = 269, normalized size = 2.04

$$-\frac{ad^3x^3\mathrm{Ei}\left(-dx\right)e^{(-c)}-ad^3x^3\mathrm{Ei}\left(dx\right)e^c-3\,bd^2x^3\mathrm{Ei}\left(-dx\right)e^{(-c)}-3\,bd^2x^3\mathrm{Ei}\left(dx\right)e^c+ad^2x^2e^{(dx+c)}+ad^2x^2e^{(-dx-c)}+3\,bd}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c - 3*b*d^2*x^3*Ei(-
d*x)*e^(-c) - 3*b*d^2*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e
^(-d*x - c) + 3*b*d*x^2*e^(d*x + c) - 3*b*d*x^2*e^(-d*x - c) + a*d*x*e^(d*x
 + c) - a*d*x*e^(-d*x - c) + 3*b*x*e^(d*x + c) + 3*b*x*e^(-d*x - c) + 2*a*e
^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

3.9 $\int \frac{(a+bx)\cosh(c+dx)}{x^5} dx$

Optimal. Leaf size=166

$$\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{a\cosh(c+d}{4x^4}$$

```
[Out] -(a*Cosh[c + d*x])/(4*x^4) - (b*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*
x])/(24*x^2) - (b*d^2*Cosh[c + d*x])/(6*x) + (a*d^4*Cosh[c]*CoshIntegral[d*
x])/24 + (b*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(12*x^3)
 - (b*d*Sinh[c + d*x])/(6*x^2) - (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^3*Cosh
[c]*SinhIntegral[d*x])/6 + (a*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rubi [A] time = 0.40618, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{a\cosh(c+d}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Cosh[c + d*x])/x^5,x]
```


```
[Out] -(a*Cosh[c + d*x])/(4*x^4) - (b*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*
x])/(24*x^2) - (b*d^2*Cosh[c + d*x])/(6*x) + (a*d^4*Cosh[c]*CoshIntegral[d*
x])/24 + (b*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(12*x^3)
 - (b*d*Sinh[c + d*x])/(6*x^2) - (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^3*Cosh
[c]*SinhIntegral[d*x])/6 + (a*d^4*Sinh[c]*SinhIntegral[d*x])/24
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^4}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^5}\,dx + b\int \frac{\cosh(c+dx)}{x^4}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}+\frac{1}{4}(ad)\int \frac{\sinh(c+dx)}{x^4}\,dx+\frac{1}{3}(bd)\int \frac{\sinh(c+dx)}{x^3} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{6x^2}+\frac{1}{12}\left(ad^2\right)\int \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c}{12x^3} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c}{12x^3} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}+\frac{1}{6}bd^3\mathrm{Chi}(\\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{bd^2\cosh(c+dx)}{6x}+\frac{1}{24}ad^4\cos
\end{aligned}$$

Mathematica [A] time = 0.318458, size = 140, normalized size = 0.84

$$-\frac{-d^3x^4\mathrm{Chi}(dx)(ad\cosh(c)+4b\sinh(c))-d^3x^4\mathrm{Shi}(dx)(ad\sinh(c)+4b\cosh(c))+ad^3x^3\sinh(c+dx)+ad^2x^2\cosh(}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -(6*a*Cosh[c + d*x] + 8*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] + 4*b*d
^2*x^3*Cosh[c + d*x] - d^3*x^4*CoshIntegral[d*x]*(a*d*Cosh[c] + 4*b*Sinh[c]
) + 2*a*d*x*Sinh[c + d*x] + 4*b*d*x^2*Sinh[c + d*x] + a*d^3*x^3*Sinh[c + d*
x] - d^3*x^4*(4*b*Cosh[c] + a*d*Sinh[c])*SinhIntegral[d*x])/(24*x^4)
```

Maple [A] time = 0.062, size = 271, normalized size = 1.6

$$\frac{ad^3\mathrm{e}^{-dx-c}}{48\,x}-\frac{ad^2\mathrm{e}^{-dx-c}}{48\,x^2}+\frac{da\mathrm{e}^{-dx-c}}{24\,x^3}-\frac{a\mathrm{e}^{-dx-c}}{8\,x^4}-\frac{d^4a\mathrm{e}^{-c}\mathrm{Ei}\,(1,dx)}{48}-\frac{bd^2\mathrm{e}^{-dx-c}}{12\,x}+\frac{bd\mathrm{e}^{-dx-c}}{12\,x^2}-\frac{b\mathrm{e}^{-dx-c}}{6\,x^3}+\frac{d^3b\mathrm{e}^{-c}\mathrm{Ei}\,(1,dx)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*cosh(d*x+c)/x^5,x)
```

```
[Out] 1/48*d^3*a*exp(-d*x-c)/x-1/48*d^2*a*exp(-d*x-c)/x^2+1/24*d*a*exp(-d*x-c)/x^
3-1/8*a*exp(-d*x-c)/x^4-1/48*d^4*a*exp(-c)*Ei(1,d*x)-1/12*d^2*b*exp(-d*x-c)
/x+1/12*d*b*exp(-d*x-c)/x^2-1/6*b*exp(-d*x-c)/x^3+1/12*d^3*b*exp(-c)*Ei(1,d
*x)-1/8*a/x^4*exp(d*x+c)-1/24*d*a/x^3*exp(d*x+c)-1/48*d^2*a/x^2*exp(d*x+c)-
1/48*d^3*a/x*exp(d*x+c)-1/48*d^4*a*exp(c)*Ei(1,-d*x)-1/6*b/x^3*exp(d*x+c)-1
/12*d*b/x^2*exp(d*x+c)-1/12*d^2*b/x*exp(d*x+c)-1/12*d^3*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.39888, size = 111, normalized size = 0.67

$$\frac{1}{24}\left(3\,ad^{3}e^{(-c)}\Gamma\left(-3,dx\right)+3\,ad^{3}e^{c}\Gamma\left(-3,-dx\right)+4\,bd^{2}e^{(-c)}\Gamma\left(-2,dx\right)-4\,bd^{2}e^{c}\Gamma\left(-2,-dx\right)\right)d-\frac{(4\,bx+3\,a)\cosh\left(dx+c\right)}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a*d^3*e^(-c)*gamma(-3, d*x) + 3*a*d^3*e^c*gamma(-3, -d*x) + 4*b*d^2
*e^(-c)*gamma(-2, d*x) - 4*b*d^2*e^c*gamma(-2, -d*x))*d - 1/12*(4*b*x + 3*a
)*cosh(d*x + c)/x^4
```

Fricas [A] time = 2.02097, size = 366, normalized size = 2.2

$$-\frac{2\left(4\,bd^{2}x^{3}+ad^{2}x^{2}+8\,bx+6\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^{4}+4\,bd^{3}\right)x^{4}\mathrm{Ei}\left(dx\right)+\left(ad^{4}-4\,bd^{3}\right)x^{4}\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)+2\left(ad^{3}x^{3}\right.}{48\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(4*b*d^2*x^3 + a*d^2*x^2 + 8*b*x + 6*a)*cosh(d*x + c) - ((a*d^4 +
4*b*d^3)*x^4*Ei(d*x) + (a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*cosh(c) + 2*(a*d^3*x
^3 + 4*b*d*x^2 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 4*b*d^3)*x^4*Ei(d*x) -
(a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26538, size = 359, normalized size = 2.16

$$ad^{4}x^{4}\mathrm{Ei}\left(-dx\right)e^{(-c)}+ad^{4}x^{4}\mathrm{Ei}\left(dx\right)e^{c}-4\,bd^{3}x^{4}\mathrm{Ei}\left(-dx\right)e^{(-c)}+4\,bd^{3}x^{4}\mathrm{Ei}\left(dx\right)e^{c}-ad^{3}x^{3}e^{(dx+c)}+ad^{3}x^{3}e^{(-dx-c)}-4\,bd^{2}x^{3}e^{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c - 4*b*d^3*x^4*Ei(-d
*x)*e^(-c) + 4*b*d^3*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*e^
(-d*x - c) - 4*b*d^2*x^3*e^(d*x + c) - 4*b*d^2*x^3*e^(-d*x - c) - a*d^2*x^2
*e^(d*x + c) - a*d^2*x^2*e^(-d*x - c) - 4*b*d*x^2*e^(d*x + c) + 4*b*d*x^2*e
^(-d*x - c) - 2*a*d*x*e^(d*x + c) + 2*a*d*x*e^(-d*x - c) - 8*b*x*e^(d*x + c
) - 8*b*x*e^(-d*x - c) - 6*a*e^(d*x + c) - 6*a*e^(-d*x - c))/x^4
```

3.10 $\int x^2(a+bx)^2\cosh(c+dx)\,dx$

Optimal. Leaf size=184

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c}{d^4}$$

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[
c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])/d
^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*Sin
h[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/
d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rubi [A] time = 0.346497, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6742, 3296, 2637, 2638}

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[
c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])/d
^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*Sin
h[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/
d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = \int \left(a^2x^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)dx$$
$$= a^2\int x^2\cosh(c+dx)\,dx+(2ab)\int x^3\cosh(c+dx)\,dx+b^2\int x^4\cosh(c+dx)\,dx$$
$$= \frac{a^2x^2\sinh(c+dx)}{d}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^4\sinh(c+dx)}{d}-\frac{\left(2a^2\right)\int x\sinh(c+dx)\,d}{d}$$
$$= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}+$$
$$= -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{2a^2\sinh(c+dx)}{d^3}+$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-$$
$$= -\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2}-$$

Mathematica [A] time = 0.241922, size = 100, normalized size = 0.54

$$\frac{\left(a^2d^2\left(d^2x^2+2\right)+2abd^2x\left(d^2x^2+6\right)+b^2\left(d^4x^4+12d^2x^2+24\right)\right)\sinh(c+dx)-2d(a+2bx)\left(ad^2x+b\left(d^2x^2+6\right)\right)\cosh(}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*(a + 2*b*x)*(a*d^2*x + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^2*(2 +
 d^2*x^2) + 2*a*b*d^2*x*(6 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Si
nh[c + d*x])/d^5
```

Maple [B] time = 0.007, size = 463, normalized size = 2.5

$$\frac{1}{d^3}\left(\frac{b^2\left((dx+c)^4\sinh(dx+c)-4(dx+c)^3\cosh(dx+c)+12(dx+c)^2\sinh(dx+c)-24(dx+c)\cosh(dx+c)+24\,\mathrm{s}\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d^3*(b^2/d^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*
sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-4*b^2/d^2*c*((d*x+c)^3*s
inh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+6*b
^2*c^2/d^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4*b^
2/d^2*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+2*b/d*a*((d*x+c)^3*sinh(d*x+c)-
3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-6*b*c/d*a*((d*
x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+6*b/d*a*c^2*((d*x+c
)*sinh(d*x+c)-cosh(d*x+c))+b^2*c^4/d^2*sinh(d*x+c)-2*b*c^3/d*a*sinh(d*x+c)+
a^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*a^2*c*((d
*x+c)*sinh(d*x+c)-cosh(d*x+c))+a^2*c^2*sinh(d*x+c))
```

Maxima [A] time = 1.18353, size = 444, normalized size = 2.41

$$-\frac{1}{60}d\left(\frac{10\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)a^2e^{(dx)}}{d^4}+\frac{10\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)a^2e^{(-dx-c)}}{d^4}+\frac{15\left(d^4x^4e^c-4\,d^3x^3e^c+12\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/60*d*(10*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/d
^4 + 10*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 15*(d^4*x^
4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b*e^(d*x)/d
^5 + 15*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*b*e^(-d*x - c)/d
^5 + 6*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120
*d*x*e^c - 120*e^c)*b^2*e^(d*x)/d^6 + 6*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 +
 60*d^2*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6) + 1/30*(6*b^2*x^5 + 15*a
*b*x^4 + 10*a^2*x^3)*cosh(d*x + c)
```

Fricas [A] time = 2.01887, size = 273, normalized size = 1.48

$$-\frac{2\left(2b^2d^3x^3+3abd^3x^2+6abd+\left(a^2d^3+12b^2d\right)x\right)\cosh\left(dx+c\right)-\left(b^2d^4x^4+2abd^4x^3+12abd^2x+2a^2d^2+\left(a^2d^4\right.\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 + 6*a*b*d + (a^2*d^3 + 12*b^2*d)*x)*cosh
(d*x + c) - (b^2*d^4*x^4 + 2*a*b*d^4*x^3 + 12*a*b*d^2*x + 2*a^2*d^2 + (a^2*
d^4 + 12*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^5
```

Sympy [A] time = 3.53062, size = 228, normalized size = 1.24

$$\begin{cases} \frac{a^2x^2\sinh(c+dx)}{d}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{2a^2\sinh(c+dx)}{d^3}+\frac{2abx^3\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{b^2 \ldots}{} \\ \left(\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}\right)\cosh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2
*sinh(c + d*x)/d**3 + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)
/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**4
*sinh(c + d*x)/d - 4*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d
*x)/d**3 - 24*b**2*x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d,
 0)), ((a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*cosh(c), True))
```

Giac [A] time = 1.17721, size = 319, normalized size = 1.73

$$\frac{\left(b^2d^4x^4+2abd^4x^3+a^2d^4x^2-4b^2d^3x^3-6abd^3x^2-2a^2d^3x+12b^2d^2x^2+12abd^2x+2a^2d^2-24b^2dx-12abd+24\ldots\right.}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 4*b^2*d^3*x^3 - 6*a*b*d^3*
x^2 - 2*a^2*d^3*x + 12*b^2*d^2*x^2 + 12*a*b*d^2*x + 2*a^2*d^2 - 24*b^2*d*x
- 12*a*b*d + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a
^2*d^4*x^2 + 4*b^2*d^3*x^3 + 6*a*b*d^3*x^2 + 2*a^2*d^3*x + 12*b^2*d^2*x^2 +
 12*a*b*d^2*x + 2*a^2*d^2 + 24*b^2*d*x + 12*a*b*d + 24*b^2)*e^(-d*x - c)/d^
5
```

3.11 $\int x(a+bx)^2\cosh(c+dx)\,dx$

Optimal. Leaf size=134

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{3b^2x^2\cosh(c+\cdots}{d^2}$$

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*
x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^
2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x]
)/d + (b^2*x^3*Sinh[c + d*x])/d
```

Rubi [A] time = 0.214288, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {6742, 3296, 2638, 2637}

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{3b^2x^2\cosh(c+\cdots}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*
x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^
2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x]
)/d + (b^2*x^3*Sinh[c + d*x])/d
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \int \left(a^2x\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^3\cosh(c+dx)\right)dx$$
$$= a^2\int x\cosh(c+dx)\,dx+(2ab)\int x^2\cosh(c+dx)\,dx+b^2\int x^3\cosh(c+dx)\,dx$$
$$= \frac{a^2x\sinh(c+dx)}{d}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{a^2\int\sinh(c+dx)\,dx}{d}-\frac{(4$$
$$= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{2abx$$
$$= -\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}+\frac{6b^2x$$
$$= -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{4ab\,s$$

Mathematica [A] time = 0.188351, size = 87, normalized size = 0.65

$$\frac{d\left(a^2d^2x+2ab\left(d^2x^2+2\right)+b^2x\left(d^2x^2+6\right)\right)\sinh(c+dx)-\left(a^2d^2+4abd^2x+3b^2\left(d^2x^2+2\right)\right)\cosh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a^2*d^
2*x + 2*a*b*(2 + d^2*x^2) + b^2*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Maple [B] time = 0.01, size = 283, normalized size = 2.1

$$\frac{1}{d^2}\left(\frac{b^2\left((dx+c)^3\sinh(dx+c)-3\,(dx+c)^2\cosh(dx+c)+6\,(dx+c)\sinh(dx+c)-6\,\cosh(dx+c)\right)}{d^2}-3\,\frac{cb^2\left((dx+c)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d^2*(b^2/d^2*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sin
h(d*x+c)-6*cosh(d*x+c))-3*b^2/d^2*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d
*x+c)+2*sinh(d*x+c))+2*b/d*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2
*sinh(d*x+c))+3*b^2*c^2/d^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-4*b*c/d*a*((d
*x+c)*sinh(d*x+c)-cosh(d*x+c))+a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-b^2*c^
3/d^2*sinh(d*x+c)+2*b*c^2/d*a*sinh(d*x+c)-c*a^2*sinh(d*x+c))
```

Maxima [B] time = 1.17618, size = 371, normalized size = 2.77

$$-\frac{1}{24}\,d\left(\frac{6\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)a^2e^{(dx)}}{d^3}+\frac{6\left(d^2x^2+2\,dx+2\right)a^2e^{(-dx-c)}}{d^3}+\frac{8\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)abe^{(dx)}}{d^4}+\frac{8\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(6*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 6*(d^2*x^2 +
 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 8*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e
^c - 6*e^c)*a*b*e^(d*x)/d^4 + 8*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d
*x - c)/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c
+ 24*e^c)*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x +
24)*b^2*e^(-d*x - c)/d^5) + 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)*cosh(d
*x + c)
```

Fricas [A] time = 2.00752, size = 203, normalized size = 1.51

$$-\frac{\left(3\,b^2d^2x^2+4\,abd^2x+a^2d^2+6\,b^2\right)\cosh\left(dx+c\right)-\left(b^2d^3x^3+2\,abd^3x^2+4\,abd+\left(a^2d^3+6\,b^2d\right)x\right)\sinh\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2)*cosh(d*x + c) - (b^2*d^3*
x^3 + 2*a*b*d^3*x^2 + 4*a*b*d + (a^2*d^3 + 6*b^2*d)*x)*sinh(d*x + c))/d^4
```

Sympy [A] time = 2.11423, size = 172, normalized size = 1.28

$$\begin{cases}\frac{a^2x\sinh\left(c+dx\right)}{d}-\frac{a^2\cosh\left(c+dx\right)}{d^2}+\frac{2abx^2\sinh\left(c+dx\right)}{d}-\frac{4abx\cosh\left(c+dx\right)}{d^2}+\frac{4ab\sinh\left(c+dx\right)}{d^3}+\frac{b^2x^3\sinh\left(c+dx\right)}{d}-\frac{3b^2x^2\cosh\left(c+dx\right)}{d^2}+\frac{6b^2x\sin}{d} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}\right)\cosh\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**2*si
nh(c + d*x)/d - 4*a*b*x*cosh(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**
2*x**3*sinh(c + d*x)/d - 3*b**2*x**2*cosh(c + d*x)/d**2 + 6*b**2*x*sinh(c +
 d*x)/d**3 - 6*b**2*cosh(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x*
*3/3 + b**2*x**4/4)*cosh(c), True))
```

Giac [A] time = 1.14316, size = 231, normalized size = 1.72

$$\frac{\left(b^2d^3x^3+2\,abd^3x^2+a^2d^3x-3\,b^2d^2x^2-4\,abd^2x-a^2d^2+6\,b^2dx+4\,abd-6\,b^2\right)e^{(dx+c)}}{2\,d^4}-\frac{\left(b^2d^3x^3+2\,abd^3x^2+a^2d^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 3*b^2*d^2*x^2 - 4*a*b*d^2*x
- a^2*d^2 + 6*b^2*d*x + 4*a*b*d - 6*b^2)*e^(d*x + c)/d^4 - 1/2*(b^2*d^3*x^3
 + 2*a*b*d^3*x^2 + a^2*d^3*x + 3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^
2*d*x + 4*a*b*d + 6*b^2)*e^(-d*x - c)/d^4
```

3.12 $\int (a+bx)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=49

$$-\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2b^2\sinh(c+dx)}{d^3}$$

```
[Out] (-2*b*(a + b*x)*Cosh[c + d*x])/d^2 + (2*b^2*Sinh[c + d*x])/d^3 + ((a + b*x)
^2*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0479988, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3296, 2637}

$$-\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2b^2\sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*b*(a + b*x)*Cosh[c + d*x])/d^2 + (2*b^2*Sinh[c + d*x])/d^3 + ((a + b*x)
^2*Sinh[c + d*x])/d
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\cosh(c+dx)\,dx &= \frac{(a+bx)^2\sinh(c+dx)}{d}-\frac{(2b)\int (a+bx)\sinh(c+dx)\,dx}{d}\\ &= -\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{\left(2b^2\right)\int \cosh(c+dx)\,dx}{d^2}\\ &= -\frac{2b(a+bx)\cosh(c+dx)}{d^2}+\frac{2b^2\sinh(c+dx)}{d^3}+\frac{(a+bx)^2\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.142182, size = 56, normalized size = 1.14

$$\frac{\left(a^2d^2+2abd^2x+b^2\left(d^2x^2+2\right)\right)\sinh(c+dx)-2bd(a+bx)\cosh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*b*d*(a + b*x)*Cosh[c + d*x] + (a^2*d^2 + 2*a*b*d^2*x + b^2*(2 + d^2*x^2
))*Sinh[c + d*x])/d^3
```

Maple [B] time = 0.007, size = 147, normalized size = 3.

$$\frac{1}{d}\left(\frac{b^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-2\frac{cb^2\left((dx+c)\sinh(dx+c)-\cosh(dx+\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d*(b^2/d^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*
b^2/d^2*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+2*b/d*a*((d*x+c)*sinh(d*x+c)-co
sh(d*x+c))+b^2*c^2/d^2*sinh(d*x+c)-2*b*c/d*a*sinh(d*x+c)+a^2*sinh(d*x+c))
```

Maxima [B] time = 1.20621, size = 182, normalized size = 3.71

$$\frac{a^2e^{(dx+c)}}{2d}+\frac{(dxe^c-e^c)abe^{(dx)}}{d^2}-\frac{(dx+1)abe^{(-dx-c)}}{d^2}-\frac{a^2e^{(-dx-c)}}{2d}+\frac{\left(d^2x^2e^c-2dxe^c+2e^c\right)b^2e^{(dx)}}{2d^3}-\frac{\left(d^2x^2+2dx+2\right)b^2e}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d + (d*x*e^c - e^c)*a*b*e^(d*x)/d^2 - (d*x + 1)*a*b*e^(
-d*x - c)/d^2 - 1/2*a^2*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e
^c)*b^2*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3
```

Fricas [A] time = 1.94877, size = 140, normalized size = 2.86

$$-\frac{2\left(b^2dx+abd\right)\cosh(dx+c)-\left(b^2d^2x^2+2abd^2x+a^2d^2+2b^2\right)\sinh(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(b^2*d*x + a*b*d)*cosh(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
+ 2*b^2)*sinh(d*x + c))/d^3
```

Sympy [A] time = 1.0641, size = 112, normalized size = 2.29

$$\begin{cases}\frac{a^2\sinh(c+dx)}{d}+\frac{2abx\sinh(c+dx)}{d}-\frac{2ab\cosh(c+dx)}{d^2}+\frac{b^2x^2\sinh(c+dx)}{d}-\frac{2b^2x\cosh(c+dx)}{d^2}+\frac{2b^2\sinh(c+dx)}{d^3} & \text{for } d\neq 0\\ \left(a^2x+abx^2+\frac{b^2x^3}{3}\right)\cosh(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x*sinh(c + d*x)/d - 2*a*b*cosh(c +
d*x)/d**2 + b**2*x**2*sinh(c + d*x)/d - 2*b**2*x*cosh(c + d*x)/d**2 + 2*b**
2*sinh(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*cosh(c)
, True))
```

Giac [B] time = 1.21508, size = 151, normalized size = 3.08

$$\frac{\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2 - 2\,b^2dx - 2\,abd + 2\,b^2\right)e^{(dx+c)}}{2\,d^3} - \frac{\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2 + 2\,b^2dx + 2\,abd + 2\,b^2\right)e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2*d*x - 2*a*b*d + 2*b^2)*e^(
d*x + c)/d^3 - 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2*d*x + 2*a*b
*d + 2*b^2)*e^(-d*x - c)/d^3
```

3.13 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx$

Optimal. Leaf size=62

$$a^2 \cosh(c)\mathbf{Chi}(dx) + a^2 \sinh(c)\mathbf{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

```
[Out] -((b^2*Cosh[c + d*x])/d^2) + a^2*Cosh[c]*CoshIntegral[d*x] + (2*a*b*Sinh[c
+ d*x])/d + (b^2*x*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.184752, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2637, 3303, 3298, 3301, 3296, 2638}

$$a^2 \cosh(c)\mathbf{Chi}(dx) + a^2 \sinh(c)\mathbf{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x,x]
```


```
[Out] -((b^2*Cosh[c + d*x])/d^2) + a^2*Cosh[c]*CoshIntegral[d*x] + (2*a*b*Sinh[c
+ d*x])/d + (b^2*x*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = \int \left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x}+b^2x\cosh(c+dx)\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x}\,dx+(2ab)\int \cosh(c+dx)\,dx+b^2\int x\cosh(c+dx)\,dx$$
$$= \frac{2ab\sinh(c+dx)}{d}+\frac{b^2x\sinh(c+dx)}{d}-\frac{b^2\int \sinh(c+dx)\,dx}{d}+\left(a^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2}+a^2\cosh(c)\mathbf{Chi}(dx)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x\sinh(c+dx)}{d}+a^2\sinh($$

Mathematica [A] time = 0.234957, size = 51, normalized size = 0.82

$$a^2\cosh(c)\mathbf{Chi}(dx)+a^2\sinh(c)\mathbf{Shi}(dx)+\frac{b(d(2a+bx)\sinh(c+dx)-b\cosh(c+dx))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

```
[Out] a^2*Cosh[c]*CoshIntegral[d*x] + (b*(-(b*Cosh[c + d*x]) + d*(2*a + b*x)*Sinh
[c + d*x]))/d^2 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.036, size = 121, normalized size = 2.

$$-\frac{a^2e^{-c}\text{Ei}(1,dx)}{2}-\frac{b^2e^{-dx-c}x}{2d}-\frac{b^2e^{-dx-c}}{2d^2}-\frac{abe^{-dx-c}}{d}-\frac{a^2e^{c}\text{Ei}(1,-dx)}{2}+\frac{e^{dx+c}b^2x}{2d}-\frac{e^{dx+c}b^2}{2d^2}+\frac{abe^{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x,x)
```

```
[Out] -1/2*a^2*exp(-c)*Ei(1,d*x)-1/2*b^2/d*exp(-d*x-c)*x-1/2*b^2/d^2*exp(-d*x-c)-
a*b/d*exp(-d*x-c)-1/2*a^2*exp(c)*Ei(1,-d*x)+1/2*b^2/d*exp(d*x+c)*x-1/2*b^2/
d^2*exp(d*x+c)+a*b/d*exp(d*x+c)
```

Maxima [B] time = 1.34019, size = 236, normalized size = 3.81

$$-\frac{1}{4}\left(4ab\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)+b^2\left(\frac{\left(d^2x^2e^c-2dxe^c+2e^c\right)e^{(dx)}}{d^3}+\frac{\left(d^2x^2+2dx+2\right)e^{(-dx-c)}}{d^3}\right)+\frac{4a^2\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + b^
2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^
(-d*x - c)/d^3) + 4*a^2*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x)*e^(-c) + Ei(d*
x)*e^c)*a^2/d)*d + 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*cosh(d*x + c)
```

Fricas [A] time = 1.97115, size = 221, normalized size = 3.56

$$-\frac{2\,b^2\cosh\left(dx+c\right)-\left(a^2d^2\mathrm{Ei}\left(dx\right)+a^2d^2\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-2\left(b^2dx+2\,abd\right)\sinh\left(dx+c\right)-\left(a^2d^2\mathrm{Ei}\left(dx\right)-a^2d^2\mathrm{Ei}\left(\right.\right.}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei(d*x) + a^2*d^2*Ei(-d*x))*cosh(c) -
2*(b^2*d*x + 2*a*b*d)*sinh(d*x + c) - (a^2*d^2*Ei(d*x) - a^2*d^2*Ei(-d*x))*
sinh(c))/d^2
```

Sympy [A] time = 5.07679, size = 73, normalized size = 1.18

$$a^2\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a^2\cosh\left(c\right)\mathrm{Chi}\left(dx\right)+2ab\left(\begin{cases}\frac{\sinh\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x\cosh\left(c\right) & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}\frac{x\sinh\left(c+dx\right)}{d}-\frac{\cosh\left(c+dx\right)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cosh\left(c\right)}{2} & \text{otherwi}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((sinh(c + d
*x)/d, Ne(d, 0)), (x*cosh(c), True)) + b**2*Piecewise((x*sinh(c + d*x)/d -
cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Giac [A] time = 1.26265, size = 153, normalized size = 2.47

$$\frac{a^2d^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+a^2d^2\mathrm{Ei}\left(dx\right)e^c+b^2dxe^{(dx+c)}-b^2dxe^{(-dx-c)}+2\,abde^{(dx+c)}-2\,abde^{(-dx-c)}-b^2e^{(dx+c)}-b^2e^{(-dx-c)}}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d^2*Ei(-d*x)*e^(-c) + a^2*d^2*Ei(d*x)*e^c + b^2*d*x*e^(d*x + c) -
b^2*d*x*e^(-d*x - c) + 2*a*b*d*e^(d*x + c) - 2*a*b*d*e^(-d*x - c) - b^2*e^(
d*x + c) - b^2*e^(-d*x - c))/d^2
```

3.14 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} dx$

Optimal. Leaf size=70

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c)\text{Chi}(dx) + 2ab \sinh(c)\text{Shi}(dx) + \frac{b^2 \sinh(c+d}{d}$$

```
[Out] -((a^2*Cosh[c + d*x])/x) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + a^2*d*CoshInte
gral[d*x]*Sinh[c] + (b^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
 + 2*a*b*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.246734, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c)\text{Chi}(dx) + 2ab \sinh(c)\text{Shi}(dx) + \frac{b^2 \sinh(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + a^2*d*CoshInte
gral[d*x]*Sinh[c] + (b^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
 + 2*a*b*Sinh[c]*SinhIntegral[d*x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = \int \left(b^2\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^2}+\frac{2ab\cosh(c+dx)}{x}\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx + (2ab)\int \frac{\cosh(c+dx)}{x}\,dx + b^2\int \cosh(c+dx)\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{x} + \frac{b^2\sinh(c+dx)}{d} + \left(a^2d\right)\int \frac{\sinh(c+dx)}{x}\,dx + (2ab\cosh(c))\int \frac{\text{co}}{}$$
$$= -\frac{a^2\cosh(c+dx)}{x} + 2ab\cosh(c)\mathbf{Chi}(dx) + \frac{b^2\sinh(c+dx)}{d} + 2ab\sinh(c)\mathbf{Shi}(dx) + \left(a\right.$$
$$= -\frac{a^2\cosh(c+dx)}{x} + 2ab\cosh(c)\mathbf{Chi}(dx) + a^2d\mathbf{Chi}(dx)\sinh(c) + \frac{b^2\sinh(c+dx)}{d} + a^2$$

Mathematica [A] time = 0.229813, size = 62, normalized size = 0.89

$$-\frac{a^2\cosh(c+dx)}{x} + a\mathbf{Chi}(dx)(ad\sinh(c)+2b\cosh(c)) + a\mathbf{Shi}(dx)(ad\cosh(c)+2b\sinh(c)) + \frac{b^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) + a*CoshIntegral[d*x]*(2*b*Cosh[c] + a*d*Sinh[c])
+ (b^2*Sinh[c + d*x])/d + a*(a*d*Cosh[c] + 2*b*Sinh[c])*SinhIntegral[d*x]
```

Maple [A] time = 0.049, size = 118, normalized size = 1.7

$$-\frac{a^2e^{-dx-c}}{2x} + \frac{da^2e^{-c}\mathrm{Ei}(1,dx)}{2} - \frac{b^2e^{-dx-c}}{2d} - abe^{-c}\mathrm{Ei}(1,dx) - \frac{e^{dx+c}a^2}{2x} - \frac{da^2e^{c}\mathrm{Ei}(1,-dx)}{2} + \frac{e^{dx+c}b^2}{2d} - abe^{c}\mathrm{Ei}(1,-dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^2,x)
```

```
[Out] -1/2*a^2*exp(-d*x-c)/x+1/2*d*a^2*exp(-c)*Ei(1,d*x)-1/2*b^2/d*exp(-d*x-c)-a*
b*exp(-c)*Ei(1,d*x)-1/2*a^2/x*exp(d*x+c)-1/2*d*a^2*exp(c)*Ei(1,-d*x)+1/2*b^
2/d*exp(d*x+c)-a*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.39303, size = 184, normalized size = 2.63

$$-\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)e^{(-c)} - \mathrm{Ei}(dx)e^{c}\right)a^2 + b^2\left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + \frac{4ab\cosh(dx+c)\log(x)}{d} - \frac{2\left(\mathrm{Ei}(-dx)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a^2 + b^2*((d*x*e^c - e^c)*e^(d*x)/d^
2 + (d*x + 1)*e^(-d*x - c)/d^2) + 4*a*b*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x
)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + (b^2*x + 2*a*b*log(x) - a^2/x)*cosh(d*x
+ c)
```

Fricas [A] time = 1.99353, size = 284, normalized size = 4.06

$$-\frac{2\,a^2d\cosh\left(dx+c\right)-2\,b^2x\sinh\left(dx+c\right)-\left(\left(a^2d^2+2\,abd\right)x\mathrm{Ei}\left(dx\right)-\left(a^2d^2-2\,abd\right)x\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(\left(a^2d^2+2\,abc\right.\right.}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*d*cosh(d*x + c) - 2*b^2*x*sinh(d*x + c) - ((a^2*d^2 + 2*a*b*d)*
x*Ei(d*x) - (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*cosh(c) - ((a^2*d^2 + 2*a*b*d)*
x*Ei(d*x) + (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**2, x)
```

Giac [A] time = 1.23853, size = 161, normalized size = 2.3

$$-\frac{a^2d^2x\mathrm{Ei}\left(-dx\right)e^{(-c)}-a^2d^2x\mathrm{Ei}\left(dx\right)e^{c}-2\,abdx\mathrm{Ei}\left(-dx\right)e^{(-c)}-2\,abdx\mathrm{Ei}\left(dx\right)e^{c}+a^2de^{(dx+c)}-b^2xe^{(dx+c)}+a^2de^{(-dx-c)}+b^2}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^2*x*Ei(-d*x)*e^(-c) - a^2*d^2*x*Ei(d*x)*e^c - 2*a*b*d*x*Ei(-d*x
)*e^(-c) - 2*a*b*d*x*Ei(d*x)*e^c + a^2*d*e^(d*x + c) - b^2*x*e^(d*x + c) +
a^2*d*e^(-d*x - c) + b^2*x*e^(-d*x - c))/(d*x)
```

3.15 $\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=121

$$\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2abd\sinh(c)\mathrm{Chi}(dx)+2abd\,c$$

```
[Out] -(a^2*Cosh[c + d*x])/(2*x^2) - (2*a*b*Cosh[c + d*x])/x + b^2*Cosh[c]*CoshIn
tegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + 2*a*b*d*CoshIntegral[
d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(2*x) + 2*a*b*d*Cosh[c]*SinhIntegral[d
*x] + b^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] time = 0.338688, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\mathrm{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2abd\sinh(c)\mathrm{Chi}(dx)+2abd\,c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] -(a^2*Cosh[c + d*x])/(2*x^2) - (2*a*b*Cosh[c + d*x])/x + b^2*Cosh[c]*CoshIn
tegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + 2*a*b*d*CoshIntegral[
d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(2*x) + 2*a*b*d*Cosh[c]*SinhIntegral[d
*x] + b^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^3}+\frac{2ab\cosh(c+dx)}{x^2}+\frac{b^2\cosh(c+dx)}{x}\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int \frac{\cosh(c+dx)}{x}\,dx\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+\frac{1}{2}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^2}\,dx+(2abd)\int \frac{\sinh(c+}{x}\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\mathbf{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{2x}+b^2\sinh(\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\mathbf{Chi}(dx)+2abd\mathbf{Chi}(dx)\sinh(c)-\frac{a^2d}{}\\
&= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\mathbf{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\mathbf{Chi}(dx)+2a
\end{aligned}$$

Mathematica [A] time = 0.373391, size = 93, normalized size = 0.77

$$\frac{1}{2}\left(\mathbf{Chi}(dx)\left(\cosh(c)\left(a^2d^2+2b^2\right)+4abd\sinh(c)\right)+\mathbf{Shi}(dx)\left(\sinh(c)\left(a^2d^2+2b^2\right)+4abd\cosh(c)\right)-\frac{a((a+4bx)\cosh(c}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] (CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[c] + 4*a*b*d*Sinh[c]) - (a*((a +
 4*b*x)*Cosh[c + d*x] + a*d*x*Sinh[c + d*x]))/x^2 + (4*a*b*d*Cosh[c] + (2*b
^2 + a^2*d^2)*Sinh[c])*SinhIntegral[d*x])/2
```

Maple [A] time = 0.052, size = 181, normalized size = 1.5

$$-\frac{d^2a^2e^{-c}\mathrm{Ei}\,(1,dx)}{4}-\frac{abe^{-dx-c}}{x}+dabe^{-c}\mathrm{Ei}\,(1,dx)-\frac{b^2e^{-c}\mathrm{Ei}\,(1,dx)}{2}+\frac{da^2e^{-dx-c}}{4\,x}-\frac{a^2e^{-dx-c}}{4\,x^2}-\frac{abe^{dx+c}}{x}-dabe^{c}\mathrm{Ei}\,(1,-dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^3,x)
```

```
[Out] -1/4*d^2*a^2*exp(-c)*Ei(1,d*x)-a*b*exp(-d*x-c)/x+d*a*b*exp(-c)*Ei(1,d*x)-1/
2*b^2*exp(-c)*Ei(1,d*x)+1/4*d*a^2*exp(-d*x-c)/x-1/4*a^2*exp(-d*x-c)/x^2-a*b
/x*exp(d*x+c)-d*a*b*exp(c)*Ei(1,-d*x)-1/2*b^2*exp(c)*Ei(1,-d*x)-1/4*d^2*a^2
*exp(c)*Ei(1,-d*x)-1/4*a^2/x^2*exp(d*x+c)-1/4*d*a^2/x*exp(d*x+c)
```

Maxima [A] time = 1.37234, size = 170, normalized size = 1.4

$$\frac{1}{4}\left(\left(de^{(-c)}\Gamma\left(-1,dx\right)+de^{c}\Gamma\left(-1,-dx\right)\right)a^2-4\left(\mathrm{Ei}\left(-dx\right)e^{(-c)}-\mathrm{Ei}\left(dx\right)e^{c}\right)ab-\frac{4\,b^2\cosh\left(dx+c\right)\log\left(x\right)}{d}+\frac{2\left(\mathrm{Ei}\left(-dx\right)e^{(-c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - 4*(Ei(-d*x)*e^
(-c) - Ei(d*x)*e^c)*a*b - 4*b^2*cosh(d*x + c)*log(x)/d + 2*(Ei(-d*x)*e^(-c)
 + Ei(d*x)*e^c)*b^2/d)*d + 1/2*(2*b^2*log(x) - (4*a*b*x + a^2)/x^2)*cosh(d*
x + c)
```

Fricas [A] time = 1.94994, size = 351, normalized size = 2.9

$$-\frac{2\,a^2dx\sinh\left(dx+c\right)+2\left(4\,abx+a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^2+4\,abd+2\,b^2\right)x^2\mathrm{Ei}\left(dx\right)+\left(a^2d^2-4\,abd+2\,b^2\right)x^2\mathrm{Ei}\left(-\right.\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*sinh(d*x + c) + 2*(4*a*b*x + a^2)*cosh(d*x + c) - ((a^2*d^2
 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x))
*cosh(c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 - 4*a*b*d +
2*b^2)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**3, x)
```

Giac [A] time = 1.17773, size = 244, normalized size = 2.02

$$\frac{a^2d^2x^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+a^2d^2x^2\mathrm{Ei}\left(dx\right)e^{c}-4\,abdx^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+4\,abdx^2\mathrm{Ei}\left(dx\right)e^{c}+2\,b^2x^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+2\,b^2x^2\mathrm{Ei}\left(d\right.}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c - 4*a*b*d*x^2*Ei
(-d*x)*e^(-c) + 4*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) + 2*b^2
*x^2*Ei(d*x)*e^c - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x - c) - 4*a*b*x*e^(
d*x + c) - 4*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*e^(-d*x - c))/x^2
```

3.16 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} dx$

Optimal. Leaf size=172

$$\frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{a^2\cosh(c+dx)}{3x^3} + abd^2\cosh(c$$

```
[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (a*b*Cosh[c + d*x])/x^2 - (b^2*Cosh[c + d*x]
)/x - (a^2*d^2*Cosh[c + d*x])/(6*x) + a*b*d^2*Cosh[c]*CoshIntegral[d*x] + b
^2*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a
^2*d*Sinh[c + d*x])/(6*x^2) - (a*b*d*Sinh[c + d*x])/x + b^2*d*Cosh[c]*SinhI
ntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + a*b*d^2*Sinh[c]*Sinh
Integral[d*x]
```

Rubi [A] time = 0.42641, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{a^2\cosh(c+dx)}{3x^3} + abd^2\cosh(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```


```
[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (a*b*Cosh[c + d*x])/x^2 - (b^2*Cosh[c + d*x]
)/x - (a^2*d^2*Cosh[c + d*x])/(6*x) + a*b*d^2*Cosh[c]*CoshIntegral[d*x] + b
^2*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a
^2*d*Sinh[c + d*x])/(6*x^2) - (a*b*d*Sinh[c + d*x])/x + b^2*d*Cosh[c]*SinhI
ntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + a*b*d^2*Sinh[c]*Sinh
Integral[d*x]
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^4} + \frac{2ab\cosh(c+dx)}{x^3} + \frac{b^2\cosh(c+dx)}{x^2}\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx + (2ab)\int \frac{\cosh(c+dx)}{x^3}\,dx + b^2\int \frac{\cosh(c+dx)}{x^2}\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} + \frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^3}\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{abd\sinh}{x}$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d^2\cosh(c+dx)}{6x} + b^2d\mathrm{Chi}$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d^2\cosh(c+dx)}{6x} + abd^2\mathrm{co}$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{ab\cosh(c+dx)}{x^2} - \frac{b^2\cosh(c+dx)}{x} - \frac{a^2d^2\cosh(c+dx)}{6x} + abd^2\mathrm{co}$$

Mathematica [A] time = 0.44756, size = 154, normalized size = 0.9

$$-\frac{-dx^3\mathrm{Chi}(dx)\left(\sinh(c)\left(a^2d^2+6b^2\right)+6abd\cosh(c)\right) - dx^3\mathrm{Shi}(dx)\left(a^2d^2\cosh(c)+6abd\sinh(c)+6b^2\cosh(c)\right) + a^2d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(2*a^2*Cosh[c + d*x] + 6*a*b*x*Cosh[c + d*x] + 6*b^2*x^2*Cosh[c + d*x] + a
^2*d^2*x^2*Cosh[c + d*x] - d*x^3*CoshIntegral[d*x]*(6*a*b*d*Cosh[c] + (6*b^
2 + a^2*d^2)*Sinh[c]) + a^2*d*x*Sinh[c + d*x] + 6*a*b*d*x^2*Sinh[c + d*x] -
 d*x^3*(6*b^2*Cosh[c] + a^2*d^2*Cosh[c] + 6*a*b*d*Sinh[c])*SinhIntegral[d*x
])/(6*x^3)
```

Maple [A] time = 0.06, size = 287, normalized size = 1.7

$$\frac{d^3a^2e^{-c}\mathrm{Ei}\,(1,dx)}{12} - \frac{a^2d^2e^{-dx-c}}{12\,x} - \frac{a^2e^{-dx-c}}{6\,x^3} + \frac{da^2e^{-dx-c}}{12\,x^2} - \frac{b^2e^{-dx-c}}{2\,x} + \frac{bdae^{-dx-c}}{2\,x} - \frac{abe^{-dx-c}}{2\,x^2} - \frac{d^2abe^{-c}\mathrm{Ei}\,(1,dx)}{2} + \frac{db^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^4,x)
```

```
[Out] 1/12*d^3*a^2*exp(-c)*Ei(1,d*x)-1/12*d^2*a^2*exp(-d*x-c)/x-1/6*a^2*exp(-d*x-
c)/x^3+1/12*d*a^2*exp(-d*x-c)/x^2-1/2*b^2*exp(-d*x-c)/x+1/2*d*a*b*exp(-d*x-
c)/x-1/2*a*b*exp(-d*x-c)/x^2-1/2*d^2*a*b*exp(-c)*Ei(1,d*x)+1/2*d*b^2*exp(-c
)*Ei(1,d*x)-1/6*a^2/x^3*exp(d*x+c)-1/12*d*a^2/x^2*exp(d*x+c)-1/12*d^2*a^2/x
*exp(d*x+c)-1/2*d^2*a*b*exp(c)*Ei(1,-d*x)-1/2*a*b/x^2*exp(d*x+c)-1/2*d*a*b/
x*exp(d*x+c)-1/12*d^3*a^2*exp(c)*Ei(1,-d*x)-1/2*b^2/x*exp(d*x+c)-1/2*d*b^2*
```

```
exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.43103, size = 158, normalized size = 0.92

$$\frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma\left(-2,dx\right)-a^2d^2e^c\Gamma\left(-2,-dx\right)+3\,abde^{(-c)}\Gamma\left(-1,dx\right)+3\,abde^c\Gamma\left(-1,-dx\right)-3\,b^2\text{Ei}\left(-dx\right)e^{(-c)}+3\,b^2\text{Ei}\left(dx\right)e^c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) + 3*a*b*d*
e^(-c)*gamma(-1, d*x) + 3*a*b*d*e^c*gamma(-1, -d*x) - 3*b^2*Ei(-d*x)*e^(-c)
 + 3*b^2*Ei(d*x)*e^c)*d - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*cosh(d*x + c)/x^3
```

Fricas [A] time = 2.01559, size = 431, normalized size = 2.51

$$-\frac{2\left(6\,abx+\left(a^2d^2+6\,b^2\right)x^2+2\,a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^3+6\,abd^2+6\,b^2d\right)x^3\text{Ei}\left(dx\right)-\left(a^2d^3-6\,abd^2+6\,b^2d\right)x^3\text{Ei}\left(-dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(6*a*b*x + (a^2*d^2 + 6*b^2)*x^2 + 2*a^2)*cosh(d*x + c) - ((a^2*d^
3 + 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) - (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*
Ei(-d*x))*cosh(c) + 2*(6*a*b*d*x^2 + a^2*d*x)*sinh(d*x + c) - ((a^2*d^3 + 6
*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) + (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(-d
*x))*sinh(c))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**4, x)
```

Giac [A] time = 1.15162, size = 385, normalized size = 2.24

$$-\frac{a^2d^3x^3\text{Ei}\left(-dx\right)e^{(-c)}-a^2d^3x^3\text{Ei}\left(dx\right)e^c-6\,abd^2x^3\text{Ei}\left(-dx\right)e^{(-c)}-6\,abd^2x^3\text{Ei}\left(dx\right)e^c+6\,b^2dx^3\text{Ei}\left(-dx\right)e^{(-c)}-6\,b^2dx^3\text{Ei}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^3*x^3*Ei(-d*x)*e^(-c) - a^2*d^3*x^3*Ei(d*x)*e^c - 6*a*b*d^2*x^
3*Ei(-d*x)*e^(-c) - 6*a*b*d^2*x^3*Ei(d*x)*e^c + 6*b^2*d*x^3*Ei(-d*x)*e^(-c)
 - 6*b^2*d*x^3*Ei(d*x)*e^c + a^2*d^2*x^2*e^(d*x + c) + a^2*d^2*x^2*e^(-d*x
- c) + 6*a*b*d*x^2*e^(d*x + c) - 6*a*b*d*x^2*e^(-d*x - c) + a^2*d*x*e^(d*x
+ c) + 6*b^2*x^2*e^(d*x + c) - a^2*d*x*e^(-d*x - c) + 6*b^2*x^2*e^(-d*x - c
) + 6*a*b*x*e^(d*x + c) + 6*a*b*x*e^(-d*x - c) + 2*a^2*e^(d*x + c) + 2*a^2*
e^(-d*x - c))/x^3
```

3.17 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal. Leaf size=248

$$\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2\cosh}{4}$$

```
[Out] -(a^2*Cosh[c + d*x])/(4*x^4) - (2*a*b*Cosh[c + d*x])/(3*x^3) - (b^2*Cosh[c
+ d*x])/(2*x^2) - (a^2*d^2*Cosh[c + d*x])/(24*x^2) - (a*b*d^2*Cosh[c + d*x]
)/(3*x) + (b^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a^2*d^4*Cosh[c]*CoshInte
gral[d*x])/24 + (a*b*d^3*CoshIntegral[d*x]*Sinh[c])/3 - (a^2*d*Sinh[c + d*x
])/(12*x^3) - (a*b*d*Sinh[c + d*x])/(3*x^2) - (b^2*d*Sinh[c + d*x])/(2*x) -
 (a^2*d^3*Sinh[c + d*x])/(24*x) + (a*b*d^3*Cosh[c]*SinhIntegral[d*x])/3 + (
b^2*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/
24
```

Rubi [A] time = 0.520699, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2\cosh}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -(a^2*Cosh[c + d*x])/(4*x^4) - (2*a*b*Cosh[c + d*x])/(3*x^3) - (b^2*Cosh[c
+ d*x])/(2*x^2) - (a^2*d^2*Cosh[c + d*x])/(24*x^2) - (a*b*d^2*Cosh[c + d*x]
)/(3*x) + (b^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a^2*d^4*Cosh[c]*CoshInte
gral[d*x])/24 + (a*b*d^3*CoshIntegral[d*x]*Sinh[c])/3 - (a^2*d*Sinh[c + d*x
])/(12*x^3) - (a*b*d*Sinh[c + d*x])/(3*x^2) - (b^2*d*Sinh[c + d*x])/(2*x) -
 (a^2*d^3*Sinh[c + d*x])/(24*x) + (a*b*d^3*Cosh[c]*SinhIntegral[d*x])/3 + (
b^2*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/
24
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}dx = \int \left(\frac{a^2\cosh(c+dx)}{x^5} + \frac{2ab\cosh(c+dx)}{x^4} + \frac{b^2\cosh(c+dx)}{x^3}\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x^5}dx + (2ab)\int \frac{\cosh(c+dx)}{x^4}dx + b^2\int \frac{\cosh(c+dx)}{x^3}dx$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} + \frac{1}{4}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^4}dx$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{abd\sin}{3}$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{abd^2 c}{}$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{abd^2 c}{}$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{abd^2 c}{}$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{3x^3} - \frac{b^2\cosh(c+dx)}{2x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{abd^2 c}{}$$

Mathematica [A] time = 0.523918, size = 206, normalized size = 0.83

$$-\frac{-d^2x^4\text{Chi}(dx)\left(\cosh(c)\left(a^2d^2+12b^2\right)+8abd\sinh(c)\right)-d^2x^4\text{Shi}(dx)\left(a^2d^2\sinh(c)+8abd\cosh(c)+12b^2\sinh(c)\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -(6*a^2*Cosh[c + d*x] + 16*a*b*x*Cosh[c + d*x] + 12*b^2*x^2*Cosh[c + d*x] +
 a^2*d^2*x^2*Cosh[c + d*x] + 8*a*b*d^2*x^3*Cosh[c + d*x] - d^2*x^4*CoshInte
gral[d*x]*((12*b^2 + a^2*d^2)*Cosh[c] + 8*a*b*d*Sinh[c]) + 2*a^2*d*x*Sinh[c
 + d*x] + 8*a*b*d*x^2*Sinh[c + d*x] + 12*b^2*d*x^3*Sinh[c + d*x] + a^2*d^3*
x^3*Sinh[c + d*x] - d^2*x^4*(8*a*b*d*Cosh[c] + 12*b^2*Sinh[c] + a^2*d^2*Sin
h[c])*SinhIntegral[d*x])/(24*x^4)
```

Maple [A] time = 0.072, size = 396, normalized size = 1.6

$$-\frac{d^4a^2e^{-c}\text{Ei}(1,dx)}{48} + \frac{da^2e^{-dx-c}}{24x^3} - \frac{a^2e^{-dx-c}}{8x^4} + \frac{a^2d^3e^{-dx-c}}{48x} - \frac{a^2d^2e^{-dx-c}}{48x^2} + \frac{d^3abe^{-c}\text{Ei}(1,dx)}{6} - \frac{abd^2e^{-dx-c}}{6x} + \frac{bdae^{-dx-c}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*cosh(d*x+c)/x^5,x)
```

```
[Out] -1/48*d^4*a^2*exp(-c)*Ei(1,d*x)+1/24*d*a^2*exp(-d*x-c)/x^3-1/8*a^2*exp(-d*x
-c)/x^4+1/48*d^3*a^2*exp(-d*x-c)/x-1/48*d^2*a^2*exp(-d*x-c)/x^2+1/6*d^3*a*b
*exp(-c)*Ei(1,d*x)-1/6*d^2*a*b*exp(-d*x-c)/x+1/6*d*a*b*exp(-d*x-c)/x^2-1/3*
a*b*exp(-d*x-c)/x^3+1/4*d*b^2*exp(-d*x-c)/x-1/4*b^2*exp(-d*x-c)/x^2-1/4*d^2
*b^2*exp(-c)*Ei(1,d*x)-1/4*b^2/x^2*exp(d*x+c)-1/4*d*b^2/x*exp(d*x+c)-1/4*d^
2*b^2*exp(c)*Ei(1,-d*x)-1/48*d^4*a^2*exp(c)*Ei(1,-d*x)-1/8*a^2/x^4*exp(d*x+
c)-1/24*d*a^2/x^3*exp(d*x+c)-1/48*d^2*a^2/x^2*exp(d*x+c)-1/48*d^3*a^2/x*exp
(d*x+c)-1/6*d^2*a*b/x*exp(d*x+c)-1/6*d^3*a*b*exp(c)*Ei(1,-d*x)-1/3*a*b/x^3*
exp(d*x+c)-1/6*d*a*b/x^2*exp(d*x+c)
```

Maxima [A] time = 1.39841, size = 173, normalized size = 0.7

$$\frac{1}{24}\left(3\,a^2d^3e^{(-c)}\Gamma\left(-3,dx\right)+3\,a^2d^3e^c\Gamma\left(-3,-dx\right)+8\,abd^2e^{(-c)}\Gamma\left(-2,dx\right)-8\,abd^2e^c\Gamma\left(-2,-dx\right)+6\,b^2de^{(-c)}\Gamma\left(-1,dx\right)+6\,b^2d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/24*(3*a^2*d^3*e^(-c)*gamma(-3, d*x) + 3*a^2*d^3*e^c*gamma(-3, -d*x) + 8*a
*b*d^2*e^(-c)*gamma(-2, d*x) - 8*a*b*d^2*e^c*gamma(-2, -d*x) + 6*b^2*d*e^(-
c)*gamma(-1, d*x) + 6*b^2*d*e^c*gamma(-1, -d*x))*d - 1/12*(6*b^2*x^2 + 8*a*
b*x + 3*a^2)*cosh(d*x + c)/x^4
```

Fricas [A] time = 1.97258, size = 510, normalized size = 2.06

$$-\frac{2\left(8\,abd^2x^3+16\,abx+\left(a^2d^2+12\,b^2\right)x^2+6\,a^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^4+8\,abd^3+12\,b^2d^2\right)x^4\mathrm{Ei}\left(dx\right)+\left(a^2d^4-8\,abd^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(8*a*b*d^2*x^3 + 16*a*b*x + (a^2*d^2 + 12*b^2)*x^2 + 6*a^2)*cosh(d
*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(d*x) + (a^2*d^4 - 8*a*
b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*(8*a*b*d*x^2 + 2*a^2*d*x + (a
^2*d^3 + 12*b^2*d)*x^3)*sinh(d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)
*x^4*Ei(d*x) - (a^2*d^4 - 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*sinh(c))/x^
4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx)^2\cosh\left(c+dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x)**2*cosh(c + d*x)/x**5, x)
```

Giac [A] time = 1.11518, size = 533, normalized size = 2.15

$$\frac{a^2 d^4 x^4 \mathrm{Ei}\left(-dx\right) e^{(-c)} + a^2 d^4 x^4 \mathrm{Ei}\left(dx\right) e^c - 8\,abd^3 x^4 \mathrm{Ei}\left(-dx\right) e^{(-c)} + 8\,abd^3 x^4 \mathrm{Ei}\left(dx\right) e^c + 12\,b^2 d^2 x^4 \mathrm{Ei}\left(-dx\right) e^{(-c)} + 12\,b^2 d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c - 8*a*b*d^3*x^4
*Ei(-d*x)*e^(-c) + 8*a*b*d^3*x^4*Ei(d*x)*e^c + 12*b^2*d^2*x^4*Ei(-d*x)*e^(-
c) + 12*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) + a^2*d^3*x^3*e^(
-d*x - c) - 8*a*b*d^2*x^3*e^(d*x + c) - 8*a*b*d^2*x^3*e^(-d*x - c) - a^2*d^
2*x^2*e^(d*x + c) - 12*b^2*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c) + 1
2*b^2*d*x^3*e^(-d*x - c) - 8*a*b*d*x^2*e^(d*x + c) + 8*a*b*d*x^2*e^(-d*x -
c) - 2*a^2*d*x*e^(d*x + c) - 12*b^2*x^2*e^(d*x + c) + 2*a^2*d*x*e^(-d*x - c
) - 12*b^2*x^2*e^(-d*x - c) - 16*a*b*x*e^(d*x + c) - 16*a*b*x*e^(-d*x - c)
- 6*a^2*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4
```

3.18 $\int \frac{x^4 \cosh(c+dx)}{a+bx} dx$

Optimal. Leaf size=219

$$\frac{a^4 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{a^2 \cosh(c+dx)}{b^3 d^2} + \frac{a^4 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{a^3 \sinh(c+dx)}{b^4 d} + \frac{a^2 x \sinh(c+dx)}{b^3 d}$$

```
[Out] (-6*Cosh[c + d*x])/(b*d^4) - (a^2*Cosh[c + d*x])/(b^3*d^2) + (2*a*x*Cosh[c
+ d*x])/(b^2*d^2) - (3*x^2*Cosh[c + d*x])/(b*d^2) + (a^4*Cosh[c - (a*d)/b]*
CoshIntegral[(a*d)/b + d*x])/b^5 - (2*a*Sinh[c + d*x])/(b^2*d^3) - (a^3*Sin
h[c + d*x])/(b^4*d) + (6*x*Sinh[c + d*x])/(b*d^3) + (a^2*x*Sinh[c + d*x])/(
b^3*d) - (a*x^2*Sinh[c + d*x])/(b^2*d) + (x^3*Sinh[c + d*x])/(b*d) + (a^4*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^5
```

Rubi [A] time = 0.484027, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^4 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{a^2 \cosh(c+dx)}{b^3 d^2} + \frac{a^4 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{a^3 \sinh(c+dx)}{b^4 d} + \frac{a^2 x \sinh(c+dx)}{b^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x),x]
```


```
[Out] (-6*Cosh[c + d*x])/(b*d^4) - (a^2*Cosh[c + d*x])/(b^3*d^2) + (2*a*x*Cosh[c
+ d*x])/(b^2*d^2) - (3*x^2*Cosh[c + d*x])/(b*d^2) + (a^4*Cosh[c - (a*d)/b]*
CoshIntegral[(a*d)/b + d*x])/b^5 - (2*a*Sinh[c + d*x])/(b^2*d^3) - (a^3*Sin
h[c + d*x])/(b^4*d) + (6*x*Sinh[c + d*x])/(b*d^3) + (a^2*x*Sinh[c + d*x])/(
b^3*d) - (a*x^2*Sinh[c + d*x])/(b^2*d) + (x^3*Sinh[c + d*x])/(b*d) + (a^4*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^5
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = \int \left(-\frac{a^3\cosh(c+dx)}{b^4} + \frac{a^2x\cosh(c+dx)}{b^3} - \frac{ax^2\cosh(c+dx)}{b^2} + \frac{x^3\cosh(c+dx)}{b} + \frac{a^4\cosh(c}{b^4(a+}\right.$$

$$= -\frac{a^3\int\cosh(c+dx)\,dx}{b^4} + \frac{a^4\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^2\int x\cosh(c+dx)\,dx}{b^3} - \frac{a\int x^2\cosh(c+dx)\,d}{b^2}$$

$$= -\frac{a^3\sinh(c+dx)}{b^4d} + \frac{a^2x\sinh(c+dx)}{b^3d} - \frac{ax^2\sinh(c+dx)}{b^2d} + \frac{x^3\sinh(c+dx)}{bd} - \frac{a^2\int\sinh(c+}{b^3d}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5}$$

$$= -\frac{6\cosh(c+dx)}{bd^4} - \frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{a}{b}\right.}{}$$

Mathematica [A] time = 0.631353, size = 159, normalized size = 0.73

$$\frac{-b\left(d\left(-a^2bd^2x+a^3d^2+ab^2\left(d^2x^2+2\right)-b^3x\left(d^2x^2+6\right)\right)\sinh(c+dx)+b\left(a^2d^2-2abd^2x+3b^2\left(d^2x^2+2\right)\right)\cosh(c+}{b^5d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a^4*d^4*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - b*(b*(a^2*d^2 - 2*a*
b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x] + d*(a^3*d^2 - a^2*b*d^2*x + a
*b^2*(2 + d^2*x^2) - b^3*x*(6 + d^2*x^2))*Sinh[c + d*x]) + a^4*d^4*Sinh[c -
 (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^5*d^4)
```

Maple [A] time = 0.133, size = 442, normalized size = 2.

$$-\frac{a^4}{2b^5}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{3e^{-dx-c}x^2}{2bd^2} - \frac{e^{-dx-c}a^2}{2d^2b^3} - 3\frac{e^{-dx-c}}{bd^4} + \frac{e^{-dx-c}ax}{d^2b^2} - \frac{e^{-dx-c}x^3}{2bd} + \frac{e^{-dx-c}a^3}{2db^4} - 3\frac{e^{-dx-c}x}{d^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] -1/2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4-3/2/d^2*exp(-d*x-c)/b
*x^2-1/2/d^2*exp(-d*x-c)/b^3*a^2-3/d^4*exp(-d*x-c)/b+1/d^2*exp(-d*x-c)/b^2*
a*x-1/2/d*exp(-d*x-c)/b*x^3+1/2/d*exp(-d*x-c)/b^4*a^3-3/d^3*exp(-d*x-c)/b*x
+1/d^3*exp(-d*x-c)/b^2*a+1/2/d*exp(-d*x-c)/b^2*a*x^2-1/2/d*exp(-d*x-c)/b^3*
a^2*x-3/d^4/b*exp(d*x+c)-1/2/d/b^2*a*exp(d*x+c)*x^2+1/d^2/b^2*a*exp(d*x+c)*
x-1/2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4+1/2/d/b*exp(d*x+c)
*x^3-3/2/d^2/b*exp(d*x+c)*x^2+3/d^3/b*exp(d*x+c)*x-1/2/d/b^4*a^3*exp(d*x+c)
-1/d^3/b^2*a*exp(d*x+c)-1/2/d^2/b^3*a^2*exp(d*x+c)+1/2/d/b^3*a^2*exp(d*x+c)
*x
```

Maxima [A] time = 1.44775, size = 590, normalized size = 2.69

$$-\frac{1}{24}d\left(\frac{12\,a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^4d}-\frac{12\,a^3\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^4}+\frac{6\,a^2\left(\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+\cdots}{\cdots}\right)}{b^3}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*d*(12*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c -
 a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^4*d) - 12*a^3*((d*x*e^c - e
^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^4 + 6*a^2*((d^2*x^2*e^c - 2
*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3
 - 4*a*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^
3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b^2 + 3*((d^4*x^4*e^c - 4*
d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4
+ 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^(-d*x - c)/d^5)/b + 24*a^4*cosh(d
*x + c)*log(b*x + a)/(b^5*d)) + 1/12*(12*a^4*log(b*x + a)/b^5 + (3*b^3*x^4
- 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4)*cosh(d*x + c)
```

Fricas [A] time = 2.09321, size = 481, normalized size = 2.2

$$-\frac{2\left(3\,b^4d^2x^2-2\,ab^3d^2x+a^2b^2d^2+6\,b^4\right)\cosh\left(dx+c\right)-\left(a^4d^4\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^4d^4\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^4d^3x^3\cdots\right.}{2\,b^5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 + 6*b^4)*cosh(d*x + c)
 - (a^4*d^4*Ei((b*d*x + a*d)/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c
- a*d)/b) - 2*(b^4*d^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 - 2*a*b^3*d + (a^2*b
^2*d^3 + 6*b^4*d)*x)*sinh(d*x + c) + (a^4*d^4*Ei((b*d*x + a*d)/b) - a^4*d^4
*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x), x)
```

Giac [A] time = 1.14898, size = 84, normalized size = 0.38

$$\frac{a^4 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^4 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)}}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^4*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b))/b^5
```

3.19 $\int \frac{x^3 \cosh(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=150

$$-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2\sinh(c+dx)}{b^3d}+\frac{a\cosh(c+dx)}{b^2d^2}-\frac{ax\sinh(c+dx)}{b^2d}$$

```
[Out] (a*Cosh[c + d*x])/(b^2*d^2) - (2*x*Cosh[c + d*x])/(b*d^2) - (a^3*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 + (2*Sinh[c + d*x])/(b*d^3) + (a^2
*Sinh[c + d*x])/(b^3*d) - (a*x*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d*x])
/(b*d) - (a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rubi [A] time = 0.328539, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2\sinh(c+dx)}{b^3d}+\frac{a\cosh(c+dx)}{b^2d^2}-\frac{ax\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a*Cosh[c + d*x])/(b^2*d^2) - (2*x*Cosh[c + d*x])/(b*d^2) - (a^3*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 + (2*Sinh[c + d*x])/(b*d^3) + (a^2
*Sinh[c + d*x])/(b^3*d) - (a*x*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d*x])
/(b*d) - (a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{b^3} - \frac{ax\cosh(c+dx)}{b^2} + \frac{x^2\cosh(c+dx)}{b} - \frac{a^3\cosh(c+dx)}{b^3(a+bx)}\right) dx$$

$$= \frac{a^2\int\cosh(c+dx)\,dx}{b^3} - \frac{a^3\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^3} - \frac{a\int x\cosh(c+dx)\,dx}{b^2} + \frac{\int x^2\cosh(c+dx)\,dx}{b}$$

$$= \frac{a^2\sinh(c+dx)}{b^3d} - \frac{ax\sinh(c+dx)}{b^2d} + \frac{x^2\sinh(c+dx)}{bd} + \frac{a\int\sinh(c+dx)\,dx}{b^2d} - \frac{2\int x\sinh(c+\ldots}{bd}$$

$$= \frac{a\cosh(c+dx)}{b^2d^2} - \frac{2x\cosh(c+dx)}{bd^2} - \frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{a^2\sinh(c+dx)}{b^3d} - \frac{ax\ldots}{\ldots}$$

$$= \frac{a\cosh(c+dx)}{b^2d^2} - \frac{2x\cosh(c+dx)}{bd^2} - \frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{2\sinh(c+dx)}{bd^3} + \frac{a^2\ldots}{\ldots}$$

Mathematica [A] time = 0.430323, size = 118, normalized size = 0.79

$$\frac{b\left(\left(a^2d^2-abd^2x+b^2\left(d^2x^2+2\right)\right)\sinh(c+dx)+bd(a-2bx)\cosh(c+dx)\right)-a^3d^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)-a^3\ldots}{b^4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-(a^3*d^3*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*(b*d*(a - 2*b*x
)*Cosh[c + d*x] + (a^2*d^2 - a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])
- a^3*d^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^4*d^3)
```

Maple [A] time = 0.034, size = 292, normalized size = 2.

$$-\frac{e^{-dx-c}x^2}{2\,bd} + \frac{e^{-dx-c}ax}{2\,db^2} - \frac{e^{-dx-c}a^2}{2\,db^3} - \frac{e^{-dx-c}x}{bd^2} + \frac{e^{-dx-c}a}{2\,d^2b^2} - \frac{e^{-dx-c}}{d^3b} + \frac{a^3}{2\,b^4}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{e^{dx+c}x}{bd^2} + \frac{e^{dx+c}\ldots}{2\,b\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x^2+1/2/d*exp(-d*x-c)/b^2*a*x-1/2/d*exp(-d*x-c)/b^3*a^
2-1/d^2*exp(-d*x-c)/b*x+1/2/d^2*exp(-d*x-c)/b^2*a-1/d^3*exp(-d*x-c)/b+1/2/b
^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3-1/d^2/b*exp(d*x+c)*x+1/2/d/
```

```
b*exp(d*x+c)*x^2+1/d^3/b*exp(d*x+c)+1/2/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(
a*d-b*c)/b)*a^3-1/2/d/b^2*a*exp(d*x+c)*x+1/2/d^2/b^2*a*exp(d*x+c)+1/2/d/b^3
*a^2*exp(d*x+c)
```

Maxima [B] time = 1.27553, size = 443, normalized size = 2.95

$$\frac{1}{12}d\left(\frac{6\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^3d}-\frac{6\,a^2\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^3}+\frac{3\,a\left(\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2}{d^3}\right.}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*d*(6*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^3*d) - 6*a^2*((d*x*e^c - e^c)
*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^3 + 3*a*((d^2*x^2*e^c - 2*d*x*
e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^2 - 2*
((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 +
 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b + 12*a^3*cosh(d*x + c)*log(b*x
+ a)/(b^4*d)) - 1/6*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^
2*x)/b^3)*cosh(d*x + c)
```

Fricas [A] time = 2.09382, size = 392, normalized size = 2.61

$$-\frac{2\left(2\,b^3dx-ab^2d\right)\cosh\left(dx+c\right)+\left(a^3d^3\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^3d^3\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^3d^2x^2-ab^2d^2x+a^2bd^2+2\right.}{2\,b^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b^3*d*x - a*b^2*d)*cosh(d*x + c) + (a^3*d^3*Ei((b*d*x + a*d)/b)
+ a^3*d^3*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*d^2*x^2 - a*b
^2*d^2*x + a^2*b*d^2 + 2*b^3)*sinh(d*x + c) - (a^3*d^3*Ei((b*d*x + a*d)/b)
- a^3*d^3*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\cosh\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x), x)
```

Giac [A] time = 1.1756, size = 84, normalized size = 0.56

$$-\frac{a^3\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^3\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^3*Ei(-(b*d*x + a*d)/b)*e^(-
c + a*d/b))/b^4
```

3.20 $\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx$

Optimal. Leaf size=100

$$\frac{a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{a \sinh(c+dx)}{b^2 d}-\frac{\cosh(c+dx)}{bd^2}+\frac{x \sinh(c+dx)}{bd}$$

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + (a^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*
x])/b^3 - (a*Sinh[c + d*x])/(b^2*d) + (x*Sinh[c + d*x])/(b*d) + (a^2*Sinh[c
 - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rubi [A] time = 0.26226, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{a \sinh(c+dx)}{b^2 d}-\frac{\cosh(c+dx)}{bd^2}+\frac{x \sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + (a^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*
x])/b^3 - (a*Sinh[c + d*x])/(b^2*d) + (x*Sinh[c + d*x])/(b*d) + (a^2*Sinh[c
 - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx &= \int \left(-\frac{a\cosh(c+dx)}{b^2}+\frac{x\cosh(c+dx)}{b}+\frac{a^2\cosh(c+dx)}{b^2(a+bx)}\right)dx\\ &= -\frac{a\int\cosh(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^2}+\frac{\int x\cosh(c+dx)\,dx}{b}\\ &= -\frac{a\sinh(c+dx)}{b^2d}+\frac{x\sinh(c+dx)}{bd}-\frac{\int\sinh(c+dx)\,dx}{bd}+\frac{\left(a^2\cosh\left(c-\frac{ad}{b}\right)\right)\int\frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}\\ &= -\frac{\cosh(c+dx)}{bd^2}+\frac{a^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x\sinh(c+dx)}{bd}+\frac{a^2\sin}{}\end{aligned}$$

Mathematica [A] time = 0.283274, size = 89, normalized size = 0.89

$$\frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+a^2d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)+b(d(bx-a)\sinh(c+dx)-b\cosh(c+dx))}{b^3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (a^2*d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + b*(-(b*Cosh[c + d*x]
) + d*(-a + b*x)*Sinh[c + d*x]) + a^2*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*
(a/b + x)])/(b^3*d^2)
```

Maple [A] time = 0.03, size = 184, normalized size = 1.8

$$-\frac{e^{-dx-c}x}{2\,bd}+\frac{e^{-dx-c}a}{2\,db^2}-\frac{e^{-dx-c}}{2\,bd^2}-\frac{a^2}{2\,b^3}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)+\frac{e^{dx+c}x}{2\,bd}-\frac{e^{dx+c}}{2\,bd^2}-\frac{ae^{dx+c}}{2\,db^2}-\frac{a^2}{2\,b^3}e^{-\frac{da-cb}{b}}\mathrm{Ei}\left(1,-dx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x+1/2/d*exp(-d*x-c)/b^2*a-1/2/d^2*exp(-d*x-c)/b-1/2/b^
3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+1/2/d/b*exp(d*x+c)*x-1/2/d^2
/b*exp(d*x+c)-1/2/d/b^2*a*exp(d*x+c)-1/2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-
(a*d-b*c)/b)*a^2
```

Maxima [B] time = 1.37828, size = 315, normalized size = 3.15

$$-\frac{1}{4}d\left(\frac{2a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d}-\frac{2a\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^2}+\frac{\frac{(d^2x^2e^c-2dxe^c+2e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2dx+2)e^{(-dx}}{d^3}}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*d*(2*a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^2*d) - 2*a*((d*x*e^c - e^c)*e
^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2 + ((d^2*x^2*e^c - 2*d*x*e^c +
2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b + 4*a^2*cosh
(d*x + c)*log(b*x + a)/(b^3*d)) + 1/2*(2*a^2*log(b*x + a)/b^3 + (b*x^2 - 2*
a*x)/b^2)*cosh(d*x + c)
```

Fricas [A] time = 2.15575, size = 327, normalized size = 3.27

$$-\frac{2b^2\cosh(dx+c)-\left(a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\left(b^2dx-abd\right)\sinh(dx+c)+\left(a^2d^2\mathrm{Ei}\left(\frac{bdx+}{b}\right.\right.}{2b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei((b*d*x + a*d)/b) + a^2*d^2*Ei(-(b*d
*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^2*d*x - a*b*d)*sinh(d*x + c) + (a
^2*d^2*Ei((b*d*x + a*d)/b) - a^2*d^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d
)/b))/(b^3*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\cosh(c+dx)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x), x)
```

Giac [A] time = 1.20668, size = 84, normalized size = 0.84

$$\frac{a^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b))/b^3
```

3.21 $\int \frac{x\cosh(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=68

$$-\frac{a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh(c+dx)}{bd}$$

```
[Out] -((a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^2) + Sinh[c + d*x]/(b
*d) - (a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rubi [A] time = 0.176147, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 2637, 3303, 3298, 3301}

$$-\frac{a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] -((a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^2) + Sinh[c + d*x]/(b
*d) - (a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cosh(c+dx)}{a+bx}\,dx &= \int\left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b(a+bx)}\right)dx \\
&= \frac{\int \cosh(c+dx)\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b} \\
&= \frac{\sinh(c+dx)}{bd}-\frac{\left(a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}-\frac{\left(a\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} \\
&= -\frac{a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^2}+\frac{\sinh(c+dx)}{bd}-\frac{a\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}
\end{aligned}
$$

Mathematica [A] time = 0.135784, size = 64, normalized size = 0.94

$$
\frac{-ad\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right)-ad\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right)+b\sinh(c+dx)}{b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x),x]
```

```
[Out] (-(a*d*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*Sinh[c + d*x] - a*d
*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^2*d)
```

Maple [A] time = 0.023, size = 114, normalized size = 1.7

$$
-\frac{e^{-dx-c}}{2bd}+\frac{a}{2b^2}e^{\frac{da-cb}{b}}\operatorname{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)+\frac{e^{dx+c}}{2bd}+\frac{a}{2b^2}e^{-\frac{da-cb}{b}}\operatorname{Ei}\left(1,-dx-c-\frac{da-cb}{b}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b+1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/2
/d/b*exp(d*x+c)+1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a
```

Maxima [B] time = 1.33287, size = 211, normalized size = 3.1

$$
\frac{1}{2}d\left(\frac{a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{bd}-\frac{\frac{(dxe^{c}-e^{c})e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}}{b}+\frac{2a\cosh(dx+c)\log(bx+a)}{b^2d}\right)+\left(\frac{x}{b}-\frac{a\log
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)
*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2
```

```
+ (d*x + 1)*e^(-d*x - c)/d^2)/b + 2*a*cosh(d*x + c)*log(b*x + a)/(b^2*d)) +
 (x/b - a*log(b*x + a)/b^2)*cosh(d*x + c)
```

Fricas [A] time = 1.9967, size = 251, normalized size = 3.69

$$-\frac{\left(ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\,b\sinh\left(dx+c\right)-\left(ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*Ei((b*d*x + a*d)/b) + a*d*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - 2*b*sinh(d*x + c) - (a*d*Ei((b*d*x + a*d)/b) - a*d*Ei(-(b*d*x + a*d)
/b))*sinh(-(b*c - a*d)/b))/(b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(c+dx\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x), x)
```

Giac [A] time = 1.18626, size = 78, normalized size = 1.15

$$-\frac{a\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*Ei(-(b*d*x + a*d)/b)*e^(-c +
a*d/b))/b^2
```

3.22 $\int \frac{\cosh(c+dx)}{a+bx}\,dx$

Optimal. Leaf size=51

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}$$

```
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b
```

Rubi [A] time = 0.0795504, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3303, 3298, 3301}

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x),x]
```

```
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+bx}\,dx &= \cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx+\sinh\left(c-\frac{ad}{b}\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx\\ &= \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0611119, size = 49, normalized size = 0.96

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)+\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x),x]
```

```
[Out] (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x] + Sinh[c - (a*d)/b]*SinhInte
gral[(a*d)/b + d*x])/b
```

Maple [A] time = 0.019, size = 81, normalized size = 1.6

$$-\frac{1}{2b}\mathrm{e}^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)-\frac{1}{2b}\mathrm{e}^{-\frac{da-cb}{b}}\mathrm{Ei}\left(1,-dx-c-\frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x+a),x)
```

```
[Out] -1/2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b*exp(-(a*d-b*c)/b)*Ei(
1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.31556, size = 77, normalized size = 1.51

$$-\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{2b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - 1/2*e^(c - a*d/b)*
exp_integral_e(1, -(b*x + a)*d/b)/b
```

Fricas [A] time = 2.08618, size = 193, normalized size = 3.78

$$\frac{\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - (E
i((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*x), x)
```

Giac [A] time = 1.17093, size = 76, normalized size = 1.49

$$\frac{\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
))/b
```

3.23 $\int \frac{\cosh(c+dx)}{x(a+bx)} \, dx$

Optimal. Leaf size=73

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\cosh(c)\text{Chi}(dx)}{a}+\frac{\sinh(c)\text{Shi}(dx)}{a}$$

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d
*x])/a + (Sinh[c]*SinhIntegral[d*x])/a - (Sinh[c - (a*d)/b]*SinhIntegral[(a
*d)/b + d*x])/a
```

Rubi [A] time = 0.258935, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6742, 3303, 3298, 3301}

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a}+\frac{\cosh(c)\text{Chi}(dx)}{a}+\frac{\sinh(c)\text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d
*x])/a + (Sinh[c]*SinhIntegral[d*x])/a - (Sinh[c - (a*d)/b]*SinhIntegral[(a
*d)/b + d*x])/a
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c+dx)}{a(a+bx)}\right)dx$$
$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a}$$
$$= \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a} - \frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a} - \frac{\left(b\sinh\left(c-\right.\right.}{}$$
$$= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right.}{a}$$

Mathematica [A] time = 0.140752, size = 63, normalized size = 0.86

$$\frac{-\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)-\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)+\cosh(c)\text{Chi}(dx)+\sinh(c)\text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x] - Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] +
Sinh[c]*SinhIntegral[d*x] - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a
```

Maple [A] time = 0.031, size = 108, normalized size = 1.5

$$-\frac{e^{-c}\text{Ei}(1,dx)}{2a}+\frac{1}{2a}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)-\frac{e^{c}\text{Ei}(1,-dx)}{2a}+\frac{1}{2a}e^{-\frac{da-cb}{b}}\text{Ei}\left(1,-dx-c-\frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x+a),x)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/2/a*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2
/a*exp(c)*Ei(1,-d*x)+1/2/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [B] time = 1.43734, size = 209, normalized size = 2.86

$$\frac{1}{2}d\left(\frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{ad}+\frac{2\cosh(dx+c)\log(bx+a)}{ad}-\frac{2\cosh(dx+c)\log(x)}{ad}+\frac{\text{Ei}(-dx)e^{(-c)}+}{ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*d*(b*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)
*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a*d) + 2*cosh(d*x + c)*log(b*x + a)/
```

```
(a*d) - 2*cosh(d*x + c)*log(x)/(a*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a*d
)) - (log(b*x + a)/a - log(x)/a)*cosh(d*x + c)
```

Fricas [A] time = 2.06579, size = 277, normalized size = 3.79

$$\frac{(\mathrm{Ei}\,(dx) + \mathrm{Ei}\,(-dx))\cosh\,(c) - \left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) + \mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right) + (\mathrm{Ei}\,(dx) - \mathrm{Ei}\,(-dx))\sinh\,(c) + \left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)\right.}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei(d*x) + Ei(-d*x))*cosh(c) - (Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d
)/b))*cosh(-(b*c - a*d)/b) + (Ei(d*x) - Ei(-d*x))*sinh(c) + (Ei((b*d*x + a*
d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\,(c + dx)}{x\,(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)), x)
```

Giac [A] time = 1.15917, size = 101, normalized size = 1.38

$$\frac{\mathrm{Ei}\,(-dx)\,e^{(-c)} - \mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + \mathrm{Ei}\,(dx)\,e^{c} - \mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(Ei(-d*x)*e^(-c) - Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(d*x)*e^c - Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/a
```

3.24 $\int \frac{\cosh(c+dx)}{x^2(a+bx)} \, dx$

Optimal. Leaf size=113

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\sinh(c)\text{C}}{a}$$

```
[Out] -(Cosh[c + d*x]/(a*x)) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (b*Cosh[c - (a
*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 + (d*CoshIntegral[d*x]*Sinh[c])/a +
 (d*Cosh[c]*SinhIntegral[d*x])/a - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (b*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^2
```

Rubi [A] time = 0.368679, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\sinh(c)\text{C}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (b*Cosh[c - (a
*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 + (d*CoshIntegral[d*x]*Sinh[c])/a +
 (d*Cosh[c]*SinhIntegral[d*x])/a - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (b*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a} - \frac{(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx}{a^2} + \frac{\left(b^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} + \frac{b\sinh\left(\right.}{}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{d\mathrm{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh}{}$$

Mathematica [A] time = 0.380891, size = 101, normalized size = 0.89

$$\frac{bx\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + \mathrm{Chi}(dx)(adx\sinh(c) - bx\cosh(c)) + bx\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + adx\cosh(c)\mathrm{Sh}}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

```
[Out] (-(a*Cosh[c + d*x]) + b*x*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + Cos
hIntegral[d*x]*(-(b*x*Cosh[c]) + a*d*x*Sinh[c]) + a*d*x*Cosh[c]*SinhIntegra
l[d*x] - b*x*Sinh[c]*SinhIntegral[d*x] + b*x*Sinh[c - (a*d)/b]*SinhIntegral
[d*(a/b + x)])/(a^2*x)
```

Maple [A] time = 0.048, size = 172, normalized size = 1.5

$$-\frac{e^{-dx-c}}{2ax} + \frac{de^{-c}\mathrm{Ei}(1,dx)}{2a} + \frac{be^{-c}\mathrm{Ei}(1,dx)}{2a^2} - \frac{b}{2a^2}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) + \frac{be^{c}\mathrm{Ei}(1,-dx)}{2a^2} - \frac{e^{dx+c}}{2ax} - \frac{de^{c}\mathrm{Ei}(1,-dx)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x+a),x)
```

```
[Out] -1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/2/a^2*exp(-c)*Ei(1,d*x)*b-
1/2*b/a^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*b/a^2*exp(c)*Ei(1,-d
*x)-1/2/a/x*exp(d*x+c)-1/2*d/a*exp(c)*Ei(1,-d*x)-1/2/a^2*b*exp(-(a*d-b*c)/b
)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.45567, size = 259, normalized size = 2.29

$$-\frac{1}{2}d\left(\frac{\mathrm{Ei}\left(-dx\right)e^{(-c)}-\mathrm{Ei}\left(dx\right)e^{c}}{a}+\frac{b^{2}\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_{1}\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_{1}\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^{2}d}+\frac{2\,b\cosh\left(dx+c\right)\log\left(bx+a\right)}{a^{2}d}-\frac{2\,b\cosh}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*d*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)/a + b^2*(e^(-c + a*d/b)*exp_integra
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(a^2*d) + 2*b*cosh(d*x + c)*log(b*x + a)/(a^2*d) - 2*b*cosh(d*x + c)*log(
x)/(a^2*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/(a^2*d)) + (b*log(b*x + a)/a
^2 - b*log(x)/a^2 - 1/(a*x))*cosh(d*x + c)
```

Fricas [A] time = 2.04736, size = 400, normalized size = 3.54

$$-\frac{2\,a\cosh\left(dx+c\right)-\left(\left(ad-b\right)x\mathrm{Ei}\left(dx\right)-\left(ad+b\right)x\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(bx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+bx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right.}{2\,a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c) - ((a*d - b)*x*Ei(d*x) - (a*d + b)*x*Ei(-d*x))*cosh
(c) - (b*x*Ei((b*d*x + a*d)/b) + b*x*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - ((a*d - b)*x*Ei(d*x) + (a*d + b)*x*Ei(-d*x))*sinh(c) + (b*x*Ei((b*d*
x + a*d)/b) - b*x*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(c+dx\right)}{x^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x)), x)
```

Giac [A] time = 1.17191, size = 174, normalized size = 1.54

$$-\frac{adx\mathrm{Ei}\left(-dx\right)e^{(-c)}-adx\mathrm{Ei}\left(dx\right)e^{c}+bx\mathrm{Ei}\left(-dx\right)e^{(-c)}-bx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+bx\mathrm{Ei}\left(dx\right)e^{c}-bx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+}{2\,a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c + b*x*Ei(-d*x)*e^(-c) - b*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b*x*Ei(d*x)*e^c - b*x*Ei(-(b*d*x + a*d
)/b)*e^(-c + a*d/b) + a*e^(d*x + c) + a*e^(-d*x - c))/(a^2*x)
```

3.25 $\int \frac{\cosh(c+dx)}{x^3(a+bx)} dx$

Optimal. Leaf size=190

$$\frac{b^2\cosh(c)\text{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{b^2\sinh(c)\text{Shi}(dx)}{a^3} - \frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{bd\sinh}{}$$

```
[Out] -Cosh[c + d*x]/(2*a*x^2) + (b*Cosh[c + d*x])/(a^2*x) + (b^2*Cosh[c]*CoshInt
egral[d*x])/a^3 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) - (b^2*Cosh[c - (a*
d)/b]*CoshIntegral[(a*d)/b + d*x])/a^3 - (b*d*CoshIntegral[d*x]*Sinh[c])/a^
2 - (d*Sinh[c + d*x])/(2*a*x) - (b*d*Cosh[c]*SinhIntegral[d*x])/a^2 + (b^2*
Sinh[c]*SinhIntegral[d*x])/a^3 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a) - (b
^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Rubi [A] time = 0.485121, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{b^2\cosh(c)\text{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3} + \frac{b^2\sinh(c)\text{Shi}(dx)}{a^3} - \frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3} - \frac{bd\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```


```
[Out] -Cosh[c + d*x]/(2*a*x^2) + (b*Cosh[c + d*x])/(a^2*x) + (b^2*Cosh[c]*CoshInt
egral[d*x])/a^3 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) - (b^2*Cosh[c - (a*
d)/b]*CoshIntegral[(a*d)/b + d*x])/a^3 - (b*d*CoshIntegral[d*x]*Sinh[c])/a^
2 - (d*Sinh[c + d*x])/(2*a*x) - (b*d*Cosh[c]*SinhIntegral[d*x])/a^2 + (b^2*
Sinh[c]*SinhIntegral[d*x])/a^3 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a) - (b
^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a^2x^2} + \frac{b^2\cosh(c+dx)}{a^3x} - \frac{b^3\cosh(c+dx)}{a^3(a+bx)}\right)dx$$
$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b^3\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3}$$
$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a} - \frac{(bd)\int \frac{\sinh(c+dx)}{x}\,dx}{a^2} + \frac{\left(b^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}$$
$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d\sinh(}{2a}$$
$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{bd\mathrm{Chi}(}{}$$
$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} - \frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Ch}}{a^3}$$

Mathematica [A] time = 0.487479, size = 178, normalized size = 0.94

$$x^2\mathrm{Chi}(dx)\left(\cosh(c)\left(a^2d^2+2b^2\right)-2abd\sinh(c)\right)+a^2d^2x^2\sinh(c)\mathrm{Shi}(dx)-a^2dx\sinh(c+dx)+a^2(-\cosh(c+dx))-2b$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

```
[Out] (-(a^2*Cosh[c + d*x]) + 2*a*b*x*Cosh[c + d*x] - 2*b^2*x^2*Cosh[c - (a*d)/b]
*CoshIntegral[d*(a/b + x)] + x^2*CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[
c] - 2*a*b*d*Sinh[c]) - a^2*d*x*Sinh[c + d*x] - 2*a*b*d*x^2*Cosh[c]*SinhInt
egral[d*x] + 2*b^2*x^2*Sinh[c]*SinhIntegral[d*x] + a^2*d^2*x^2*Sinh[c]*Sinh
Integral[d*x] - 2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(2*a
^3*x^2)
```

Maple [A] time = 0.05, size = 281, normalized size = 1.5

$$\frac{de^{-dx-c}}{4\,ax} + \frac{e^{-dx-c}b}{2\,a^2x} - \frac{e^{-dx-c}}{4\,ax^2} - \frac{d^2e^{-c}\mathrm{Ei}\,(1,dx)}{4\,a} - \frac{dbe^{-c}\mathrm{Ei}\,(1,dx)}{2\,a^2} - \frac{b^2e^{-c}\mathrm{Ei}\,(1,dx)}{2\,a^3} + \frac{b^2}{2\,a^3}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x+a),x)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x+1/2*exp(-d*x-c)/a^2/x*b-1/4*exp(-d*x-c)/a/x^2-1/4*d^2
/a*exp(-c)*Ei(1,d*x)-1/2*d/a^2*exp(-c)*Ei(1,d*x)*b-1/2/a^3*exp(-c)*Ei(1,d*x
```

```
)*b^2+1/2*b^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2*b^2/a^3*exp(
c)*Ei(1,-d*x)-1/4/a/x^2*exp(d*x+c)-1/4*d/a/x*exp(d*x+c)-1/4*d^2/a*exp(c)*Ei
(1,-d*x)+1/2/a^2*b/x*exp(d*x+c)+1/2*d/a^2*b*exp(c)*Ei(1,-d*x)+1/2/a^3*b^2*e
xp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.62637, size = 327, normalized size = 1.72

$$\frac{1}{4}d\left(\frac{de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)}{a}+\frac{2\left(\text{Ei}(-dx)e^{(-c)}-\text{Ei}(dx)e^{c}\right)b}{a^2}+\frac{2b^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^3d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*d*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))/a + 2*(Ei(-d*x)*e^
(-c) - Ei(d*x)*e^c)*b/a^2 + 2*b^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x +
a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a^3*d) + 4*
b^2*cosh(d*x + c)*log(b*x + a)/(a^3*d) - 4*b^2*cosh(d*x + c)*log(x)/(a^3*d)
 + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/(a^3*d)) - 1/2*(2*b^2*log(b*x + a)
/a^3 - 2*b^2*log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*cosh(d*x + c)
```

Fricas [A] time = 2.08548, size = 591, normalized size = 3.11

$$-\frac{2a^2dx\sinh(dx+c)-2\left(2abx-a^2\right)\cosh(dx+c)-\left(\left(a^2d^2-2abd+2b^2\right)x^2\text{Ei}(dx)+\left(a^2d^2+2abd+2b^2\right)x^2\text{Ei}(-d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*d*x*sinh(d*x + c) - 2*(2*a*b*x - a^2)*cosh(d*x + c) - ((a^2*d^2
 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))
*cosh(c) + 2*(b^2*x^2*Ei((b*d*x + a*d)/b) + b^2*x^2*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 +
 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))*sinh(c) - 2*(b^2*x^2*Ei((b*d*x + a*d)/b) -
b^2*x^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x)), x)
```

Giac [A] time = 1.20454, size = 335, normalized size = 1.76

$$\frac{a^2d^2x^2\text{Ei}(-dx)e^{(-c)} + a^2d^2x^2\text{Ei}(dx)e^c + 2\,abdx^2\text{Ei}(-dx)e^{(-c)} - 2\,abdx^2\text{Ei}(dx)e^c + 2\,b^2x^2\text{Ei}(-dx)e^{(-c)} - 2\,b^2x^2\text{Ei}\left(\frac{bdx+a}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c + 2*a*b*d*x^2*Ei
(-d*x)*e^(-c) - 2*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) - 2*b^2
*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^2*x^2*Ei(d*x)*e^c - 2*b^2*x^2*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x
 - c) + 2*a*b*x*e^(d*x + c) + 2*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*
e^(-d*x - c))/(a^3*x^2)
```

3.26 $\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal. Leaf size=231

$$\frac{a^4 d \sinh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{4a^3 \cosh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{4a^3 \sinh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^4 d \cosh\left(c-\right.}{}$$

```
[Out] (2*a*Cosh[c + d*x])/(b^3*d^2) - (2*x*Cosh[c + d*x])/(b^2*d^2) - (a^4*Cosh[c
 + d*x])/(b^5*(a + b*x)) - (4*a^3*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b +
d*x])/b^5 + (a^4*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^6 + (2*
Sinh[c + d*x])/(b^2*d^3) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (2*a*x*Sinh[c +
d*x])/(b^3*d) + (x^2*Sinh[c + d*x])/(b^2*d) + (a^4*d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b^6 - (4*a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/
b + d*x])/b^5
```

Rubi [A] time = 0.536144, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6742, 2637, 3296, 2638, 3297, 3303, 3298, 3301}

$$\frac{a^4 d \sinh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^6} - \frac{4a^3 \cosh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^5} - \frac{4a^3 \sinh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^5} + \frac{a^4 d \cosh\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```


```
[Out] (2*a*Cosh[c + d*x])/(b^3*d^2) - (2*x*Cosh[c + d*x])/(b^2*d^2) - (a^4*Cosh[c
 + d*x])/(b^5*(a + b*x)) - (4*a^3*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b +
d*x])/b^5 + (a^4*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^6 + (2*
Sinh[c + d*x])/(b^2*d^3) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (2*a*x*Sinh[c +
d*x])/(b^3*d) + (x^2*Sinh[c + d*x])/(b^2*d) + (a^4*d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/b^6 - (4*a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/
b + d*x])/b^5
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{3a^2\cosh(c+dx)}{b^4} - \frac{2ax\cosh(c+dx)}{b^3} + \frac{x^2\cosh(c+dx)}{b^2} + \frac{a^4\cosh(c+dx)}{b^4(a+bx)^2} - \frac{4a^3\cosh(c+ \cdots}{b^4(a+bx)}\right.$$

$$= \frac{(3a^2)\int\cosh(c+dx)\,dx}{b^4} - \frac{(4a^3)\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^4} + \frac{a^4\int\frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^4} - \frac{(2a)\int x\cosh(c+dx)\,dx}{b^3}$$

$$= -\frac{a^4\cosh(c+dx)}{b^5(a+bx)} + \frac{3a^2\sinh(c+dx)}{b^4d} - \frac{2ax\sinh(c+dx)}{b^3d} + \frac{x^2\sinh(c+dx)}{b^2d} + \frac{(2a)\int\sinh(c+d \cdots}{b^3d}$$

$$= \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{3a^2 \cdots}{}$$

$$= \frac{2a\cosh(c+dx)}{b^3d^2} - \frac{2x\cosh(c+dx)}{b^2d^2} - \frac{a^4\cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} + \frac{a^4d \cdots}{}$$

Mathematica [A] time = 1.20078, size = 173, normalized size = 0.75

$$\frac{\frac{b^2\left(3a^2d^2-2abd^2x+b^2\left(d^2x^2+2\right)\right)\sinh(c+dx)}{d^3} - \frac{b\left(-2a^2b^2+a^4d^2+2b^4x^2\right)\cosh(c+dx)}{d^2(a+bx)} + a^3\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\sinh\left(c-\frac{ad}{b}\right) - 4b\cosh\left(c-\frac{ad}{b}\right)\right) \cdots}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (-((b*(-2*a^2*b^2 + a^4*d^2 + 2*b^4*x^2)*Cosh[c + d*x])/(d^2*(a + b*x))) +
a^3*CoshIntegral[d*(a/b + x)]*(-4*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/
b]) + (b^2*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
 + a^3*(a*d*Cosh[c - (a*d)/b] - 4*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b
```

```
+ x)])/b^6
```

Maple [A] time = 0.122, size = 431, normalized size = 1.9

$$-\frac{e^{-dx-c}}{d^3b^2}+\frac{da^4}{2b^6}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)+2\frac{a^3}{b^5}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)-\frac{e^{-dx-c}x}{b^2d^2}+\frac{e^{-dx-c}a}{d^2b^3}-\frac{e^{-dx-c}x^2}{2db^2}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x+a)^2,x)
```

```
[Out] -1/d^3*exp(-d*x-c)/b^2+1/2*d/b^6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a
^4+2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3-1/d^2*exp(-d*x-c)/b^2
*x+1/d^2*exp(-d*x-c)/b^3*a-1/2/d*exp(-d*x-c)/b^2*x^2-3/2/d*exp(-d*x-c)/b^4*
a^2-1/2*d*exp(-d*x-c)/b^5/(b*d*x+a*d)*a^4+1/d*exp(-d*x-c)/b^3*a*x-1/d/b^3*a
*exp(d*x+c)*x+3/2/d/b^4*a^2*exp(d*x+c)+1/d^2/b^3*a*exp(d*x+c)+1/d^3/b^2*exp
(d*x+c)-1/2*d/b^6*exp(d*x+c)/(1/b*d*a+d*x)*a^4-1/2*d/b^6*exp(-(a*d-b*c)/b)*
Ei(1,-d*x-c-(a*d-b*c)/b)*a^4+2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/
b)*a^3+1/2/d/b^2*exp(d*x+c)*x^2-1/d^2/b^2*exp(d*x+c)*x
```

Maxima [A] time = 1.51568, size = 548, normalized size = 2.37

$$\frac{1}{6}\left(3a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^6}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^6}\right)+\frac{12a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^4d}-\frac{9a^2\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\right.}{b^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^6 - e^(c - a*
d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^6) + 12*a^3*(e^(-c + a*d/b)*exp_in
tegral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d
/b)/b)/(b^4*d) - 9*a^2*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c
)/d^2)/b^4 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2
+ 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3 - ((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*
e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d
^4)/b^2 + 24*a^3*cosh(d*x + c)*log(b*x + a)/(b^5*d))*d - 1/3*(3*a^4/(b^6*x
+ a*b^5) + 12*a^3*log(b*x + a)/b^5 - (b^2*x^3 - 3*a*b*x^2 + 9*a^2*x)/b^4)*c
osh(d*x + c)
```

Fricas [A] time = 2.07989, size = 740, normalized size = 3.2

$$-\frac{2\left(a^4bd^3+2b^5dx^2-2a^2b^3d\right)\cosh(dx+c)-\left(\left(a^5d^4-4a^4bd^3+\left(a^4bd^4-4a^3b^2d^3\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^5d^4+4a^4bd^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4*b*d^3 + 2*b^5*d*x^2 - 2*a^2*b^3*d)*cosh(d*x + c) - ((a^5*d^4 -
 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^
4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh
(-(b*c - a*d)/b) - 2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 + 2*a*b^4
 + (a^2*b^3*d^2 + 2*b^5)*x)*sinh(d*x + c) + ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*
b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^4 + 4*a^4*b*d^3 + (a
^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b
^7*d^3*x + a*b^6*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh (c + dx)}{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x)**2, x)
```

Giac [A] time = 1.18744, size = 393, normalized size = 1.7

$$\frac{a^4 bdx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^4 bdx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + a^5 d\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 4\,a^3 b^2 x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^5 d\mathrm{Ei}\left(-\frac{bdx\cdots}{b}\right.}{2\left(b^7 x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^4*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^4*b*d*x*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) + a^5*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 4*a^3*b^2
*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^5*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a
*d/b) - 4*a^3*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 4*a^4*b*Ei((b*d*x
 + a*d)/b)*e^(c - a*d/b) - 4*a^4*b*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^
4*b*e^(d*x + c) - a^4*b*e^(-d*x - c))/(b^7*x + a*b^6)
```

3.27 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} dx$

Optimal. Leaf size=182

$$-\frac{a^3 d \sinh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{3a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{3a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^3 d \cosh\left(c\right.}{}$$

```
[Out] -(Cosh[c + d*x]/(b^2*d^2)) + (a^3*Cosh[c + d*x])/(b^4*(a + b*x)) + (3*a^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d*CoshIntegral[(a*
d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 - (2*a*Sinh[c + d*x])/(b^3*d) + (x*Sinh[
c + d*x])/(b^2*d) - (a^3*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b
^5 + (3*a^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Rubi [A] time = 0.422891, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {6742, 2637, 3296, 2638, 3297, 3303, 3298, 3301}

$$-\frac{a^3 d \sinh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{3a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{3a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^3 d \cosh\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```


```
[Out] -(Cosh[c + d*x]/(b^2*d^2)) + (a^3*Cosh[c + d*x])/(b^4*(a + b*x)) + (3*a^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d*CoshIntegral[(a*
d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 - (2*a*Sinh[c + d*x])/(b^3*d) + (x*Sinh[
c + d*x])/(b^2*d) - (a^3*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b
^5 + (3*a^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^2}\,dx = \int\left(-\frac{2a\cosh(c+dx)}{b^3}+\frac{x\cosh(c+dx)}{b^2}-\frac{a^3\cosh(c+dx)}{b^3(a+bx)^2}+\frac{3a^2\cosh(c+dx)}{b^3(a+bx)}\right)dx$$

$$= -\frac{(2a)\int\cosh(c+dx)\,dx}{b^3}+\frac{\left(3a^2\right)\int\frac{\cosh(c+dx)}{a+bx}\,dx}{b^3}-\frac{a^3\int\frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^3}+\frac{\int x\cosh(c+dx)\,dx}{b^2}$$

$$= \frac{a^3\cosh(c+dx)}{b^4(a+bx)}-\frac{2a\sinh(c+dx)}{b^3d}+\frac{x\sinh(c+dx)}{b^2d}-\frac{\int\sinh(c+dx)\,dx}{b^2d}-\frac{\left(a^3d\right)\int\frac{\sinh(c+dx)}{a+bx}\,dx}{b^4}$$

$$= -\frac{\cosh(c+dx)}{b^2d^2}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{2a\sinh(c+dx)}{b^3d}+\frac{x\sin}{}$$

$$= -\frac{\cosh(c+dx)}{b^2d^2}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}-\frac{a^3d\,\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(\right.}{b^5}$$

Mathematica [A] time = 0.914422, size = 156, normalized size = 0.86

$$\frac{\frac{b\left(\left(a^3d^2-ab^2-b^3x\right)\cosh(c+dx)+bd\left(-2a^2-abx+b^2x^2\right)\sinh(c+dx)\right)}{d^2(a+bx)}+a^2\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(3b\cosh\left(c-\frac{ad}{b}\right)-ad\sinh\left(c-\frac{ad}{b}\right)\right)-a^2\mathrm{Shi}\left(d\left(\right.\right.}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a^2*CoshIntegral[d*(a/b + x)]*(3*b*Cosh[c - (a*d)/b] - a*d*Sinh[c - (a*d)/
b]) + (b*((-(a*b^2) + a^3*d^2 - b^3*x)*Cosh[c + d*x] + b*d*(-2*a^2 - a*b*x
+ b^2*x^2)*Sinh[c + d*x]))/(d^2*(a + b*x)) - a^2*(a*d*Cosh[c - (a*d)/b] - 3
*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^5
```

Maple [A] time = 0.046, size = 325, normalized size = 1.8

$$\frac{d\mathrm{e}^{-dx-c}a^3}{2\,b^4\,(bdx+da)}-\frac{da^3}{2\,b^5}\mathrm{e}^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)-\frac{3\,a^2}{2\,b^4}\mathrm{e}^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)-\frac{\mathrm{e}^{-dx-c}x}{2\,db^2}+\frac{\mathrm{e}^{-dx-c}a}{db^3}-\frac{\mathrm{e}^{-dx-}}{2\,d^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/2*d*exp(-d*x-c)/b^4/(b*d*x+a*d)*a^3-1/2*d/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c
+(a*d-b*c)/b)*a^3-3/2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2-1/2/
d*exp(-d*x-c)/b^2*x+1/d*exp(-d*x-c)/b^3*a-1/2/d^2*exp(-d*x-c)/b^2+1/2/d/b^2
*exp(d*x+c)*x+1/2*d/b^5*exp(d*x+c)/(1/b*d*a+d*x)*a^3+1/2*d/b^5*exp(-(a*d-b*
c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3-1/d/b^3*a*exp(d*x+c)-1/2/d^2/b^2*exp(d*x
+c)-3/2/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2
```

Maxima [A] time = 1.3432, size = 421, normalized size = 2.31

$$-\frac{1}{4}\left(2\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^5}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^5}\right)+\frac{6\,a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^3d}-\frac{4\,a\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\right.}{b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^5 - e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^5) + 6*a^2*(e^(-c + a*d/b)*exp_in
tegral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d
/b)/b)/(b^3*d) - 4*a*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/
d^2)/b^3 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*
x + 2)*e^(-d*x - c)/d^3)/b^2 + 12*a^2*cosh(d*x + c)*log(b*x + a)/(b^4*d))*d
 + 1/2*(2*a^3/(b^5*x + a*b^4) + 6*a^2*log(b*x + a)/b^4 + (b*x^2 - 4*a*x)/b^
3)*cosh(d*x + c)
```

Fricas [A] time = 2.03256, size = 663, normalized size = 3.64

$$2\left(a^3bd^2-b^4x-ab^3\right)\cosh\left(dx+c\right)-\left(\left(a^4d^3-3\,a^3bd^2+\left(a^3bd^3-3\,a^2b^2d^2\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^4d^3+3\,a^3bd^2+\left(a^3bd^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3*b*d^2 - b^4*x - a*b^3)*cosh(d*x + c) - ((a^4*d^3 - 3*a^3*b*d^2
+ (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^3 + 3*a^3*b*d
^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)
/b) + 2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*sinh(d*x + c) + ((a^4*d^3 - 3
*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^3
```

```
+ 3*a^3*b*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-
(b*c - a*d)/b))/(b^6*d^2*x + a*b^5*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**2, x)
```

Giac [A] time = 1.18621, size = 393, normalized size = 2.16

$$-\frac{a^3bdx\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^3bdx\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + a^4d\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 3\,a^2b^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^4d\text{Ei}\left(-\frac{bd}{}\right.}{2\left(b^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^3*b*d*x*Ei(-(b*d*x +
a*d)/b)*e^(-c + a*d/b) + a^4*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 3*a^2*b^
2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^4*d*Ei(-(b*d*x + a*d)/b)*e^(-c +
a*d/b) - 3*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 3*a^3*b*Ei((b*d*
x + a*d)/b)*e^(c - a*d/b) - 3*a^3*b*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a
^3*b*e^(d*x + c) - a^3*b*e^(-d*x - c))/(b^6*x + a*b^5)
```

3.28 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\, dx$

Optimal. Leaf size=147

$$\frac{a^2 d \sinh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2 d \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \cosh(c+dx)}{b^3(a+bx)}-\frac{2a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{a}{b}\right.}{b^3}$$

```
[Out] -((a^2*Cosh[c + d*x])/(b^3*(a + b*x))) - (2*a*Cosh[c - (a*d)/b]*CoshIntegra
l[(a*d)/b + d*x])/b^3 + (a^2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b
])/b^4 + Sinh[c + d*x]/(b^2*d) + (a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d
)/b + d*x])/b^4 - (2*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rubi [A] time = 0.374579, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$\frac{a^2 d \sinh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2 d \cosh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2 \cosh(c+dx)}{b^3(a+bx)}-\frac{2a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{a}{b}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/(b^3*(a + b*x))) - (2*a*Cosh[c - (a*d)/b]*CoshIntegra
l[(a*d)/b + d*x])/b^3 + (a^2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b
])/b^4 + Sinh[c + d*x]/(b^2*d) + (a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d
)/b + d*x])/b^4 - (2*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{b^2} + \frac{a^2\cosh(c+dx)}{b^2(a+bx)^2} - \frac{2a\cosh(c+dx)}{b^2(a+bx)}\right) dx$$

$$= \frac{\int \cosh(c+dx)\,dx}{b^2} - \frac{(2a)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^2} + \frac{a^2\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^2}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)} + \frac{\sinh(c+dx)}{b^2 d} + \frac{(a^2 d)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^3} - \frac{\left(2a\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b^2}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)} - \frac{2a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\sinh(c+dx)}{b^2 d} - \frac{2a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+\right.}{b^3}$$

$$= -\frac{a^2\cosh(c+dx)}{b^3(a+bx)} - \frac{2a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2 d\,\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sinh(c}{b^2}$$

Mathematica [A] time = 0.692046, size = 115, normalized size = 0.78

$$\frac{b\left(\frac{b\sinh(c+dx)}{d} - \frac{a^2\cosh(c+dx)}{a+bx}\right) + a\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\sinh\left(c-\frac{ad}{b}\right) - 2b\cosh\left(c-\frac{ad}{b}\right)\right) + a\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cosh\left(c-\frac{ad}{b}\right.\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*CoshIntegral[d*(a/b + x)]*(-2*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b
]) + b*(-((a^2*Cosh[c + d*x])/(a + b*x)) + (b*Sinh[c + d*x])/d) + a*(a*d*Co
sh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^4
```

Maple [A] time = 0.039, size = 254, normalized size = 1.7

$$-\frac{e^{-dx-c}}{2\,db^2} - \frac{de^{-dx-c}a^2}{2\,b^3\,(bdx+da)} + \frac{da^2}{2\,b^4}e^{\frac{da-cb}{b}}\text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) + \frac{a}{b^3}e^{\frac{da-cb}{b}}\text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) + \frac{e^{dx+c}}{2\,db^2} - \frac{de^{dx+c}a^2}{2\,b^4}\left(\frac{d}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x+a)^2,x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b^2-1/2*d*exp(-d*x-c)/b^3/(b*d*x+a*d)*a^2+1/2*d/b^4*exp(
(a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+1/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+
(a*d-b*c)/b)*a+1/2/d/b^2*exp(d*x+c)-1/2*d/b^4*exp(d*x+c)/(1/b*d*a+d*x)*a^2-
```

```
1/2*d/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2+1/b^3*exp(-(a*d-b*
c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a
```

Maxima [A] time = 1.34762, size = 319, normalized size = 2.17

$$\frac{1}{2}\left(a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^4}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^4}\right)+\frac{2a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d}-\frac{\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-}}{d^2}}{b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^4 - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/b^4) + 2*a*(e^(-c + a*d/b)*exp_integra
l_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b
)/(b^2*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2
+ 4*a*cosh(d*x + c)*log(b*x + a)/(b^3*d))*d - (a^2/(b^4*x + a*b^3) - x/b^2
+ 2*a*log(b*x + a)/b^3)*cosh(d*x + c)
```

Fricas [A] time = 1.99239, size = 570, normalized size = 3.88

$$-\frac{2a^2bd\cosh(dx+c)-\left(\left(a^3d^2-2a^2bd+\left(a^2bd^2-2ab^2d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^3d^2+2a^2bd+\left(a^2bd^2+2ab^2d\right)x\right)\mathrm{Ei}\left(-\frac{b}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b*d*cosh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^
2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d
)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*x + a*b^2)*sinh(d*
x + c) + ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d
)/b) + (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/
b))*sinh(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**2, x)
```

Giac [A] time = 1.22872, size = 387, normalized size = 2.63

$$\frac{a^2bdx\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^2bdx\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + a^3d\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 2\,ab^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - a^3d\text{Ei}\left(-\frac{bdx+ad}{b}\right. \cdots}{2\left(b^5x \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^2*b*d*x*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) + a^3*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*a*b^2*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^3*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d
/b) - 2*a*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a^2*b*Ei((b*d*x + a
*d)/b)*e^(c - a*d/b) - 2*a^2*b*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b*
e^(d*x + c) - a^2*b*e^(-d*x - c))/(b^5*x + a*b^4)
```

3.29 $\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=125

$$-\frac{ad\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}}{b^3}$$

```
[Out] (a*Cosh[c + d*x])/(b^2*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b
 + d*x])/b^2 - (a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 - (a
*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (Sinh[c - (a*d)/b]*
SinhIntegral[(a*d)/b + d*x])/b^2
```

Rubi [A] time = 0.297248, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{ad\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] (a*Cosh[c + d*x])/(b^2*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b
 + d*x])/b^2 - (a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 - (a
*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (Sinh[c - (a*d)/b]*
SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \int \left(-\frac{a\cosh(c+dx)}{b(a+bx)^2}+\frac{\cosh(c+dx)}{b(a+bx)}\right)dx$$
$$= \frac{\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b}$$
$$= \frac{a\cosh(c+dx)}{b^2(a+bx)} - \frac{(ad)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^2} + \frac{\cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} + \frac{\sinh\left(c-\frac{ad}{b}\right)\int \frac{\sinh\left(\frac{ad}{b}+\cdots\right)}{a+bx}}{b}$$
$$= \frac{a\cosh(c+dx)}{b^2(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{\left(ad\cosh\left(c-\frac{ad}{b}\right.\right.\cdots}{\cdots}$$
$$= \frac{a\cosh(c+dx)}{b^2(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{ad\,\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} - \frac{ad\cosh\left(c-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.41379, size = 97, normalized size = 0.78

$$\frac{\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right)-ad\sinh\left(c-\frac{ad}{b}\right)\right)+\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\sinh\left(c-\frac{ad}{b}\right)-ad\cosh\left(c-\frac{ad}{b}\right)\right)+\frac{ab\cosh(c+dx)}{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

```
[Out] ((a*b*Cosh[c + d*x])/(a + b*x) + CoshIntegral[d*(a/b + x)]*(b*Cosh[c - (a*d
)/b] - a*d*Sinh[c - (a*d)/b]) + (-(a*d*Cosh[c - (a*d)/b]) + b*Sinh[c - (a*d
)/b])*SinhIntegral[d*(a/b + x)])/b^3
```

Maple [A] time = 0.027, size = 215, normalized size = 1.7

$$\frac{d\mathrm{e}^{-dx-c}a}{2\,b^2\,(bdx+da)} - \frac{da}{2\,b^3}\mathrm{e}^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{1}{2\,b^2}\mathrm{e}^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) + \frac{d\mathrm{e}^{dx+c}a}{2\,b^3}\left(\frac{da}{b}+dx\right)^{-1} + \frac{da}{2\,b^3}\mathrm{e}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x+a)^2,x)
```

```
[Out] 1/2*d*exp(-d*x-c)/b^2/(b*d*x+a*d)*a-1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(
a*d-b*c)/b)*a-1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*d/b^3*ex
p(d*x+c)/(1/b*d*a+d*x)*a+1/2*d/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/
b)*a-1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.36264, size = 240, normalized size = 1.92

$$-\frac{1}{2}\left(a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^3}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^3}\right)+\frac{\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}}{bd}+\frac{2\cosh(dx+c)\log(bx+}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^3 - e^(c - a*d/b
)*exp_integral_e(1, -(b*x + a)*d/b)/b^3) + (e^(-c + a*d/b)*exp_integral_e(1
, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*
d) + 2*cosh(d*x + c)*log(b*x + a)/(b^2*d))*d + (a/(b^3*x + a*b^2) + log(b*x
 + a)/b^2)*cosh(d*x + c)
```

Fricas [A] time = 2.03375, size = 414, normalized size = 3.31

$$\frac{2\,ab\cosh(dx+c)-\left(\left(a^2d-ab+\left(abd-b^2\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^2d+ab+\left(abd+b^2\right)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+\left(\right.}{2\left(b^4x+ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cosh(d*x + c) - ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d
)/b) - (a^2*d + a*b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a
*d)/b) + ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d + a*
b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*x + a
*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x)**2, x)
```

Giac [B] time = 1.24044, size = 373, normalized size = 2.98

$$-\frac{abdx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-abdx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+a^2d\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-b^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-a^2d\mathrm{Ei}\left(-\frac{bdx+}{b}\right.}{2\left(b^3x+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a*b*d*x*Ei(-(b*d*x + a*d)
/b)*e^(-c + a*d/b) + a^2*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - b^2*x*Ei((b*
d*x + a*d)/b)*e^(c - a*d/b) - a^2*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - b
^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b*Ei((b*d*x + a*d)/b)*e^(c - a
*d/b) - a*b*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b*e^(d*x + c) - a*b*e^(
-d*x - c))/((b^3*x + a*b^2)*b)
```

3.30 $\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal. Leaf size=71

$$\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\cosh(c+dx)}{b(a+bx)}$$

```
[Out] -(Cosh[c + d*x]/(b*(a + b*x))) + (d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a
*d)/b])/b^2 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rubi [A] time = 0.109993, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3298, 3301}

$$\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}-\frac{\cosh(c+dx)}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Cosh[c + d*x]/(b*(a + b*x))) + (d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a
*d)/b])/b^2 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b}$$
$$= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b} + \frac{\left(d\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{b}$$
$$= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^2} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

Mathematica [A] time = 0.201944, size = 65, normalized size = 0.92

$$\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)-\frac{b\cosh(c+dx)}{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^2,x]
```

```
[Out] (-((b*Cosh[c + d*x])/(a + b*x)) + d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d
)/b] + d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b^2
```

Maple [A] time = 0.025, size = 132, normalized size = 1.9

$$-\frac{de^{-dx-c}}{2b(bdx+da)}+\frac{d}{2b^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)-\frac{de^{dx+c}}{2b^2}\left(\frac{da}{b}+dx\right)^{-1}-\frac{d}{2b^2}e^{-\frac{da-cb}{b}}\text{Ei}\left(1,-dx-c-\frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x+a)^2,x)
```

```
[Out] -1/2*d*exp(-d*x-c)/b/(b*d*x+a*d)+1/2*d/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d
-b*c)/b)-1/2*d/b^2*exp(d*x+c)/(1/b*d*a+d*x)-1/2*d/b^2*exp(-(a*d-b*c)/b)*Ei(
1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.20947, size = 109, normalized size = 1.54

$$\frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{2b}-\frac{\cosh(dx+c)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - e^(c - a*d/b)*ex
p_integral_e(1, -(b*x + a)*d/b)/b) - cosh(d*x + c)/((b*x + a)*b)
```

Fricas [B] time = 2.03666, size = 316, normalized size = 4.45

$$-\frac{2\,b\cosh\left(dx+c\right)-\left(\left(bdx+ad\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(bdx+ad\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+\left(\left(bdx+ad\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+(bdx}{2\left(b^{3}x+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cosh(d*x + c) - ((b*d*x + a*d)*Ei((b*d*x + a*d)/b) - (b*d*x + a*d
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b*d*x + a*d)*Ei((b*d*x + a
*d)/b) + (b*d*x + a*d)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^3*x +
 a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21064, size = 201, normalized size = 2.83

$$\frac{bdx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-bdx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-be^{(dx+c)}-be^{(-dx-}}{2\left(b^{3}x+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - b*d*x*Ei(-(b*d*x + a*d)/b)*e
^(-c + a*d/b) + a*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a*d*Ei(-(b*d*x + a*
d)/b)*e^(-c + a*d/b) - b*e^(d*x + c) - b*e^(-d*x - c))/(b^3*x + a*b^2)
```

3.31 $\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}}{ab}$$

```
[Out] Cosh[c + d*x]/(a*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 - (d*CoshIntegral[(a*d)/b + d*x]*S
inh[c - (a*d)/b])/(a*b) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c - (a*
d)/b]*SinhIntegral[(a*d)/b + d*x])/(a*b) - (Sinh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a^2
```

Rubi [A] time = 0.400274, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3303, 3298, 3301, 3297}

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\cosh(c)\mathrm{Chi}(dx)}{a^2}+\frac{\sinh(c)\mathrm{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] Cosh[c + d*x]/(a*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 - (d*CoshIntegral[(a*d)/b + d*x]*S
inh[c - (a*d)/b])/(a*b) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c - (a*
d)/b]*SinhIntegral[(a*d)/b + d*x])/(a*b) - (Sinh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a^2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a(a+bx)^2} - \frac{b\cosh(c+dx)}{a^2(a+bx)}\right) dx$$
$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a}$$
$$= \frac{\cosh(c+dx)}{a(a+bx)} - \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^2} - \frac{\left(b\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{a^2} +$$
$$= \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{\sinh\left(c-\right.}{}$$
$$= \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{ab}$$

Mathematica [A] time = 1.05038, size = 241, normalized size = 1.61

$$-\frac{a^2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{d(a+bx)}{b}\right) + a^2d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{d(a+bx)}{b}\right) + b^2x\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + abdx\sinh\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

```
[Out] -((-(a*b*Cosh[c + d*x]) - b*(a + b*x)*Cosh[c]*CoshIntegral[d*x] + b*(a + b*
x)*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + a^2*d*CoshIntegral[(d*(a +
 b*x))/b]*Sinh[c - (a*d)/b] + a*b*d*x*CoshIntegral[(d*(a + b*x))/b]*Sinh[c
- (a*d)/b] - a*b*Sinh[c]*SinhIntegral[d*x] - b^2*x*Sinh[c]*SinhIntegral[d*x
] + a*b*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + b^2*x*Sinh[c - (a*d)/
b]*SinhIntegral[d*(a/b + x)] + a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a +
 b*x))/b] + a*b*d*x*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b])/(a^2*b
*(a + b*x)))
```

Maple [A] time = 0.05, size = 254, normalized size = 1.7

$$\frac{e^{-dx-c}d}{2a((dx+c)b+da-cb)} - \frac{e^{-c}\text{Ei}(1,dx)}{2a^2} - \frac{d}{2ab}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) + \frac{1}{2a^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x+a)^2,x)
```

```
[Out] 1/2*exp(-d*x-c)*d/a/((d*x+c)*b+d*a-c*b)-1/2/a^2*exp(-c)*Ei(1,d*x)-1/2/a/b*e
xp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a^2*exp((a*d-b*c)/b)*Ei(1,d*x
+c+(a*d-b*c)/b)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/2/b*d/a*exp(d*x+c)/(1/b*d*a+d*x
```

```
)+1/2/b*d/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2/a^2*exp(-(a*d-b*
c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.43188, size = 306, normalized size = 2.04

$$-\frac{1}{2}d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{ab}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{ab}-\frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d}-\frac{2\cosh\left(dx+c\right)\log\left(bx+a\right)}{a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/(a*b) - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/(a*b) - b*(e^(-c + a*d/b)*exp_integral
_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)
/(a^2*d) - 2*cosh(d*x + c)*log(b*x + a)/(a^2*d) + 2*cosh(d*x + c)*log(x)/(a
^2*d) - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a^2*d)) + (1/(a*b*x + a^2) - log(b
*x + a)/a^2 + log(x)/a^2)*cosh(d*x + c)
```

Fricas [A] time = 2.15082, size = 579, normalized size = 3.86

$$\frac{2\,ab\cosh\left(dx+c\right)+\left(\left(b^2x+ab\right)\mathrm{Ei}\left(dx\right)+\left(b^2x+ab\right)\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(\left(a^2d+ab+\left(abd+b^2\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^2d-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*cosh(d*x + c) + ((b^2*x + a*b)*Ei(d*x) + (b^2*x + a*b)*Ei(-d*x))
*cosh(c) - ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) - (a^2*d -
a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b^2*x
 + a*b)*Ei(d*x) - (b^2*x + a*b)*Ei(-d*x))*sinh(c) + ((a^2*d + a*b + (a*b*d
+ b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x
+ a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(c+dx\right)}{x\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)**2,x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**2), x)
```

Giac [B] time = 1.21112, size = 439, normalized size = 2.93

$$-\frac{\left(abdx\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - abdx\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} - b^2x\text{Ei}\left(-dx\right)e^{(-c)} + a^2d\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + b^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a*b*d*x*Ei(-(b*d*x + a*d)
/b)*e^(-c + a*d/b) - b^2*x*Ei(-d*x)*e^(-c) + a^2*d*Ei((b*d*x + a*d)/b)*e^(c
 - a*d/b) + b^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - b^2*x*Ei(d*x)*e^c - a
^2*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b) - a*b*Ei(-d*x)*e^(-c) + a*b*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a
*b*Ei(d*x)*e^c + a*b*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b*e^(d*x + c)
- a*b*e^(-d*x - c))*b/(a^2*b^3*x + a^3*b^2)
```

3.32 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx$

Optimal. Leaf size=186

$$\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{2b\cosh(c)\text{Chi}(dx)}{a^3}+\frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\sinh(c)\text{Shi}(dx)}{a^3}+\frac{2b\sinh\left(c\right.}{}$$

```
[Out] -(Cosh[c + d*x]/(a^2*x)) - (b*Cosh[c + d*x])/(a^2*(a + b*x)) - (2*b*Cosh[c]
*CoshIntegral[d*x])/a^3 + (2*b*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x
])/a^3 + (d*CoshIntegral[d*x]*Sinh[c])/a^2 + (d*CoshIntegral[(a*d)/b + d*x]
*Sinh[c - (a*d)/b])/a^2 + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (2*b*Sinh[c]*
SinhIntegral[d*x])/a^3 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/
a^2 + (2*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Rubi [A] time = 0.501074, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{2b\cosh(c)\text{Chi}(dx)}{a^3}+\frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\sinh(c)\text{Shi}(dx)}{a^3}+\frac{2b\sinh\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```



```
[Out] -(Cosh[c + d*x]/(a^2*x)) - (b*Cosh[c + d*x])/(a^2*(a + b*x)) - (2*b*Cosh[c]
*CoshIntegral[d*x])/a^3 + (2*b*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x
])/a^3 + (d*CoshIntegral[d*x]*Sinh[c])/a^2 + (d*CoshIntegral[(a*d)/b + d*x]
*Sinh[c - (a*d)/b])/a^2 + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (2*b*Sinh[c]*
SinhIntegral[d*x])/a^3 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/
a^2 + (2*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{2b\cosh(c+dx)}{a^3x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^2} + \frac{2b^2\cosh(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^2} - \frac{(2b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} + \frac{\left(2b^2\right)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3} + \frac{b^2\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^2} + \frac{(bd)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^2} - \frac{(2b\cosh(c))\int \frac{\cosh}{x}}{a^3}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\text{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{2b\,\text{s}}{}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\text{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d\text{Cl}}{}$$

Mathematica [A] time = 1.41446, size = 183, normalized size = 0.98

$$ad\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + 2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + 2b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right) + ad\cosh\left(c-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```

```
[Out] (-((a*(a + 2*b*x)*Cosh[c]*Cosh[d*x])/(x*(a + b*x))) - 2*b*Cosh[c]*CoshInteg
ral[d*x] + 2*b*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + a*d*CoshIntegr
al[d*x]*Sinh[c] + a*d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d)/b] - (a*(a +
 2*b*x)*Sinh[c]*Sinh[d*x])/(x*(a + b*x)) + a*d*Cosh[c]*SinhIntegral[d*x] -
2*b*Sinh[c]*SinhIntegral[d*x] + a*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b +
 x)] + 2*b*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a^3
```

Maple [A] time = 0.058, size = 312, normalized size = 1.7

$$-\frac{de^{-dx-c}b}{a^2(bdx+da)} - \frac{de^{-dx-c}}{2ax(bdx+da)} + \frac{de^{-c}\text{Ei}(1,dx)}{2a^2} + \frac{be^{-c}\text{Ei}(1,dx)}{a^3} + \frac{d}{2a^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{b}{a^3}e^{\frac{da-cb}{b}}\text{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x+a)^2,x)
```

```
[Out] -d*exp(-d*x-c)/a^2/(b*d*x+a*d)*b-1/2*d*exp(-d*x-c)/a/x/(b*d*x+a*d)+1/2*d/a^
2*exp(-c)*Ei(1,d*x)+1/a^3*exp(-c)*Ei(1,d*x)*b+1/2*d/a^2*exp((a*d-b*c)/b)*Ei
```

```
(1,d*x+c+(a*d-b*c)/b)-1/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+1/a^
3*b*exp(c)*Ei(1,-d*x)-1/2/a^2/x*exp(d*x+c)-1/2*d/a^2*exp(c)*Ei(1,-d*x)-1/2*
d/a^2*exp(d*x+c)/(1/b*d*a+d*x)-1/2*d/a^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d
-b*c)/b)-b/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^2*x^2), x)
```

Fricas [A] time = 2.11014, size = 802, normalized size = 4.31

$$-\frac{2\left(2\,abx+a^2\right)\cosh(dx+c)-\left(\left(\left(abd-2\,b^2\right)x^2+\left(a^2d-2\,ab\right)x\right)\mathrm{Ei}\,(dx)-\left(\left(abd+2\,b^2\right)x^2+\left(a^2d+2\,ab\right)x\right)\mathrm{Ei}\,(-dx)\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*a*b*x + a^2)*cosh(d*x + c) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*
a*b)*x)*Ei(d*x) - ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*cosh(
c) - (((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) - ((a*b
*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d
)/b) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(d*x) + ((a*b*d + 2*b^2
)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*sinh(c) + (((a*b*d + 2*b^2)*x^2 + (a^2
*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) + ((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)
*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b*x^2 + a^4*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.28598, size = 576, normalized size = 3.1

$$-\frac{abdx^2\mathrm{Ei}\,(-dx)\,e^{(-c)}-abdx^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-abdx^2\mathrm{Ei}\,(dx)\,e^{c}+abdx^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+a^2dx\mathrm{Ei}\,(-dx)\,e^{(-c)}+2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d*x^2*Ei(-d*x)*e^(-c) - a*b*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/
b) - a*b*d*x^2*Ei(d*x)*e^c + a*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b)
+ a^2*d*x*Ei(-d*x)*e^(-c) + 2*b^2*x^2*Ei(-d*x)*e^(-c) - a^2*d*x*Ei((b*d*x +
 a*d)/b)*e^(c - a*d/b) - 2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^2*
d*x*Ei(d*x)*e^c + 2*b^2*x^2*Ei(d*x)*e^c + a^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-
c + a*d/b) - 2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a*b*x*Ei(-d*
x)*e^(-c) - 2*a*b*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a*b*x*Ei(d*x)*e^c
 - 2*a*b*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a*b*x*e^(d*x + c) + 2*a*
b*x*e^(-d*x - c) + a^2*e^(d*x + c) + a^2*e^(-d*x - c))/(a^3*b*x^2 + a^4*x)
```

3.33 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{a^3d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{3a^2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{3a^2d\cosh\left(\cdots\right.}{\cdots}$$

```
[Out] (a^3*Cosh[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Cosh[c + d*x])/(b^4*(a + b
*x)) - (3*a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^6) + (3*a^2*d*CoshIntegr
al[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 + Sinh[c + d*x]/(b^3*d) + (a^3*d*S
inh[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(
a*d)/b + d*x])/b^5 - (3*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^
4 - (a^3*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^6)
```

Rubi [A] time = 0.618072, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$-\frac{a^3d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{3a^2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{3a^2d\cosh\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a^3*Cosh[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Cosh[c + d*x])/(b^4*(a + b
*x)) - (3*a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d^2*C
osh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^6) + (3*a^2*d*CoshIntegr
al[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 + Sinh[c + d*x]/(b^3*d) + (a^3*d*S
inh[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(
a*d)/b + d*x])/b^5 - (3*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^
4 - (a^3*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^6)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{b^3} - \frac{a^3\cosh(c+dx)}{b^3(a+bx)^3} + \frac{3a^2\cosh(c+dx)}{b^3(a+bx)^2} - \frac{3a\cosh(c+dx)}{b^3(a+bx)}\right)dx$$

$$= \frac{\int \cosh(c+dx)\,dx}{b^3} - \frac{(3a)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^3} + \frac{\left(3a^2\right)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^3} - \frac{a^3\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b^3}$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} + \frac{\sinh(c+dx)}{b^3d} + \frac{\left(3a^2d\right)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^4} - \frac{\left(a^3d\right)\int \frac{\sinh(c}{(a+b}}{2b^4}$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{\sinh(c+dx)}{b^3d} + \frac{a^3d}{2}$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} + \frac{3a^2d\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5}$$

$$= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2\cosh(c+dx)}{b^4(a+bx)} - \frac{3a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} - \frac{a^3d^2\cosh\left(c-\frac{ad}{b}\right)}{2b^6}$$

Mathematica [A] time = 0.999999, size = 236, normalized size = 0.89

$$-\frac{ad(a+bx)^2\left(\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2+6b^2\right)\cosh\left(c-\frac{ad}{b}\right)-6abd\sinh\left(c-\frac{ad}{b}\right)\right)+\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2+6b^2\right)\sinh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(b*Cosh[d*x]*(a^2*b*d*(5*a + 6*b*x)*Cosh[c] - (a + b*x)*(2*a*b^2 + a^3*d^2
 + 2*b^3*x)*Sinh[c]) - b*((a + b*x)*(2*a*b^2 + a^3*d^2 + 2*b^3*x)*Cosh[c] -
 a^2*b*d*(5*a + 6*b*x)*Sinh[c])*Sinh[d*x] + a*d*(a + b*x)^2*(CoshIntegral[d
*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 6*a*b*d*Sinh[c - (a*d)/b
]) + (-6*a*b*d*Cosh[c - (a*d)/b] + (6*b^2 + a^2*d^2)*Sinh[c - (a*d)/b])*Sin
hIntegral[d*(a/b + x)]))/(2*b^6*d*(a + b*x)^2)
```

Maple [B] time = 0.117, size = 571, normalized size = 2.2

$$-\frac{e^{-dx-c}}{2db^3}+\frac{d^2a^3}{4b^6}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)+\frac{3da^2}{2b^5}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)+\frac{3a}{2b^4}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+c+\frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x+a)^3,x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b^3+1/4*d^2/b^6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)
*a^3+3/2*d/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2+3/2/b^4*exp((a*
d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-5/4*d^2*exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*
a*b*d^2*x+a^2*d^2)*a^3-3/2*d^2*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2
*d^2)*a^2*x-1/4*d^3*exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3*x
-1/4*d^3*exp(-d*x-c)/b^5/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^4-3/2*d/b^5*ex
p(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2+3/2/b^4*exp(-(a*d-b*c)/b)*Ei(1
,-d*x-c-(a*d-b*c)/b)*a+1/2/d/b^3*exp(d*x+c)+1/4*d^2/b^6*exp(d*x+c)/(1/b*d*a
+d*x)*a^3+1/4*d^2/b^6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3-3/2*d/
b^5*exp(d*x+c)/(1/b*d*a+d*x)*a^2+1/4*d^2/b^6*exp(d*x+c)/(1/b*d*a+d*x)^2*a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2}a^2d\int\frac{xe^{(dx+c)}}{b^5d^2x^4+4ab^4d^2x^3+6a^2b^3d^2x^2+4a^3b^2d^2x+a^4bd^2}\,dx-\frac{3}{2}a^2d\int\frac{x}{b^5d^2x^4e^{(dx+c)}+4ab^4d^2x^3e^{(dx+c)}+6a^2b^3d^2x^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/2*a^2*d*integrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^
3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3/2*a^2*d*integrate(x/(b^5*d
^2*x^4*e^(d*x + c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x
 + c) + 4*a^3*b^2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) - 3*a*b*in
tegrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x^2 +
4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3*a*b*integrate(x/(b^5*d^2*x^4*e^(d*x +
c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x + c) + 4*a^3*b^
2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) + 1/2*((b*d*x^3*e^(2*c) -
3*a*x*e^(2*c))*e^(d*x) - (b*d*x^3 + 3*a*x)*e^(-d*x))/(b^4*d^2*x^3*e^c + 3*a
*b^3*d^2*x^2*e^c + 3*a^2*b^2*d^2*x*e^c + a^3*b*d^2*e^c) - 3/2*a^2*e^(-c + a
*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b^2*d^2) - 3/2*a^2*e^(c
 - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b^2*d^2)
```

Fricas [B] time = 2.50905, size = 1142, normalized size = 4.33

$$-\frac{2\left(6a^2b^3dx+5a^3b^2d\right)\cosh(dx+c)+\left(\left(a^5d^3-6a^4bd^2+6a^3b^2d+\left(a^3b^2d^3-6a^2b^3d^2+6ab^4d\right)x^2+2\left(a^4bd^3-6a^3b^2d \cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*cosh(d*x + c) + ((a^5*d^3 - 6*a^4*b*d
^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b
*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^3 + 6*a
^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*
(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b
*c - a*d)/b) - 2*(a^4*b*d^2 + 2*b^5*x^2 + 2*a^2*b^3 + (a^3*b^2*d^2 + 4*a*b^
4)*x)*sinh(d*x + c) - ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3
- 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3
*d)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^
2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a
^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b
^7*d*x + a^2*b^6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**3, x)
```

Giac [B] time = 1.16071, size = 1019, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^3*b^2*d^2*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^4*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c
 - a*d/b) - 6*a^2*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^4*b*d^2
*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a^2*b^3*d*x^2*Ei(-(b*d*x + a*d)/
b)*e^(-c + a*d/b) + a^5*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^3*b^2*
d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a*b^4*x^2*Ei((b*d*x + a*d)/b)*e^(
c - a*d/b) + a^5*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^3*b^2*d*x*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b) - a^3*b^2*d*x*e^(d*x + c) + a^3*b^2*d*x*e^(-d*x - c) - 6*a^4*b*d*
Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a^2*b^3*x*Ei((b*d*x + a*d)/b)*e^(c -
 a*d/b) + 6*a^4*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^2*b^3*x*Ei(-
(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^4*b*d*e^(d*x + c) + 6*a^2*b^3*x*e^(d*x
+ c) + a^4*b*d*e^(-d*x - c) + 6*a^2*b^3*x*e^(-d*x - c) + 6*a^3*b^2*Ei((b*d*
x + a*d)/b)*e^(c - a*d/b) + 6*a^3*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) +
 5*a^3*b^2*e^(d*x + c) + 5*a^3*b^2*e^(-d*x - c))/(b^8*x^2 + 2*a*b^7*x + a^2
*b^6)
```

3.34 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx$

Optimal. Leaf size=241

$$\frac{a^2d^2 \cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^5}+\frac{a^2d^2 \sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d\sinh(c+dx)}{2b^4(a+bx)}-\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2}-\frac{2ad\sinh\left(c\right.}{}$$

```
[Out] -(a^2*Cosh[c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Cosh[c + d*x])/(b^3*(a + b*
x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Cosh[c
 - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^5) - (2*a*d*CoshIntegral[(a*d
)/b + d*x]*Sinh[c - (a*d)/b])/b^4 - (a^2*d*Sinh[c + d*x])/(2*b^4*(a + b*x))
 - (2*a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4 + (Sinh[c - (a
*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Sinh[c - (a*d)/b]*SinhIn
tegral[(a*d)/b + d*x])/(2*b^5)
```

Rubi [A] time = 0.530363, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{a^2d^2 \cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^5}+\frac{a^2d^2 \sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d\sinh(c+dx)}{2b^4(a+bx)}-\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2}-\frac{2ad\sinh\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```


```
[Out] -(a^2*Cosh[c + d*x])/(2*b^3*(a + b*x)^2) + (2*a*Cosh[c + d*x])/(b^3*(a + b*
x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Cosh[c
 - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^5) - (2*a*d*CoshIntegral[(a*d
)/b + d*x]*Sinh[c - (a*d)/b])/b^4 - (a^2*d*Sinh[c + d*x])/(2*b^4*(a + b*x))
 - (2*a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4 + (Sinh[c - (a
*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Sinh[c - (a*d)/b]*SinhIn
tegral[(a*d)/b + d*x])/(2*b^5)
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{b^2(a+bx)^3} - \frac{2a\cosh(c+dx)}{b^2(a+bx)^2} + \frac{\cosh(c+dx)}{b^2(a+bx)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{a+bx}\,dx}{b^2} - \frac{(2a)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b^2} + \frac{a^2\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b^2}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} - \frac{(2ad)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^3} + \frac{\left(a^2d\right)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^3} + \frac{\cosh\left(\right.}{}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} + \frac{\text{si}}{}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{2ad\text{Chi}\left(\frac{ad}{b}+dx\right)\sin}{b^4}$$

$$= -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a\cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\text{Ch}}{2b^5}$$

Mathematica [A] time = 0.928054, size = 153, normalized size = 0.63

$$\frac{\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2+2b^2\right)\cosh\left(c-\frac{ad}{b}\right)-4abd\sinh\left(c-\frac{ad}{b}\right)\right)+\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2+2b^2\right)\sinh\left(c-\frac{ad}{b}\right)-4ab\right.}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (CoshIntegral[d*(a/b + x)]*((2*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 4*a*b*d*S
inh[c - (a*d)/b]) - (a*b*(-(b*(3*a + 4*b*x)*Cosh[c + d*x]) + a*d*(a + b*x)*
Sinh[c + d*x]))/(a + b*x)^2 + (-4*a*b*d*Cosh[c - (a*d)/b] + (2*b^2 + a^2*d^
2)*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/(2*b^5)
```

Maple [B] time = 0.047, size = 527, normalized size = 2.2

$$\frac{d^3 e^{-dx-c}a^2x}{4\,b^3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} + \frac{d^3 e^{-dx-c}a^3}{4\,b^4\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} + \frac{d^2 e^{-dx-c}ax}{b^2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} + \frac{3\,d^2 e^{-dx-}}{4\,b^3\left(b^2d^2x^2+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x+a)^3,x)
```

```
[Out] 1/4*d^3*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2*x+1/4*d^3*exp
(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3+d^2*exp(-d*x-c)/b^2/(b^2
*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x+3/4*d^2*exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*
b*d^2*x+a^2*d^2)*a^2-1/4*d^2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a
^2-d/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-1/2/b^3*exp((a*d-b*c)/b
)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b
)-1/4*d^2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2-1/4*d^2/b^5*ex
p(d*x+c)/(1/b*d*a+d*x)^2*a^2-1/4*d^2/b^5*exp(d*x+c)/(1/b*d*a+d*x)*a^2+d/b^4
*exp(d*x+c)/(1/b*d*a+d*x)*a+d/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b
)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{2}\,ad\int\frac{xe^{(dx+c)}}{b^4d^2x^4+4\,ab^3d^2x^3+6\,a^2b^2d^2x^2+4\,a^3bd^2x+a^4d^2}\,dx+\frac{3}{2}\,ad\int\frac{x}{b^4d^2x^4e^{(dx+c)}+4\,ab^3d^2x^3e^{(dx+c)}+6\,a^2b^2d^2x^2e^{(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -3/2*a*d*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2
*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 3/2*a*d*integrate(x/(b^4*d^2*x^4*
e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) +
 4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + b*integrate(x*e^(d*
x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^2*x +
 a^4*d^2), x) + b*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^
(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4
*d^2*e^(d*x + c)), x) + 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 -
 x)*e^(-d*x))/(b^3*d^2*x^3*e^c + 3*a*b^2*d^2*x^2*e^c + 3*a^2*b*d^2*x*e^c +
a^3*d^2*e^c) + 1/2*a*e^(-c + a*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x
+ a)^3*b*d^2) + 1/2*a*e^(c - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x
 + a)^3*b*d^2)
```

Fricas [A] time = 2.47278, size = 973, normalized size = 4.04

$$\frac{2\left(4\,ab^3x+3\,a^2b^2\right)\cosh\left(dx+c\right)+\left(\left(a^4d^2-4\,a^3bd+2\,a^2b^2+\left(a^2b^2d^2-4\,ab^3d+2\,b^4\right)x^2+2\left(a^3bd^2-4\,a^2b^2d+2\,ab^3\right)x\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(4*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + ((a^4*d^2 - 4*a^3*b*d + 2*a^
2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d
+ 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2
*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x
)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a^2*b^2*d*x + a^3*b*d)*si
nh(d*x + c) - ((a^4*d^2 - 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d
+ 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b)
 - (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2
 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*
c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x)**3, x)
```

Giac [B] time = 1.18058, size = 1000, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c
- a*d/b) - 4*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^
(-c + a*d/b) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 8*a^2*b^2*d*x*Ei
((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/
b) + a^4*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 8*a^2*b^2*d*x*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) -
 a^2*b^2*d*x*e^(d*x + c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a^3*b*d*Ei((b*d*x +
 a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^
3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*x*Ei(-(b*d*x + a*d)/b)*
e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) + 4*a*b^3*x*e^(d*x + c) + a^3*b*d*e^(-
d*x - c) + 4*a*b^3*x*e^(-d*x - c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*
d/b) + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 3*a^2*b^2*e^(d*x + c
) + 3*a^2*b^2*e^(-d*x - c))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

3.35 $\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal. Leaf size=178

$$-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{S}}{b^3}$$

```
[Out] (a*Cosh[c + d*x])/(2*b^2*(a + b*x)^2) - Cosh[c + d*x]/(b^2*(a + b*x)) - (a*
d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^4) + (d*CoshIntegra
l[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 + (a*d*Sinh[c + d*x])/(2*b^3*(a + b
*x)) + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 - (a*d^2*Sinh[
c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^4)
```

Rubi [A] time = 0.369989, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{S}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] (a*Cosh[c + d*x])/(2*b^2*(a + b*x)^2) - Cosh[c + d*x]/(b^2*(a + b*x)) - (a*
d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^4) + (d*CoshIntegra
l[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 + (a*d*Sinh[c + d*x])/(2*b^3*(a + b
*x)) + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 - (a*d^2*Sinh[
c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^4)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \int \left(-\frac{a\cosh(c+dx)}{b(a+bx)^3} + \frac{\cosh(c+dx)}{b(a+bx)^2}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{b}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{b^2} - \frac{(ad)\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b^2}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} + \frac{ad\sinh(c+dx)}{2b^3(a+bx)} - \frac{\left(ad^2\right)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^3} + \frac{\left(d\cosh\left(c-\frac{ad}{b}\right)\right)}{b^2}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} + \frac{d\,\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} + \frac{ad\sinh(c+dx)}{2b^3(a+bx)} + \frac{d\cosh\left(\right.}{}$$

$$= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} - \frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2b^4} + \frac{d\,\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\right.}{b^3}$$

Mathematica [A] time = 0.578165, size = 158, normalized size = 0.89

$$-\frac{d(a+bx)^2\left(\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\cosh\left(c-\frac{ad}{b}\right)-2b\sinh\left(c-\frac{ad}{b}\right)\right)+\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)\left(ad\sinh\left(c-\frac{ad}{b}\right)-2b\cosh\left(c-\right.\right.\right.}{2b^4(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

```
[Out] -(b*Cosh[d*x]*(b*(a + 2*b*x)*Cosh[c] - a*d*(a + b*x)*Sinh[c]) - b*(a*d*(a +
 b*x)*Cosh[c] - b*(a + 2*b*x)*Sinh[c])*Sinh[d*x] + d*(a + b*x)^2*(CoshInteg
ral[d*(a/b + x)]*(a*d*Cosh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b]) + (-2*b*Co
sh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)]))/(2*b^4
*(a + b*x)^2)
```

Maple [B] time = 0.036, size = 435, normalized size = 2.4

$$-\frac{d^3 e^{-dx-c} ax}{4\,b^2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} - \frac{d^3 e^{-dx-c} a^2}{4\,b^3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} - \frac{d^2 e^{-dx-c} x}{2\,b\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} - \frac{d^2 e^{-d}}{4\,b^2\left(b^2d^2x^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x+a)^3,x)
```

```
[Out] -1/4*d^3*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x-1/4*d^3*exp(
-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/2*d^2*exp(-d*x-c)/b/(b^
2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x-1/4*d^2*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b
```

```
*d^2*x+a^2*d^2)*a+1/4*d^2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a+1/
2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4*d^2/b^4*exp(d*x+c)/(1/
b*d*a+d*x)^2*a+1/4*d^2/b^4*exp(d*x+c)/(1/b*d*a+d*x)*a+1/4*d^2/b^4*exp(-(a*d
-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a-1/2*d/b^3*exp(d*x+c)/(1/b*d*a+d*x)-1/2*
d/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b\int \frac{xe^{(dx+c)}}{b^4dx^4+4\,ab^3dx^3+6\,a^2b^2dx^2+4\,a^3bdx+a^4d}\,dx-b\int \frac{x}{b^4dx^4e^{(dx+c)}+4\,ab^3dx^3e^{(dx+c)}+6\,a^2b^2dx^2e^{(dx+c)}+4\,a^3bdxe^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b*integrate(x*e^(d*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4*
a^3*b*d*x + a^4*d), x) - b*integrate(x/(b^4*d*x^4*e^(d*x + c) + 4*a*b^3*d*x
^3*e^(d*x + c) + 6*a^2*b^2*d*x^2*e^(d*x + c) + 4*a^3*b*d*x*e^(d*x + c) + a^
4*d*e^(d*x + c)), x) + 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^3*e^c +
3*a*b^2*d*x^2*e^c + 3*a^2*b*d*x*e^c + a^3*d*e^c) - 1/2*a*e^(-c + a*d/b)*exp
_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d) + 1/2*a*e^(c - a*d/b)*exp_i
ntegral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d)
```

Fricas [B] time = 2.23575, size = 770, normalized size = 4.33

$$-\frac{2\left(2\,b^3x+ab^2\right)\cosh\left(dx+c\right)+\left(\left(a^3d^2-2\,a^2bd+\left(ab^2d^2-2\,b^3d\right)x^2+2\left(a^2bd^2-2\,ab^2d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(a^3d^2+2\,a^2bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*b^3*x + a*b^2)*cosh(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^
2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) + (a^3*
d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*
Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*sinh(d
*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2
- 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2
*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c
 - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23384, size = 714, normalized size = 4.01

$$-\frac{ab^2d^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+ab^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+2\,a^2bd^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-2\,b^3dx^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*b^2*d^2*x^2*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^2*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a
*d/b) - 2*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d^2*x*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a
*d/b) + a^3*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 4*a*b^2*d*x*Ei((b*d*x +
 a*d)/b)*e^(c - a*d/b) + a^3*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*
b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b^2*d*x*e^(d*x + c) + a*b^2
*d*x*e^(-d*x - c) - 2*a^2*b*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b*d*e^(d*x + c) + 2*b^3*x*e^(d*x
 + c) + a^2*b*d*e^(-d*x - c) + 2*b^3*x*e^(-d*x - c) + a*b^2*e^(d*x + c) + a
*b^2*e^(-d*x - c))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

3.36 $\int \frac{\cosh(c+dx)}{(a+bx)^3} dx$

Optimal. Leaf size=104

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^3}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^3}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}-\frac{\cosh(c+dx)}{2b(a+bx)^2}$$

```
[Out] -Cosh[c + d*x]/(2*b*(a + b*x)^2) + (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d
)/b + d*x])/(2*b^3) - (d*Sinh[c + d*x])/(2*b^2*(a + b*x)) + (d^2*Sinh[c - (
a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^3)
```

Rubi [A] time = 0.146297, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3297, 3303, 3298, 3301}

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^3}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^3}-\frac{d\sinh(c+dx)}{2b^2(a+bx)}-\frac{\cosh(c+dx)}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x)^3,x]
```


```
[Out] -Cosh[c + d*x]/(2*b*(a + b*x)^2) + (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d
)/b + d*x])/(2*b^3) - (d*Sinh[c + d*x])/(2*b^2*(a + b*x)) + (d^2*Sinh[c - (
a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^3)
```


Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2b}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx)^2} - \frac{d\sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2\int \frac{\cosh(c+dx)}{a+bx}\,dx}{2b^2}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx)^2} - \frac{d\sinh(c+dx)}{2b^2(a+bx)} + \frac{\left(d^2\cosh\left(c-\frac{ad}{b}\right)\right)\int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2} + \frac{\left(d^2\sinh\left(c-\frac{ad}{b}\right)\right)\int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx}\,dx}{2b^2}$$

$$= -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{2b^3} - \frac{d\sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

Mathematica [A] time = 0.467275, size = 88, normalized size = 0.85

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + d^2\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right) - \frac{b(d(a+bx)\sinh(c+dx)+b\cosh(c+dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^3,x]
```

```
[Out] (d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - (b*(b*Cosh[c + d*x] + d*
(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + d^2*Sinh[c - (a*d)/b]*SinhIntegral[
d*(a/b + x)])/(2*b^3)
```

Maple [B] time = 0.028, size = 276, normalized size = 2.7

$$\frac{d^3 e^{-dx-c} x}{4\,b\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} + \frac{d^3 e^{-dx-c} a}{4\,b^2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} - \frac{d^2 e^{-dx-c}}{4\,b\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)} - \frac{d^2}{4\,b^3} e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x+a)^3,x)
```

```
[Out] 1/4*d^3*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x+1/4*d^3*exp(-d*x-
c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a-1/4*d^2*exp(-d*x-c)/b/(b^2*d^2*x
^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b
)-1/4*d^2/b^3*exp(d*x+c)/(1/b*d*a+d*x)^2-1/4*d^2/b^3*exp(d*x+c)/(1/b*d*a+d*
x)-1/4*d^2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [A] time = 1.18995, size = 128, normalized size = 1.23

$$\frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_2\left(\frac{(bx+a)d}{b}\right)}{(bx+a)b} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_2\left(-\frac{(bx+a)d}{b}\right)}{(bx+a)b}\right)}{4\,b} - \frac{\cosh\left(dx+c\right)}{2\left(bx+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d*(e^(-c + a*d/b)*exp_integral_e(2, (b*x + a)*d/b)/((b*x + a)*b) - e^(c
 - a*d/b)*exp_integral_e(2, -(b*x + a)*d/b)/((b*x + a)*b))/b - 1/2*cosh(d*x
 + c)/((b*x + a)^2*b)
```

Fricas [B] time = 1.95569, size = 518, normalized size = 4.98

$$-\frac{2\,b^2\cosh\left(dx+c\right)-\left(\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+ \cdots}{4\left(b^5x^2+\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*cosh(d*x + c) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*
x + a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) + 2*(b^2*d*x + a*b*d)*sinh(d*x + c) + ((b^2*d^2*x^2 + 2
*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^
2*d^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a
^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18275, size = 402, normalized size = 3.87

$$\frac{b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+2\,abd^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+2\,abd^2x\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}+a^2d\cdots}{4\left(b^5x^2+2\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b^2*d^2*x^2*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) + 2*a*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) +
 2*a*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*d^2*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - b^2*d*x*e
^(d*x + c) + b^2*d*x*e^(-d*x - c) - a*b*d*e^(d*x + c) + a*b*d*e^(-d*x - c)
- b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

3.37 $\int \frac{\cosh(c+dx)}{x(a+bx)^3} dx$

Optimal. Leaf size=262

$$-\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2b}-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(x\right.}{a^2b}$$

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x)^2) + Cosh[c + d*x]/(a^2*(a + b*x)) + (Cosh[c]*
CoshIntegral[d*x])/a^3 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^
3 - (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a*b^2) - (d*Cosh
Integral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a^2*b) + (d*Sinh[c + d*x])/(2*a
*b*(a + b*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 - (d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/(a^2*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b
+ d*x])/a^3 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a*b^2)
```

Rubi [A] time = 0.561227, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3303, 3298, 3301, 3297}

$$-\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2b}-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(x\right.}{a^2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x)^2) + Cosh[c + d*x]/(a^2*(a + b*x)) + (Cosh[c]*
CoshIntegral[d*x])/a^3 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^
3 - (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a*b^2) - (d*Cosh
Integral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a^2*b) + (d*Sinh[c + d*x])/(2*a
*b*(a + b*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 - (d*Cosh[c - (a*d)/b]*Sinh
Integral[(a*d)/b + d*x])/(a^2*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b
+ d*x])/a^3 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a*b^2)
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{a(a+bx)^3} - \frac{b\cosh(c+dx)}{a^2(a+bx)^2} - \frac{b\cosh(c+dx)}{a^3(a+bx)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} - \frac{d\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{(a+bx)^2}\,dx}{2a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^3} - \frac{(b\,\mathrm{c}\ldots}{\ldots}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d\sinh(c+dx)}{2ab(a+bx)} \ldots$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\ldots}{a^2b\ldots}$$

$$= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^3} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} - \frac{d^2\cosh\left(c-\frac{ad}{b}\ldots\right.}{2a\ldots}$$

Mathematica [B] time = 4.59218, size = 614, normalized size = 2.34

$$-\frac{a^2b^2d^2x^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{d(a+bx)}{b}\right) + 4a^2b^2dx\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{d(a+bx)}{b}\right) + a^2b^2d^2x^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{d(a+bx)}{b}\right) - 2a^2\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

```
[Out] -(-3*a^2*b^2*Cosh[c + d*x] - 2*a*b^3*x*Cosh[c + d*x] - 2*b^2*(a + b*x)^2*Co
sh[c]*CoshIntegral[d*x] + 2*b^2*(a + b*x)^2*Cosh[c - (a*d)/b]*CoshIntegral[
d*(a/b + x)] + a^4*d^2*Cosh[c - (a*d)/b]*CoshIntegral[(d*(a + b*x))/b] + 2*
a^3*b*d^2*x*Cosh[c - (a*d)/b]*CoshIntegral[(d*(a + b*x))/b] + a^2*b^2*d^2*x
^2*Cosh[c - (a*d)/b]*CoshIntegral[(d*(a + b*x))/b] + 2*a^3*b*d*CoshIntegral
[(d*(a + b*x))/b]*Sinh[c - (a*d)/b] + 4*a^2*b^2*d*x*CoshIntegral[(d*(a + b*
x))/b]*Sinh[c - (a*d)/b] + 2*a*b^3*d*x^2*CoshIntegral[(d*(a + b*x))/b]*Sinh
[c - (a*d)/b] - a^3*b*d*Sinh[c + d*x] - a^2*b^2*d*x*Sinh[c + d*x] - 2*a^2*b
^2*Sinh[c]*SinhIntegral[d*x] - 4*a*b^3*x*Sinh[c]*SinhIntegral[d*x] - 2*b^4*
x^2*Sinh[c]*SinhIntegral[d*x] + 2*a^2*b^2*Sinh[c - (a*d)/b]*SinhIntegral[d*
(a/b + x)] + 4*a*b^3*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*b^4*
x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d*Cosh[c - (a*d)/
b]*SinhIntegral[(d*(a + b*x))/b] + 4*a^2*b^2*d*x*Cosh[c - (a*d)/b]*SinhInte
gral[(d*(a + b*x))/b] + 2*a*b^3*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a
+ b*x))/b] + a^4*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + 2*a^
```

```
3*b*d^2*x*Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + a^2*b^2*d^2*x^2
*Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b])/(2*a^3*b^2*(a + b*x)^2)
```

Maple [A] time = 0.055, size = 488, normalized size = 1.9

$$-\frac{e^{-dx-c}d\left((dx+c)\,abd+a^2d^2-cdba-2\,b^2\,(dx+c)-3\,bda+2\,cb^2\right)}{4\,a^2b\left(b^2\,(dx+c)^2+2\,(dx+c)\,abd-2\,(dx+c)\,b^2c+a^2d^2-2\,cdba+c^2b^2\right)}-\frac{e^{-c}\mathrm{Ei}\,(1,dx)}{2\,a^3}+\frac{d^2}{4\,ab^2}e^{\frac{da-cb}{b}}\mathrm{Ei}\left(1,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x+a)^3,x)
```

```
[Out] -1/4*exp(-d*x-c)*d*((d*x+c)*a*b*d+a^2*d^2-c*d*b*a-2*b^2*(d*x+c)-3*b*d*a+2*c
*b^2)/a^2/b/(b^2*(d*x+c)^2+2*(d*x+c)*a*b*d-2*(d*x+c)*b^2*c+a^2*d^2-2*c*d*b*
a+c^2*b^2)-1/2/a^3*exp(-c)*Ei(1,d*x)+1/4/a/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+
(a*d-b*c)/b)*d^2-1/2/a^2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a
^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/a^3*exp(c)*Ei(1,-d*x)+1/4*d
^2/b^2/a*exp(d*x+c)/(1/b*d*a+d*x)^2+1/4*d^2/b^2/a*exp(d*x+c)/(1/b*d*a+d*x)+
1/4*d^2/b^2/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2*d/b/a^2*exp(d*
x+c)/(1/b*d*a+d*x)+1/2*d/b/a^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1
/2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x), x)
```

Fricas [B] time = 2.17532, size = 1243, normalized size = 4.74

$$\frac{2\left(2\,ab^3x+3\,a^2b^2\right)\cosh(dx+c)+2\left(\left(b^4x^2+2\,ab^3x+a^2b^2\right)\mathrm{Ei}\,(dx)+\left(b^4x^2+2\,ab^3x+a^2b^2\right)\mathrm{Ei}\,(-dx)\right)\cosh(c)-\left(\left(a^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(2*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^
2*b^2)*Ei(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*cosh(c) - ((a^4*
d^2 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^
3*b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 - 2*a^3*
b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*
a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2
*b^2*d*x + a^3*b*d)*sinh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(d
*x) - (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*sinh(c) + ((a^4*d^2 + 2*a^3
```

```
*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 2
*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^2 - 2*a^3*b*d + 2*a^2
*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*a^2*b^2*d +
 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b^4*x^2 + 2*a
^4*b^3*x + a^5*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)**3,x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x)**3), x)
```

Giac [B] time = 1.2106, size = 1130, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c
 - a*d/b) + 2*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e
^(-c + a*d/b) - 2*b^4*x^2*Ei(-d*x)*e^(-c) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(
c - a*d/b) + 4*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei
((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*b^4*x^2*Ei(d*x)*e^c + a^4*d^2*Ei(-(b*d*
x + a*d)/b)*e^(-c + a*d/b) - 4*a^2*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d
/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x*e^(d*x +
c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a*b^3*x*Ei(-d*x)*e^(-c) + 2*a^3*b*d*Ei((b
*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b)
- 4*a*b^3*x*Ei(d*x)*e^c - 2*a^3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4
*a*b^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) - 2*a*b^
3*x*e^(d*x + c) + a^3*b*d*e^(-d*x - c) - 2*a*b^3*x*e^(-d*x - c) - 2*a^2*b^2
*Ei(-d*x)*e^(-c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*a^2*b^2*
Ei(d*x)*e^c + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 3*a^2*b^2*e^(
d*x + c) - 3*a^2*b^2*e^(-d*x - c))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

3.38 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^3} \, dx$

Optimal. Leaf size=298

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^2b}+\frac{2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\right.}{a^4}$$

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (b*Cosh[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Cos
h[c + d*x])/(a^3*(a + b*x)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (3*b*Co
sh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^4 + (d^2*Cosh[c - (a*d)/b]*C
oshIntegral[(a*d)/b + d*x])/(2*a^2*b) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 +
 (2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^3 - (d*Sinh[c + d*x]
)/(2*a^2*(a + b*x)) + (d*Cosh[c]*SinhIntegral[d*x])/a^3 - (3*b*Sinh[c]*Sinh
Integral[d*x])/a^4 + (2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^
3 + (3*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^4 + (d^2*Sinh[c -
 (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^2*b)
```

Rubi [A] time = 0.689996, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^2b}+\frac{2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\right.}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]
```


```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (b*Cosh[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Cos
h[c + d*x])/(a^3*(a + b*x)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (3*b*Co
sh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^4 + (d^2*Cosh[c - (a*d)/b]*C
oshIntegral[(a*d)/b + d*x])/(2*a^2*b) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 +
 (2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^3 - (d*Sinh[c + d*x]
)/(2*a^2*(a + b*x)) + (d*Cosh[c]*SinhIntegral[d*x])/a^3 - (3*b*Sinh[c]*Sinh
Integral[d*x])/a^4 + (2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^
3 + (3*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^4 + (d^2*Sinh[c -
 (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^2*b)
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^2} - \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^3} + \frac{2b^2\cosh(c+dx)}{a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)}\right) d$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x}\,dx}{a^4} + \frac{(3b^2)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^4} + \frac{(2b^2)\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx}{a^3} + \frac{b^2\int \frac{\cosh(c+}{(a+bx)}}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a^3} + \frac{(2bd)\int \frac{\sinh(c+dx)}{a+bx}\,dx}{a^3} + \frac{(b}{}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}(}{a^4}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}(}{a^4}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} - \frac{3b\cosh(c)\mathrm{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}(}{a^4}$$

Mathematica [B] time = 1.43962, size = 710, normalized size = 2.38

$$\frac{a^2b^2d^2x^3\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{d(a+bx)}{b}\right) + 8a^2b^2dx^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{d(a+bx)}{b}\right) + a^2b^2d^2x^3\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{d(a+bx)}{b}\right) + 4a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]
```

```
[Out] (-2*a^3*b*Cosh[c + d*x] - 9*a^2*b^2*x*Cosh[c + d*x] - 6*a*b^3*x^2*Cosh[c +
d*x] + 6*b^2*x*(a + b*x)^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + a^
4*d^2*x*Cosh[c - (a*d)/b]*CoshIntegral[(d*(a + b*x))/b] + 2*a^3*b*d^2*x^2*C
osh[c - (a*d)/b]*CoshIntegral[(d*(a + b*x))/b] + a^2*b^2*d^2*x^3*Cosh[c - (
a*d)/b]*CoshIntegral[(d*(a + b*x))/b] + 2*b*x*(a + b*x)^2*CoshIntegral[d*x]
*(-3*b*Cosh[c] + a*d*Sinh[c]) + 4*a^3*b*d*x*CoshIntegral[(d*(a + b*x))/b]*S
inh[c - (a*d)/b] + 8*a^2*b^2*d*x^2*CoshIntegral[(d*(a + b*x))/b]*Sinh[c - (
a*d)/b] + 4*a*b^3*d*x^3*CoshIntegral[(d*(a + b*x))/b]*Sinh[c - (a*d)/b] - a
^3*b*d*x*Sinh[c + d*x] - a^2*b^2*d*x^2*Sinh[c + d*x] + 2*a^3*b*d*x*Cosh[c]*
SinhIntegral[d*x] + 4*a^2*b^2*d*x^2*Cosh[c]*SinhIntegral[d*x] + 2*a*b^3*d*x
^3*Cosh[c]*SinhIntegral[d*x] - 6*a^2*b^2*x*Sinh[c]*SinhIntegral[d*x] - 12*a
*b^3*x^2*Sinh[c]*SinhIntegral[d*x] - 6*b^4*x^3*Sinh[c]*SinhIntegral[d*x] +
```

```
6*a^2*b^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a*b^3*x^2*Sinh
[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*b^4*x^3*Sinh[c - (a*d)/b]*SinhI
ntegral[d*(a/b + x)] + 4*a^3*b*d*x*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a + b
*x))/b] + 8*a^2*b^2*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] +
 4*a*b^3*d*x^3*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + a^4*d^2*x*
Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + 2*a^3*b*d^2*x^2*Sinh[c -
(a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + a^2*b^2*d^2*x^3*Sinh[c - (a*d)/b]*
SinhIntegral[(d*(a + b*x))/b])/(2*a^4*b*x*(a + b*x)^2)
```

Maple [B] time = 0.073, size = 643, normalized size = 2.2

$$\frac{e^{-dx-c}xd^3b}{4\,a^2\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)}+\frac{e^{-dx-c}d^3}{4\,a\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)}-\frac{3\,e^{-dx-c}xd^2b^2}{2\,a^3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)}-\frac{9\,e^{-dx-c}}{4\,a^2\left(b^2d^2x^2+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x+a)^3,x)
```

```
[Out] 1/4*exp(-d*x-c)/a^2*x*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+1/4*exp(-d*x-
c)/a*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-3/2*exp(-d*x-c)/a^3*x*d^2/(b^2*d
^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2-9/4*exp(-d*x-c)/a^2*d^2/(b^2*d^2*x^2+2*a*b*
d^2*x+a^2*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+
1/2*d/a^3*exp(-c)*Ei(1,d*x)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4/b/a^2*d^2*exp((
a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+d/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-
b*c)/b)-3/2*b/a^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+3/2/a^4*b*exp(c)
*Ei(1,-d*x)-1/2/a^3/x*exp(d*x+c)-1/2*d/a^3*exp(c)*Ei(1,-d*x)-1/4*d^2/b/a^2*
exp(d*x+c)/(1/b*d*a+d*x)^2-1/4*d^2/b/a^2*exp(d*x+c)/(1/b*d*a+d*x)-1/4*d^2/b
/a^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-d/a^3*exp(d*x+c)/(1/b*d*a+d
*x)-d/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-3/2*b/a^4*exp(-(a*d-b*
c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{\left(bx+a\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^2), x)
```

Fricas [B] time = 2.2, size = 1563, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*cosh(d*x + c) - 2*(((a*b^3*d
- 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3*b*d - 3*a^2*b^2)*x)*Ei(d*
```

```
x) - ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*b^3)*x^2 + (a^3*b*d + 3*a^
2*b^2)*x)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 + 4*a*b^3*d + 6*b^4)*x^3 + 2*(
a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 + 4*a^3*b*d + 6*a^2*b^2)*
x)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 4*a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*
d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a^3*b*d + 6*a^2*b^2)*x)*Ei(
-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2*b^2*d*x^2 + a^3*b*d*x)*sin
h(d*x + c) - 2*(((a*b^3*d - 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3
*b*d - 3*a^2*b^2)*x)*Ei(d*x) + ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*
b^3)*x^2 + (a^3*b*d + 3*a^2*b^2)*x)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 4*
a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2
 + 4*a^3*b*d + 6*a^2*b^2)*x)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 4*a*b^3*
d + 6*b^4)*x^3 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a
^3*b*d + 6*a^2*b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^4*b^3
*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.25263, size = 1358, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^3*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^3*Ei(-d*x)*e^(-c) + 2*a^3*b
*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*d*x^3*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + 2*a*b^3*d*x^3*Ei(d*x)*e^c + 2*a^3*b*d^2*x^2*Ei(-(b*d*x
+ a*d)/b)*e^(-c + a*d/b) - 4*a*b^3*d*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
) - 4*a^2*b^2*d*x^2*Ei(-d*x)*e^(-c) - 6*b^4*x^3*Ei(-d*x)*e^(-c) + a^4*d^2*x
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 8*a^2*b^2*d*x^2*Ei((b*d*x + a*d)/b)*e^
(c - a*d/b) + 6*b^4*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^2*b^2*d*x^2
*Ei(d*x)*e^c - 6*b^4*x^3*Ei(d*x)*e^c + a^4*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b) - 8*a^2*b^2*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*b^4*x^3
*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x^2*e^(d*x + c) + a^2*b^2*
d*x^2*e^(-d*x - c) - 2*a^3*b*d*x*Ei(-d*x)*e^(-c) - 12*a*b^3*x^2*Ei(-d*x)*e^
(-c) + 4*a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a*b^3*x^2*Ei((b*d
*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d*x*Ei(d*x)*e^c - 12*a*b^3*x^2*Ei(d*x)
*e^c - 4*a^3*b*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a*b^3*x^2*Ei(-(
b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x*e^(d*x + c) - 6*a*b^3*x^2*e^(d*x
 + c) + a^3*b*d*x*e^(-d*x - c) - 6*a*b^3*x^2*e^(-d*x - c) - 6*a^2*b^2*x*Ei(
-d*x)*e^(-c) + 6*a^2*b^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a^2*b^2*x*
Ei(d*x)*e^c + 6*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 9*a^2*b^2*x
*e^(d*x + c) - 9*a^2*b^2*x*e^(-d*x - c) - 2*a^3*b*e^(d*x + c) - 2*a^3*b*e^(
-d*x - c))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

3.39 $\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal. Leaf size=377

$$\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}-\frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5}-\frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{3b}{}$$

```
[Out] -Cosh[c + d*x]/(2*a^3*x^2) + (3*b*Cosh[c + d*x])/(a^4*x) + (b^2*Cosh[c + d*
x])/(2*a^3*(a + b*x)^2) + (3*b^2*Cosh[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Co
sh[c]*CoshIntegral[d*x])/a^5 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) - (6
*b^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^5 - (d^2*Cosh[c - (a*
d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a^3) - (3*b*d*CoshIntegral[d*x]*Sinh[
c])/a^4 - (3*b*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^4 - (d*Si
nh[c + d*x])/(2*a^3*x) + (b*d*Sinh[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*Cos
h[c]*SinhIntegral[d*x])/a^4 + (6*b^2*Sinh[c]*SinhIntegral[d*x])/a^5 + (d^2*
Sinh[c]*SinhIntegral[d*x])/(2*a^3) - (3*b*d*Cosh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a^4 - (6*b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])
/a^5 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^3)
```

Rubi [A] time = 0.821274, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}-\frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5}-\frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```


```
[Out] -Cosh[c + d*x]/(2*a^3*x^2) + (3*b*Cosh[c + d*x])/(a^4*x) + (b^2*Cosh[c + d*
x])/(2*a^3*(a + b*x)^2) + (3*b^2*Cosh[c + d*x])/(a^4*(a + b*x)) + (6*b^2*Co
sh[c]*CoshIntegral[d*x])/a^5 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) - (6
*b^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^5 - (d^2*Cosh[c - (a*
d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a^3) - (3*b*d*CoshIntegral[d*x]*Sinh[
c])/a^4 - (3*b*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^4 - (d*Si
nh[c + d*x])/(2*a^3*x) + (b*d*Sinh[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*Cos
h[c]*SinhIntegral[d*x])/a^4 + (6*b^2*Sinh[c]*SinhIntegral[d*x])/a^5 + (d^2*
Sinh[c]*SinhIntegral[d*x])/(2*a^3) - (3*b*d*Cosh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a^4 - (6*b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])
/a^5 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^3)
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^3} - \frac{3b\cosh(c+dx)}{a^4x^2} + \frac{6b^2\cosh(c+dx)}{a^5x} - \frac{b^3\cosh(c+dx)}{a^3(a+bx)^3} - \frac{3b^3\cosh(c+dx)}{a^4(a+bx)^2} - \right.$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a^3} - \frac{(3b)\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^4} + \frac{\left(6b^2\right)\int \frac{\cosh(c+dx)}{x}\,dx}{a^5} - \frac{\left(6b^3\right)\int \frac{\cosh(c+dx)}{a+bx}\,dx}{a^5} - \frac{\left(3b^3\right)\int \frac{\cosh}{(a+}}{a^4}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{2a^3} - \frac{(3b}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5} -$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5} -$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2\cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5} +$$

Mathematica [A] time = 1.80699, size = 627, normalized size = 1.66

$$-\frac{-x^2(a+bx)^2\text{Chi}(dx)\left(\cosh(c)\left(a^2d^2+12b^2\right)-6abd\sinh(c)\right)+x^2(a+bx)^2\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(\left(a^2d^2+12b^2\right)\cosh\left(c-\frac{ad}{b}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

```
[Out] -(a^4*Cosh[c + d*x] - 4*a^3*b*x*Cosh[c + d*x] - 18*a^2*b^2*x^2*Cosh[c + d*x
] - 12*a*b^3*x^3*Cosh[c + d*x] - x^2*(a + b*x)^2*CoshIntegral[d*x]*((12*b^2
 + a^2*d^2)*Cosh[c] - 6*a*b*d*Sinh[c]) + x^2*(a + b*x)^2*CoshIntegral[d*(a/
b + x)]*((12*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 6*a*b*d*Sinh[c - (a*d)/b])
+ a^4*d*x*Sinh[c + d*x] + a^3*b*d*x^2*Sinh[c + d*x] + 6*a^3*b*d*x^2*Cosh[c]
*SinhIntegral[d*x] + 12*a^2*b^2*d*x^3*Cosh[c]*SinhIntegral[d*x] + 6*a*b^3*d
*x^4*Cosh[c]*SinhIntegral[d*x] - 12*a^2*b^2*x^2*Sinh[c]*SinhIntegral[d*x] -
 a^4*d^2*x^2*Sinh[c]*SinhIntegral[d*x] - 24*a*b^3*x^3*Sinh[c]*SinhIntegral[
d*x] - 2*a^3*b*d^2*x^3*Sinh[c]*SinhIntegral[d*x] - 12*b^4*x^4*Sinh[c]*SinhI
```

```
ntegral[d*x] - a^2*b^2*d^2*x^4*Sinh[c]*SinhIntegral[d*x] + 6*a^3*b*d*x^2*Co
sh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*d*x^3*Cosh[c - (a*d)
/b]*SinhIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Cosh[c - (a*d)/b]*SinhIntegra
l[d*(a/b + x)] + 12*a^2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]
 + a^4*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 24*a*b^3*x^3*S
inh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^3*Sinh[c - (a*d)
/b]*SinhIntegral[d*(a/b + x)] + 12*b^4*x^4*Sinh[c - (a*d)/b]*SinhIntegral[d
*(a/b + x)] + a^2*b^2*d^2*x^4*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/
(2*a^5*x^2*(a + b*x)^2)
```

Maple [B] time = 0.082, size = 760, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x+a)^3,x)
```

```
[Out] 1/4*d^3*exp(-d*x-c)/a^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+3*d^2*exp(-d*x-
c)/a^4*x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^3+1/4*d^3*exp(-d*x-c)/a/x/(b^2
*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+9/2*d^2*exp(-d*x-c)/a^3/(b^2*d^2*x^2+2*a*b*d^
2*x+a^2*d^2)*b^2+d^2*exp(-d*x-c)/a^2/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-
1/4*d^2*exp(-d*x-c)/a/x^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/a^3*exp
(-c)*Ei(1,d*x)-3/2*d/a^4*exp(-c)*Ei(1,d*x)*b-3/a^5*exp(-c)*Ei(1,d*x)*b^2+1/
4*d^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-3/2*d/a^4*exp((a*d-b*c)/
b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+3/a^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)
*b^2-3/a^5*b^2*exp(c)*Ei(1,-d*x)-1/4/a^3/x^2*exp(d*x+c)-1/4*d/a^3/x*exp(d*x
+c)-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)+3/2/a^4*b/x*exp(d*x+c)+3/2*d/a^4*b*exp(c)
*Ei(1,-d*x)+1/4*d^2/a^3*exp(d*x+c)/(1/b*d*a+d*x)^2+1/4*d^2/a^3*exp(d*x+c)/(
1/b*d*a+d*x)+1/4*d^2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2*d/a
^4*b*exp(d*x+c)/(1/b*d*a+d*x)+3/2*d/a^4*b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*
d-b*c)/b)+3*b^2/a^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^3), x)
```

Fricas [B] time = 2.18189, size = 1845, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*cosh(d*x + c) + ((
(a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*
b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) + ((a^2*b^2*d^2
+ 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a
^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 +
6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4
*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 6
*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*
d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/
b) - 2*(a^3*b*d*x^2 + a^4*d*x)*sinh(d*x + c) + (((a^2*b^2*d^2 - 6*a*b^3*d +
 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^
3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) - ((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4
+ 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^
2*b^2)*x^2)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 6*a*b^3*d + 12*b^4)*x^4 +
2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^3*b*d + 12*a^2*
b^2)*x^2)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2
*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b
^2)*x^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^5*b^2*x^4 + 2*a^6*b
*x^3 + a^7*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31946, size = 1578, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*b^2*d^2*x^4*Ei(-d*x)*e^(-c) - a^2*b^2*d^2*x^4*Ei((b*d*x + a*d)/b)*
e^(c - a*d/b) + a^2*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*b^2*d^2*x^4*Ei(-(b*d*x +
a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x^3*Ei(-d*x)*e^(-c) + 6*a*b^3*d*x^4*Ei
(-d*x)*e^(-c) - 2*a^3*b*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a*b^3
*d*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x^3*Ei(d*x)*e^c - 6*
a*b^3*d*x^4*Ei(d*x)*e^c - 2*a^3*b*d^2*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/
b) + 6*a*b^3*d*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^4*d^2*x^2*Ei(-d*
x)*e^(-c) + 12*a^2*b^2*d*x^3*Ei(-d*x)*e^(-c) + 12*b^4*x^4*Ei(-d*x)*e^(-c) -
 a^4*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^2*b^2*d*x^3*Ei((b*d*x
 + a*d)/b)*e^(c - a*d/b) - 12*b^4*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a
^4*d^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*d*x^3*Ei(d*x)*e^c + 12*b^4*x^4*Ei(d*x)*
e^c - a^4*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^2*b^2*d*x^3*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 12*b^4*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c +
 a*d/b) + 6*a^3*b*d*x^2*Ei(-d*x)*e^(-c) + 24*a*b^3*x^3*Ei(-d*x)*e^(-c) - 6*
a^3*b*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 24*a*b^3*x^3*Ei((b*d*x + a*
d)/b)*e^(c - a*d/b) - 6*a^3*b*d*x^2*Ei(d*x)*e^c + 24*a*b^3*x^3*Ei(d*x)*e^c
```

```
+ 6*a^3*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 24*a*b^3*x^3*Ei(-(b*d
*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x^2*e^(d*x + c) + 12*a*b^3*x^3*e^(d*x
 + c) + a^3*b*d*x^2*e^(-d*x - c) + 12*a*b^3*x^3*e^(-d*x - c) + 12*a^2*b^2*x
^2*Ei(-d*x)*e^(-c) - 12*a^2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*
a^2*b^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b
) - a^4*d*x*e^(d*x + c) + 18*a^2*b^2*x^2*e^(d*x + c) + a^4*d*x*e^(-d*x - c)
 + 18*a^2*b^2*x^2*e^(-d*x - c) + 4*a^3*b*x*e^(d*x + c) + 4*a^3*b*x*e^(-d*x
- c) - a^4*e^(d*x + c) - a^4*e^(-d*x - c))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7
*x^2)
```

3.40 $\int x^3 \left(a+bx^2\right) \cosh(c+dx) \, dx$

Optimal. Leaf size=139

$$-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{ax^3\sinh(c+dx)}{d}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}$$

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c + d
*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (120
*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c + d*
x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d
```

Rubi [A] time = 0.255416, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5287, 3296, 2638}

$$-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{ax^3\sinh(c+dx)}{d}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c + d
*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (120
*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c + d*
x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^3 \left(a+bx^2\right) \cosh(c+dx)\,dx = \int \left(ax^3 \cosh(c+dx) + bx^5 \cosh(c+dx)\right) dx$$
$$= a \int x^3 \cosh(c+dx)\,dx + b \int x^5 \cosh(c+dx)\,dx$$
$$= \frac{ax^3 \sinh(c+dx)}{d} + \frac{bx^5 \sinh(c+dx)}{d} - \frac{(3a)\int x^2 \sinh(c+dx)\,dx}{d} - \frac{(5b)\int x^4 \sinh(c}{d}$$
$$= -\frac{3ax^2 \cosh(c+dx)}{d^2} - \frac{5bx^4 \cosh(c+dx)}{d^2} + \frac{ax^3 \sinh(c+dx)}{d} + \frac{bx^5 \sinh(c+dx)}{d} +$$
$$= -\frac{3ax^2 \cosh(c+dx)}{d^2} - \frac{5bx^4 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} + \frac{20bx^3 \sinh(c+dx)}{d^3}$$
$$= -\frac{6a \cosh(c+dx)}{d^4} - \frac{60bx^2 \cosh(c+dx)}{d^4} - \frac{3ax^2 \cosh(c+dx)}{d^2} - \frac{5bx^4 \cosh(c+dx)}{d^2}$$
$$= -\frac{6a \cosh(c+dx)}{d^4} - \frac{60bx^2 \cosh(c+dx)}{d^4} - \frac{3ax^2 \cosh(c+dx)}{d^2} - \frac{5bx^4 \cosh(c+dx)}{d^2}$$
$$= -\frac{120b \cosh(c+dx)}{d^6} - \frac{6a \cosh(c+dx)}{d^4} - \frac{60bx^2 \cosh(c+dx)}{d^4} - \frac{3ax^2 \cosh(c+dx)}{d^2}$$

Mathematica [A] time = 0.149123, size = 92, normalized size = 0.66

$$\frac{dx\left(ad^2\left(d^2x^2+6\right)+b\left(d^4x^4+20d^2x^2+120\right)\right)\sinh(c+dx)-\left(3ad^2\left(d^2x^2+2\right)+5b\left(d^4x^4+12d^2x^2+24\right)\right)\cosh(c+d}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-((3*a*d^2*(2 + d^2*x^2) + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])
 + d*x*(a*d^2*(6 + d^2*x^2) + b*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x]
)/d^6
```

Maple [B] time = 0.009, size = 447, normalized size = 3.2

$$\frac{1}{d^4}\left(\frac{b\left((dx+c)^5\sinh(dx+c)-5\,(dx+c)^4\cosh(dx+c)+20\,(dx+c)^3\sinh(dx+c)-60\,(dx+c)^2\cosh(dx+c)+1}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^4*(1/d^2*b*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*
sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c
))-5/d^2*b*c*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*si
nh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+10/d^2*b*c^2*((d*x+c)^3*si
nh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-10/d
^2*b*c^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+5/d^2*
b*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-1/d^2*b*c^5*sinh(d*x+c)+a*((d*x+c)^
3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-
3*a*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+3*a*c^2*(
(d*x+c)*sinh(d*x+c)-cosh(d*x+c))-a*c^3*sinh(d*x+c))
```

Maxima [A] time = 1.07114, size = 338, normalized size = 2.43

$$-\frac{1}{24}d\left(\frac{3\left(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c\right)ae^{(dx)}}{d^5} + \frac{3\left(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24\right)ae^{(-dx-c)}}{d^5} + \frac{2\,(\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/24*d*(3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3
*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b*e^(d*x)/d^7 + 2*(d^6*
x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b
*e^(-d*x - c)/d^7) + 1/12*(2*b*x^6 + 3*a*x^4)*cosh(d*x + c)
```

Fricas [A] time = 2.01632, size = 212, normalized size = 1.53

$$-\frac{\left(5\,bd^4x^4 + 6\,ad^2 + 3\left(ad^4 + 20\,bd^2\right)x^2 + 120\,b\right)\cosh\left(dx + c\right) - \left(bd^5x^5 + \left(ad^5 + 20\,bd^3\right)x^3 + 6\left(ad^3 + 20\,bd\right)x\right)\sinh\left(dx \cdots\right.}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b*d^4*x^4 + 6*a*d^2 + 3*(a*d^4 + 20*b*d^2)*x^2 + 120*b)*cosh(d*x + c)
- (b*d^5*x^5 + (a*d^5 + 20*b*d^3)*x^3 + 6*(a*d^3 + 20*b*d)*x)*sinh(d*x + c)
)/d^6
```

Sympy [A] time = 5.48512, size = 168, normalized size = 1.21

$$\begin{cases} \frac{ax^3\sinh(c+dx)}{d} - \frac{3ax^2\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} - \frac{6a\cosh(c+dx)}{d^4} + \frac{bx^5\sinh(c+dx)}{d} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{20bx^3\sinh(c+dx)}{d^3} - \frac{60bx^2\cosh(c\cdots}{d^4} \\ \left(\frac{ax^4}{4} + \frac{bx^6}{6}\right)\cosh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**5*sinh(c + d*x)/d - 5*b*x**
4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*
x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)),
 ((a*x**4/4 + b*x**6/6)*cosh(c), True))
```

Giac [A] time = 1.20411, size = 235, normalized size = 1.69

$$\frac{\left(bd^5x^5 + ad^5x^3 - 5\,bd^4x^4 - 3\,ad^4x^2 + 20\,bd^3x^3 + 6\,ad^3x - 60\,bd^2x^2 - 6\,ad^2 + 120\,bdx - 120\,b\right)e^{(dx+c)}}{2\,d^6} - \frac{\left(bd^5x^5 + ad^5x^3 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^5*x^5 + a*d^5*x^3 - 5*b*d^4*x^4 - 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a
*d^3*x - 60*b*d^2*x^2 - 6*a*d^2 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*
(b*d^5*x^5 + a*d^5*x^3 + 5*b*d^4*x^4 + 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3
*x + 60*b*d^2*x^2 + 6*a*d^2 + 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

3.41 $\int x^2 \left(a+bx^2\right)\cosh(c+dx)\,dx$

Optimal. Leaf size=109

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}-$$

```
[Out] (-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c +
 d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x^2
*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d
```

Rubi [A] time = 0.187407, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5287, 3296, 2637}

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c +
 d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x^2
*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\left(a+bx^2\right)\cosh(c+dx)\,dx = \int \left(ax^2\cosh(c+dx)+bx^4\cosh(c+dx)\right)dx$$
$$= a\int x^2\cosh(c+dx)\,dx + b\int x^4\cosh(c+dx)\,dx$$
$$= \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d} - \frac{(2a)\int x\sinh(c+dx)\,dx}{d} - \frac{(4b)\int x^3\sinh(c\cdot}{d}$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d} + \cdots$$
$$= -\frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{12bx^2\sinh(c+dx)}{d^3} + \cdots$$
$$= -\frac{24bx\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \cdots$$
$$= -\frac{24bx\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c+dx)}{d^5} + \cdots$$

Mathematica [A] time = 0.129127, size = 74, normalized size = 0.68

$$\frac{\left(ad^2\left(d^2x^2+2\right)+b\left(d^4x^4+12d^2x^2+24\right)\right)\sinh(c+dx)-2dx\left(ad^2+2b\left(d^2x^2+6\right)\right)\cosh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*d*x*(a*d^2 + 2*b*(6 + d^2*x^2))*Cosh[c + d*x] + (a*d^2*(2 + d^2*x^2) +
b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [B] time = 0.009, size = 298, normalized size = 2.7

$$\frac{1}{d^3}\left(\frac{b\left((dx+c)^4\sinh(dx+c)-4\,(dx+c)^3\cosh(dx+c)+12\,(dx+c)^2\sinh(dx+c)-24\,(dx+c)\cosh(dx+c)+24\cdots}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^2*b*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*
sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-4/d^2*b*c*((d*x+c)^3*sin
h(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+6/d^2
*b*c^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4/d^2*b*
c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^2*b*c^4*sinh(d*x+c)+a*((d*x+c)^2*
sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*a*c*((d*x+c)*sinh(d*x+c)
-cosh(d*x+c))+a*c^2*sinh(d*x+c))
```

Maxima [A] time = 1.03039, size = 289, normalized size = 2.65

$$-\frac{1}{30}\,d\left(\frac{5\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)ae^{(dx)}}{d^4}+\frac{5\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)ae^{(-dx-c)}}{d^4}+\frac{3\left(d^5x^5e^c-5\,d^4x^4e^c+20\,d\cdots\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/30*d*(5*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
+ 5*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^5*x^5*e^c -
 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b
*e^(d*x)/d^6 + 3*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x +
 120)*b*e^(-d*x - c)/d^6) + 1/15*(3*b*x^5 + 5*a*x^3)*cosh(d*x + c)
```

Fricas [A] time = 2.02551, size = 174, normalized size = 1.6

$$-\frac{2\left(2\,bd^3x^3+\left(ad^3+12\,bd\right)x\right)\cosh\left(dx+c\right)-\left(bd^4x^4+2\,ad^2+\left(ad^4+12\,bd^2\right)x^2+24\,b\right)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(2*b*d^3*x^3 + (a*d^3 + 12*b*d)*x)*cosh(d*x + c) - (b*d^4*x^4 + 2*a*d^2
 + (a*d^4 + 12*b*d^2)*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] time = 2.61537, size = 134, normalized size = 1.23

$$\begin{cases} \frac{ax^2\sinh\left(c+dx\right)}{d}-\frac{2ax\cosh\left(c+dx\right)}{d^2}+\frac{2a\sinh\left(c+dx\right)}{d^3}+\frac{bx^4\sinh\left(c+dx\right)}{d}-\frac{4bx^3\cosh\left(c+dx\right)}{d^2}+\frac{12bx^2\sinh\left(c+dx\right)}{d^3}-\frac{24bx\cosh\left(c+dx\right)}{d^4}+\frac{24b\sinh\left(c+d\right.}{d^5} \\ \left(\frac{ax^3}{3}+\frac{bx^5}{5}\right)\cosh\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**
*2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5
, Ne(d, 0)), ((a*x**3/3 + b*x**5/5)*cosh(c), True))
```

Giac [A] time = 1.19823, size = 186, normalized size = 1.71

$$\frac{\left(bd^4x^4+ad^4x^2-4\,bd^3x^3-2\,ad^3x+12\,bd^2x^2+2\,ad^2-24\,bdx+24\,b\right)e^{(dx+c)}}{2\,d^5}-\frac{\left(bd^4x^4+ad^4x^2+4\,bd^3x^3+2\,ad^3x+12\,b\right.}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^2 - 4*b*d^3*x^3 - 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d
^2 - 24*b*d*x + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*x^2 + 4*b*d^
3*x^3 + 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d^2 + 24*b*d*x + 24*b)*e^(-d*x - c)/
d^5
```

3.42 $\int x\left(a+bx^2\right)\cosh(c+dx)\,dx$

Optimal. Leaf size=79

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4}+\frac{bx^3\sinh(c+dx)}{d}$$

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/
d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d
*x])/d
```

Rubi [A] time = 0.121082, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5287, 3296, 2638}

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4}+\frac{bx^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/
d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d
*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^2\right)\cosh(c+dx)\,dx = \int \left(ax\cosh(c+dx)+bx^3\cosh(c+dx)\right)dx$$
$$= a\int x\cosh(c+dx)\,dx + b\int x^3\cosh(c+dx)\,dx$$
$$= \frac{ax\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{a\int\sinh(c+dx)\,dx}{d} - \frac{(3b)\int x^2\sinh(c+dx)\,dx}{d}$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} + \frac{(6b)\int x$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^3\sinh$$
$$= -\frac{6b\cosh(c+dx)}{d^4} - \frac{a\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{ax\sinh($$

Mathematica [A] time = 0.0997394, size = 57, normalized size = 0.72

$$\frac{dx\left(ad^2+b\left(d^2x^2+6\right)\right)\sinh(c+dx)-\left(ad^2+3b\left(d^2x^2+2\right)\right)\cosh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-((a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*x*(a*d^2 + b*(6 + d^2*x^2
))*Sinh[c + d*x])/d^4
```

Maple [B] time = 0.007, size = 183, normalized size = 2.3

$$\frac{1}{d^2}\left(\frac{b\left((dx+c)^3\sinh(dx+c)-3\,(dx+c)^2\cosh(dx+c)+6\,(dx+c)\sinh(dx+c)-6\,\cosh(dx+c)\right)}{d^2}-3\,\frac{cb\left((dx+c)^2 s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^2*b*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sin
h(d*x+c)-6*cosh(d*x+c))-3/d^2*b*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x
+c)+2*sinh(d*x+c))+3/d^2*b*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)
*sinh(d*x+c)-cosh(d*x+c))-1/d^2*b*c^3*sinh(d*x+c)-c*a*sinh(d*x+c))
```

Maxima [B] time = 1.08556, size = 288, normalized size = 3.65

$$\frac{\left(bx^2+a\right)^2\cosh(dx+c)}{4\,b}-\frac{\left(\frac{a^2e^{(dx+c)}}{d}+\frac{a^2e^{(-dx-c)}}{d}+\frac{2\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)abe^{(dx)}}{d^3}+\frac{2\left(d^2x^2+2\,dx+2\right)abe^{(-dx-c)}}{d^3}+\frac{\left(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-2\right.}{d^5}\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^2*cosh(d*x + c)/b - 1/8*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
c)/d + 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^(d*x)/d^3 + 2*(d^2*x^2 + 2
```

```
*d*x + 2)*a*b*e^(-d*x - c)/d^3 + (d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*
e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*
x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d/b
```

Fricas [A] time = 1.98437, size = 132, normalized size = 1.67

$$-\frac{\left(3\,bd^2x^2 + ad^2 + 6\,b\right)\cosh\left(dx + c\right) - \left(bd^3x^3 + \left(ad^3 + 6\,bd\right)x\right)\sinh\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((3*b*d^2*x^2 + a*d^2 + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + (a*d^3 + 6*b*d)*
x)*sinh(d*x + c))/d^4
```

Sympy [A] time = 1.31972, size = 99, normalized size = 1.25

$$\begin{cases} \frac{ax\sinh\left(c+dx\right)}{d} - \frac{a\cosh\left(c+dx\right)}{d^2} + \frac{bx^3\sinh\left(c+dx\right)}{d} - \frac{3bx^2\cosh\left(c+dx\right)}{d^2} + \frac{6bx\sinh\left(c+dx\right)}{d^3} - \frac{6b\cosh\left(c+dx\right)}{d^4} & \text{for}\: d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^4}{4}\right)\cosh\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**3*sinh(c + d*x
)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c +
 d*x)/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*cosh(c), True))
```

Giac [A] time = 1.17752, size = 136, normalized size = 1.72

$$\frac{\left(bd^3x^3 + ad^3x - 3\,bd^2x^2 - ad^2 + 6\,bdx - 6\,b\right)e^{\left(dx+c\right)}}{2\,d^4} - \frac{\left(bd^3x^3 + ad^3x + 3\,bd^2x^2 + ad^2 + 6\,bdx + 6\,b\right)e^{\left(-dx-c\right)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 + a*d^3*x - 3*b*d^2*x^2 - a*d^2 + 6*b*d*x - 6*b)*e^(d*x + c)
/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x + 3*b*d^2*x^2 + a*d^2 + 6*b*d*x + 6*b)*e^(-
d*x - c)/d^4
```

3.43 $\int \left(a+bx^2\right) \cosh(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{a\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d
+ (b*x^2*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0667648, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5277, 2637, 3296}

$$\frac{a\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d
+ (b*x^2*Sinh[c + d*x])/d
```

Rule 5277

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^2\right)\cosh(c+dx)\,dx &= \int \left(a\cosh(c+dx)+bx^2\cosh(c+dx)\right)dx\\
&= a\int \cosh(c+dx)\,dx+b\int x^2\cosh(c+dx)\,dx\\
&= \frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d}\\
&= -\frac{2bx\cosh(c+dx)}{d^2}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}+\frac{(2b)\int \cosh(c+dx)\,dx}{d^2}\\
&= -\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0667749, size = 40, normalized size = 0.78

$$\frac{\left(ad^2+b\left(d^2x^2+2\right)\right)\sinh(c+dx)-2bdx\cosh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*Cosh[c + d*x],x]
```

```
[Out] (-2*b*d*x*Cosh[c + d*x] + (a*d^2 + b*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
```

Maple [A] time = 0.008, size = 97, normalized size = 1.9

$$\frac{1}{d}\left(\frac{b\left((dx+c)^2\sinh(dx+c)-2\,(dx+c)\cosh(dx+c)+2\,\sinh(dx+c)\right)}{d^2}-2\,\frac{cb\left((dx+c)\sinh(dx+c)-\cosh(dx+c\right.}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c),x)
```

```
[Out] 1/d*(1/d^2*b*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2/
d^2*b*c*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^2*b*c^2*sinh(d*x+c)+a*sinh(d*
x+c))
```

Maxima [A] time = 1.01897, size = 116, normalized size = 2.27

$$\frac{ae^{(dx+c)}}{2\,d}-\frac{ae^{(-dx-c)}}{2\,d}+\frac{\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)be^{(dx)}}{2\,d^3}-\frac{\left(d^2x^2+2\,dx+2\right)be^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c +
 2*e^c)*b*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3
```

Fricas [A] time = 1.9949, size = 97, normalized size = 1.9

$$-\frac{2\,bdx\cosh(dx+c)-\left(bd^2x^2+ad^2+2\,b\right)\sinh(dx+c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*b*d*x*cosh(d*x + c) - (b*d^2*x^2 + a*d^2 + 2*b)*sinh(d*x + c))/d^3
```

Sympy [A] time = 0.674157, size = 65, normalized size = 1.27

$$\begin{cases} \frac{a\sinh(c+dx)}{d} + \frac{bx^2\sinh(c+dx)}{d} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^3}{3}\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**2*sinh(c + d*x)/d - 2*b*x*cosh(c + d*x)
/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*cosh(c), True)
)
```

Giac [A] time = 1.15273, size = 95, normalized size = 1.86

$$\frac{\left(bd^2x^2 + ad^2 - 2\,bdx + 2\,b\right)e^{(dx+c)}}{2\,d^3} - \frac{\left(bd^2x^2 + ad^2 + 2\,bdx + 2\,b\right)e^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + a*d^2 - 2*b*d*x + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^2*x^2 +
a*d^2 + 2*b*d*x + 2*b)*e^(-d*x - c)/d^3
```

3.44 $\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$

Optimal. Leaf size=41

$$a\cosh(c)\mathrm{Chi}(dx)+a\sinh(c)\mathrm{Shi}(dx)-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

```
[Out] -((b*Cosh[c + d*x])/d^2) + a*Cosh[c]*CoshIntegral[d*x] + (b*x*Sinh[c + d*x]
)/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.0980776, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5287, 3303, 3298, 3301, 3296, 2638}

$$a\cosh(c)\mathrm{Chi}(dx)+a\sinh(c)\mathrm{Shi}(dx)-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) + a*Cosh[c]*CoshIntegral[d*x] + (b*x*Sinh[c + d*x]
)/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x}+bx\cosh(c+dx)\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x}\,dx+b\int x\cosh(c+dx)\,dx \\
&= \frac{bx\sinh(c+dx)}{d}-\frac{b\int \sinh(c+dx)\,dx}{d}+(a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(a\sinh(c))\int \frac{\sinh
\end{aligned}
$$

$$
= -\frac{b\cosh(c+dx)}{d^2}+a\cosh(c)\mathbf{Chi}(dx)+\frac{bx\sinh(c+dx)}{d}+a\sinh(c)\mathbf{Shi}(dx)
$$

Mathematica [A] time = 0.109761, size = 55, normalized size = 1.34

$$
a\cosh(c)\mathbf{Chi}(dx)+a\sinh(c)\mathbf{Shi}(dx)+\frac{b\cosh(dx)(dx\sinh(c)-\cosh(c))}{d^2}+\frac{b\sinh(dx)(dx\cosh(c)-\sinh(c))}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*(-Cosh[c] + d*x*Sinh[c]))/d^2 +
(b*(d*x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2 + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.035, size = 81, normalized size = 2.

$$
-\frac{ae^{-c}\mathrm{Ei}\,(1,dx)}{2}-\frac{be^{-dx-c}x}{2d}-\frac{be^{-dx-c}}{2d^2}-\frac{ae^{c}\mathrm{Ei}\,(1,-dx)}{2}+\frac{be^{dx+c}x}{2d}-\frac{be^{dx+c}}{2d^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x,x)
```

```
[Out] -1/2*a*exp(-c)*Ei(1,d*x)-1/2/d*b*exp(-d*x-c)*x-1/2/d^2*b*exp(-d*x-c)-1/2*a*
exp(c)*Ei(1,-d*x)+1/2/d*b*exp(d*x+c)*x-1/2/d^2*b*exp(d*x+c)
```

Maxima [B] time = 1.16856, size = 165, normalized size = 4.02

$$
-\frac{1}{4}\left(b\left(\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}\right)+\frac{2\,a\cosh(dx+c)\log(x^2)}{d}-\frac{2\left(\mathrm{Ei}\,(-dx)\,e^{(-c)}+\mathrm{Ei}\,(dx)\right.}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x +
 2)*e^(-d*x - c)/d^3) + 2*a*cosh(d*x + c)*log(x^2)/d - 2*(Ei(-d*x)*e^(-c) +
 Ei(d*x)*e^c)*a/d)*d + 1/2*(b*x^2 + a*log(x^2))*cosh(d*x + c)
```

Fricas [A] time = 2.06274, size = 188, normalized size = 4.59

$$\frac{2\,bdx\sinh\left(dx+c\right)-2\,b\cosh\left(dx+c\right)+\left(ad^{2}\mathrm{Ei}\left(dx\right)+ad^{2}\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)+\left(ad^{2}\mathrm{Ei}\left(dx\right)-ad^{2}\mathrm{Ei}\left(-dx\right)\right)\sinh\left(c\right)}{2\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*sinh(d*x + c) - 2*b*cosh(d*x + c) + (a*d^2*Ei(d*x) + a*d^2*Ei(
-d*x))*cosh(c) + (a*d^2*Ei(d*x) - a*d^2*Ei(-d*x))*sinh(c))/d^2
```

Sympy [A] time = 3.56531, size = 49, normalized size = 1.2

$$a\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a\cosh\left(c\right)\mathrm{Chi}\left(dx\right)+b\left(\begin{cases}\frac{x\sinh\left(c+dx\right)}{d}-\frac{\cosh\left(c+dx\right)}{d^{2}} & \text{for } d\neq 0\\ \frac{x^{2}\cosh\left(c\right)}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x*sinh(c + d*x)/d -
cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Giac [A] time = 1.17627, size = 103, normalized size = 2.51

$$\frac{ad^{2}\mathrm{Ei}\left(-dx\right)e^{(-c)}+ad^{2}\mathrm{Ei}\left(dx\right)e^{c}+bdxe^{(dx+c)}-bdxe^{(-dx-c)}-be^{(dx+c)}-be^{(-dx-c)}}{2\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a*d^2*Ei(-d*x)*e^(-c) + a*d^2*Ei(d*x)*e^c + b*d*x*e^(d*x + c) - b*d*x*
e^(-d*x - c) - b*e^(d*x + c) - b*e^(-d*x - c))/d^2
```

3.45 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx$

Optimal. Leaf size=42

$$ad\sinh(c)\mathrm{Chi}(dx) + ad\cosh(c)\mathrm{Shi}(dx) - \frac{a\cosh(c+dx)}{x} + \frac{b\sinh(c+dx)}{d}$$

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.106847, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$ad\sinh(c)\mathrm{Chi}(dx) + ad\cosh(c)\mathrm{Shi}(dx) - \frac{a\cosh(c+dx)}{x} + \frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x^2}\right)dx\\
&= a\int \frac{\cosh(c+dx)}{x^2}\,dx + b\int \cosh(c+dx)\,dx\\
&= -\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}+(ad)\int \frac{\sinh(c+dx)}{x}\,dx\\
&= -\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}+(ad\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+(ad\sinh(c))\int \frac{\cos
\end{aligned}$$

$$= -\frac{a\cosh(c+dx)}{x}+ad\mathbf{Chi}(dx)\sinh(c)+\frac{b\sinh(c+dx)}{d}+ad\cosh(c)\mathbf{Shi}(dx)$$

Mathematica [A] time = 0.0881973, size = 42, normalized size = 1.

$$ad\sinh(c)\mathbf{Chi}(dx)+ad\cosh(c)\mathbf{Shi}(dx)-\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])/
d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.046, size = 81, normalized size = 1.9

$$-\frac{a\mathrm{e}^{-dx-c}}{2x}+\frac{da\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{2}-\frac{b\mathrm{e}^{-dx-c}}{2d}-\frac{a\mathrm{e}^{dx+c}}{2x}-\frac{da\mathrm{e}^{c}\mathrm{Ei}(1,-dx)}{2}+\frac{b\mathrm{e}^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^2,x)
```

```
[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2*b/d*exp(-d*x-c)-1/2*a/x*
exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)+1/2*b/d*exp(d*x+c)
```

Maxima [A] time = 1.18785, size = 108, normalized size = 2.57

$$-\frac{1}{2}\left(a\mathrm{Ei}(-dx)e^{(-c)}-a\mathrm{Ei}(dx)e^{c}+\frac{(dxe^{c}-e^{c})be^{(dx)}}{d^2}+\frac{(dx+1)be^{(-dx-c)}}{d^2}\right)d+\left(bx-\frac{a}{x}\right)\cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*Ei(-d*x)*e^(-c) - a*Ei(d*x)*e^c + (d*x*e^c - e^c)*b*e^(d*x)/d^2 + (
d*x + 1)*b*e^(-d*x - c)/d^2)*d + (b*x - a/x)*cosh(d*x + c)
```

Fricas [A] time = 2.084, size = 203, normalized size = 4.83

$$-\frac{2\,ad\cosh\left(dx+c\right)-2\,bx\sinh\left(dx+c\right)-\left(ad^2x\mathrm{Ei}\left(dx\right)-ad^2x\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(ad^2x\mathrm{Ei}\left(dx\right)+ad^2x\mathrm{Ei}\left(-dx\right)\right)\sinh\left(c\right)}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cosh(d*x + c) - 2*b*x*sinh(d*x + c) - (a*d^2*x*Ei(d*x) - a*d^2*
x*Ei(-d*x))*cosh(c) - (a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*cosh(c + d*x)/x**2, x)
```

Giac [A] time = 1.22212, size = 108, normalized size = 2.57

$$-\frac{ad^2x\mathrm{Ei}\left(-dx\right)e^{(-c)}-ad^2x\mathrm{Ei}\left(dx\right)e^{c}+ade^{(dx+c)}-bxe^{(dx+c)}+ade^{(-dx-c)}+bxe^{(-dx-c)}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^2*x*Ei(-d*x)*e^(-c) - a*d^2*x*Ei(d*x)*e^c + a*d*e^(d*x + c) - b*x
*e^(d*x + c) + a*d*e^(-d*x - c) + b*x*e^(-d*x - c))/(d*x)
```

3.46 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx) - \frac{a\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{2x} + b\cosh(c)\text{Chi}(dx) + b\sinh(c)\text{Shi}($$

```
[Out] -(a*Cosh[c + d*x])/(2*x^2) + b*Cosh[c]*CoshIntegral[d*x] + (a*d^2*Cosh[c]*C
oshIntegral[d*x])/2 - (a*d*Sinh[c + d*x])/(2*x) + b*Sinh[c]*SinhIntegral[d*
x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] time = 0.178131, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx) - \frac{a\cosh(c+dx)}{2x^2} - \frac{ad\sinh(c+dx)}{2x} + b\cosh(c)\text{Chi}(dx) + b\sinh(c)\text{Shi}($$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```


```
[Out] -(a*Cosh[c + d*x])/(2*x^2) + b*Cosh[c]*CoshIntegral[d*x] + (a*d^2*Cosh[c]*C
oshIntegral[d*x])/2 - (a*d*Sinh[c + d*x])/(2*x) + b*Sinh[c]*SinhIntegral[d*
x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^3}\,dx + b\int \frac{\cosh(c+dx)}{x}\,dx\\ &= -\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx + (b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx + (b\sinh(c))\\ &= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x}+b\sinh(c)\mathbf{Shi}(dx)+\frac{1}{2}(ad^2)\int\\ &= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x}+b\sinh(c)\mathbf{Shi}(dx)+\frac{1}{2}(ad^2\cosh\\ &= -\frac{a\cosh(c+dx)}{2x^2}+b\cosh(c)\mathbf{Chi}(dx)+\frac{1}{2}ad^2\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh(c+dx)}{2x}+b\sinh\end{aligned}$$

Mathematica [A] time = 0.147441, size = 80, normalized size = 1.08

$$\frac{1}{2}ad^2(\cosh(c)\mathbf{Chi}(dx)+\sinh(c)\mathbf{Shi}(dx))-\frac{a\cosh(dx)(dx\sinh(c)+\cosh(c))}{2x^2}-\frac{a\sinh(dx)(dx\cosh(c)+\sinh(c))}{2x^2}+b\cos$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

```
[Out] b*Cosh[c]*CoshIntegral[d*x] - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2)
 - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + b*Sinh[c]*SinhIntegral[d
*x] + (a*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Maple [A] time = 0.044, size = 110, normalized size = 1.5

$$\frac{da\mathrm{e}^{-dx-c}}{4x}-\frac{a\mathrm{e}^{-dx-c}}{4x^2}-\frac{d^2a\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{4}-\frac{b\mathrm{e}^{-c}\mathrm{Ei}(1,dx)}{2}-\frac{a\mathrm{e}^{dx+c}}{4x^2}-\frac{da\mathrm{e}^{dx+c}}{4x}-\frac{d^2a\mathrm{e}^{c}\mathrm{Ei}(1,-dx)}{4}-\frac{b\mathrm{e}^{c}\mathrm{Ei}(1,-dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^3,x)
```

```
[Out] 1/4*d*a*exp(-d*x-c)/x-1/4*a*exp(-d*x-c)/x^2-1/4*d^2*a*exp(-c)*Ei(1,d*x)-1/2
*b*exp(-c)*Ei(1,d*x)-1/4*a/x^2*exp(d*x+c)-1/4*d*a/x*exp(d*x+c)-1/4*d^2*a*ex
p(c)*Ei(1,-d*x)-1/2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.19083, size = 122, normalized size = 1.65

$$\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a-\frac{2b\cosh(dx+c)\log(x^2)}{d}+\frac{2\left(\mathrm{Ei}(-dx)e^{(-c)}+\mathrm{Ei}(dx)e^{c}\right)b}{d}\right)d+\frac{1}{2}\left(b\log(x^2)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a - 2*b*cosh(d*x + c
)*log(x^2)/d + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/2*(b*log(x^2) -
 a/x^2)*cosh(d*x + c)
```

Fricas [A] time = 2.02335, size = 254, normalized size = 3.43

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\,a\cosh\left(dx+c\right)-\left(\left(ad^{2}+2\,b\right)x^{2}\mathrm{Ei}\left(dx\right)+\left(ad^{2}+2\,b\right)x^{2}\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(\left(ad^{2}+2\,b\right)x^{2}\mathrm{Ei}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((a*d^2 + 2*b)*x^2*Ei(d*x
) + (a*d^2 + 2*b)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) - (a*d
^2 + 2*b)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.16015, size = 147, normalized size = 1.99

$$\frac{ad^{2}x^{2}\mathrm{Ei}\left(-dx\right)e^{\left(-c\right)}+ad^{2}x^{2}\mathrm{Ei}\left(dx\right)e^{c}+2\,bx^{2}\mathrm{Ei}\left(-dx\right)e^{\left(-c\right)}+2\,bx^{2}\mathrm{Ei}\left(dx\right)e^{c}-adxe^{\left(dx+c\right)}+adxe^{\left(-dx-c\right)}-ae^{\left(dx+c\right)}-ae^{(-c}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c + 2*b*x^2*Ei(-d*x)*e
^(-c) + 2*b*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) - a*e^
(d*x + c) - a*e^(-d*x - c))/x^2
```

3.47 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx$

Optimal. Leaf size=105

$$\frac{1}{6}ad^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathrm{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{a\cosh(c+dx)}{3x^3}+bd\sinh(c)\mathrm{Chi}(dx$$

```
[Out] -(a*Cosh[c + d*x])/(3*x^3) - (b*Cosh[c + d*x])/x - (a*d^2*Cosh[c + d*x])/(6
*x) + b*d*CoshIntegral[d*x]*Sinh[c] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 -
 (a*d*Sinh[c + d*x])/(6*x^2) + b*d*Cosh[c]*SinhIntegral[d*x] + (a*d^3*Cosh[
c]*SinhIntegral[d*x])/6
```

Rubi [A] time = 0.233946, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathrm{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{a\cosh(c+dx)}{3x^3}+bd\sinh(c)\mathrm{Chi}(dx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```


```
[Out] -(a*Cosh[c + d*x])/(3*x^3) - (b*Cosh[c + d*x])/x - (a*d^2*Cosh[c + d*x])/(6
*x) + b*d*CoshIntegral[d*x]*Sinh[c] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 -
 (a*d*Sinh[c + d*x])/(6*x^2) + b*d*Cosh[c]*SinhIntegral[d*x] + (a*d^3*Cosh[
c]*SinhIntegral[d*x])/6
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^2}\right)dx$$
$$= a\int \frac{\cosh(c+dx)}{x^4}\,dx + b\int \frac{\cosh(c+dx)}{x^2}\,dx$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} + \frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx + (bd)\int \frac{\sinh(c+dx)}{x}$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad\sinh(c+dx)}{6x^2} + \frac{1}{6}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x^2}\,dx +$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\,\mathrm{Chi}(dx)\sinh(c) - \frac{ad\sinh}{6}$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\,\mathrm{Chi}(dx)\sinh(c) - \frac{ad\sinh}{6}$$
$$= -\frac{a\cosh(c+dx)}{3x^3} - \frac{b\cosh(c+dx)}{x} - \frac{ad^2\cosh(c+dx)}{6x} + bd\,\mathrm{Chi}(dx)\sinh(c) + \frac{1}{6}ad^3\mathrm{C}$$

Mathematica [A] time = 0.22498, size = 95, normalized size = 0.9

$$-\frac{-dx^3\sinh(c)\left(ad^2+6b\right)\mathrm{Chi}(dx) - dx^3\cosh(c)\left(ad^2+6b\right)\mathrm{Shi}(dx) + ad^2x^2\cosh(c+dx) + adx\sinh(c+dx) + 2a\cos}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(2*a*Cosh[c + d*x] + 6*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d*(
6*b + a*d^2)*x^3*CoshIntegral[d*x]*Sinh[c] + a*d*x*Sinh[c + d*x] - d*(6*b +
 a*d^2)*x^3*Cosh[c]*SinhIntegral[d*x])/(6*x^3)
```

Maple [A] time = 0.059, size = 172, normalized size = 1.6

$$-\frac{ad^2e^{-dx-c}}{12\,x} + \frac{dae^{-dx-c}}{12\,x^2} - \frac{ae^{-dx-c}}{6\,x^3} + \frac{d^3ae^{-c}\mathrm{Ei}\,(1,dx)}{12} - \frac{be^{-dx-c}}{2\,x} + \frac{dbe^{-c}\mathrm{Ei}\,(1,dx)}{2} - \frac{ae^{dx+c}}{6\,x^3} - \frac{dae^{dx+c}}{12\,x^2} - \frac{ad^2e^{dx+c}}{12\,x} - \frac{d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^4,x)
```

```
[Out] -1/12*d^2*a*exp(-d*x-c)/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3+1/
12*d^3*a*exp(-c)*Ei(1,d*x)-1/2*b*exp(-d*x-c)/x+1/2*d*b*exp(-c)*Ei(1,d*x)-1/
6*a/x^3*exp(d*x+c)-1/12*d*a/x^2*exp(d*x+c)-1/12*d^2*a/x*exp(d*x+c)-1/12*d^3
*a*exp(c)*Ei(1,-d*x)-1/2*b/x*exp(d*x+c)-1/2*d*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.21257, size = 99, normalized size = 0.94

$$\frac{1}{6}\left(ad^2e^{(-c)}\Gamma\left(-2,dx\right) - ad^2e^c\Gamma\left(-2,-dx\right) - 3\,b\mathrm{Ei}\left(-dx\right)e^{(-c)} + 3\,b\mathrm{Ei}\left(dx\right)e^c\right)d - \frac{\left(3\,bx^2+a\right)\cosh\left(dx+c\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a*d^2*e^(-c)*gamma(-2, d*x) - a*d^2*e^c*gamma(-2, -d*x) - 3*b*Ei(-d*x)
*e^(-c) + 3*b*Ei(d*x)*e^c)*d - 1/3*(3*b*x^2 + a)*cosh(d*x + c)/x^3
```

Fricas [A] time = 1.98027, size = 298, normalized size = 2.84

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\left(\left(ad^{2}+6\,b\right)x^{2}+2\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^{3}+6\,bd\right)x^{3}\mathrm{Ei}\left(dx\right)-\left(ad^{3}+6\,bd\right)x^{3}\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)\cdots}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*((a*d^2 + 6*b)*x^2 + 2*a)*cosh(d*x + c) -
((a*d^3 + 6*b*d)*x^3*Ei(d*x) - (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*cosh(c) - ((a*
d^3 + 6*b*d)*x^3*Ei(d*x) + (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.18282, size = 230, normalized size = 2.19

$$-\frac{ad^{3}x^{3}\mathrm{Ei}\left(-dx\right)e^{(-c)}-ad^{3}x^{3}\mathrm{Ei}\left(dx\right)e^{c}+6\,bdx^{3}\mathrm{Ei}\left(-dx\right)e^{(-c)}-6\,bdx^{3}\mathrm{Ei}\left(dx\right)e^{c}+ad^{2}x^{2}e^{(dx+c)}+ad^{2}x^{2}e^{(-dx-c)}+adxe^{(dx+c)}\cdots}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + 6*b*d*x^3*Ei(-d*
x)*e^(-c) - 6*b*d*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(-d
*x - c) + a*d*x*e^(d*x + c) + 6*b*x^2*e^(d*x + c) - a*d*x*e^(-d*x - c) + 6*
b*x^2*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

3.48 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$

Optimal. Leaf size=149

$$\frac{1}{24}ad^4\cosh(c)\mathrm{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\mathrm{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{a\cosh(c}{4x^4}$$

```
[Out] -(a*Cosh[c + d*x])/(4*x^4) - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(24*x^2) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^4*Cosh[c]*CoshInt
egral[d*x])/24 - (a*d*Sinh[c + d*x])/(12*x^3) - (b*d*Sinh[c + d*x])/(2*x) -
 (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a*d^
4*Sinh[c]*SinhIntegral[d*x])/24
```

Rubi [A] time = 0.290435, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{24}ad^4\cosh(c)\mathrm{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\mathrm{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{a\cosh(c}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -(a*Cosh[c + d*x])/(4*x^4) - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d*
x])/(24*x^2) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^4*Cosh[c]*CoshInt
egral[d*x])/24 - (a*d*Sinh[c + d*x])/(12*x^3) - (b*d*Sinh[c + d*x])/(2*x) -
 (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a*d^
4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^3}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^5}\,dx + b\int \frac{\cosh(c+dx)}{x^3}\,dx \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}+\frac{1}{4}(ad)\int \frac{\sinh(c+dx)}{x^4}\,dx+\frac{1}{2}(bd)\int \frac{\sinh(c+dx)}{x^2}\,d \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+dx)}{2x}+\frac{1}{12}\left(ad^2\right)\int \frac{\text{co}}{} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{ad\sinh(c+dx)}{12x^3}-\frac{bd\sinh(c+d}{2x} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{24x^2}+\frac{1}{2}bd^2\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh}{12} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{24x^2}+\frac{1}{2}bd^2\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh}{12} \\
&= -\frac{a\cosh(c+dx)}{4x^4}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{24x^2}+\frac{1}{2}bd^2\cosh(c)\mathbf{Chi}(dx)+\frac{1}{24}ad^4\,\text{c}
\end{aligned}
$$

Mathematica [A] time = 0.274468, size = 127, normalized size = 0.85

$$
-\frac{-d^2x^4\cosh(c)\left(ad^2+12b\right)\mathbf{Chi}(dx)-d^2x^4\sinh(c)\left(ad^2+12b\right)\mathbf{Shi}(dx)+ad^3x^3\sinh(c+dx)+ad^2x^2\cosh(c+dx)+2a}{24x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

```
[Out] -(6*a*Cosh[c + d*x] + 12*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] - d^
2*(12*b + a*d^2)*x^4*Cosh[c]*CoshIntegral[d*x] + 2*a*d*x*Sinh[c + d*x] + 12
*b*d*x^3*Sinh[c + d*x] + a*d^3*x^3*Sinh[c + d*x] - d^2*(12*b + a*d^2)*x^4*S
inh[c]*SinhIntegral[d*x])/(24*x^4)
```

Maple [A] time = 0.066, size = 238, normalized size = 1.6

$$
\frac{ad^3e^{-dx-c}}{48\,x}-\frac{ad^2e^{-dx-c}}{48\,x^2}+\frac{dae^{-dx-c}}{24\,x^3}-\frac{ae^{-dx-c}}{8\,x^4}+\frac{bde^{-dx-c}}{4\,x}-\frac{be^{-dx-c}}{4\,x^2}-\frac{d^2be^{-c}\text{Ei}\,(1,dx)}{4}-\frac{d^4ae^{-c}\text{Ei}\,(1,dx)}{48}-\frac{d^4ae^{c}\text{Ei}\,(1,-}{48}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*cosh(d*x+c)/x^5,x)
```

```
[Out] 1/48*d^3*a*exp(-d*x-c)/x-1/48*d^2*a*exp(-d*x-c)/x^2+1/24*d*a*exp(-d*x-c)/x^
3-1/8*a*exp(-d*x-c)/x^4+1/4*d*b*exp(-d*x-c)/x-1/4*b*exp(-d*x-c)/x^2-1/4*d^2
*b*exp(-c)*Ei(1,d*x)-1/48*d^4*a*exp(-c)*Ei(1,d*x)-1/48*d^4*a*exp(c)*Ei(1,-d
*x)-1/8*a/x^4*exp(d*x+c)-1/24*d*a/x^3*exp(d*x+c)-1/48*d^2*a/x^2*exp(d*x+c)-
1/48*d^3*a/x*exp(d*x+c)-1/4*b/x^2*exp(d*x+c)-1/4*d*b/x*exp(d*x+c)-1/4*d^2*b
```

```
*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.21251, size = 103, normalized size = 0.69

$$\frac{1}{8}\left(ad^3e^{(-c)}\Gamma\left(-3,dx\right)+ad^3e^c\Gamma\left(-3,-dx\right)+2\,bde^{(-c)}\Gamma\left(-1,dx\right)+2\,bde^c\Gamma\left(-1,-dx\right)\right)d-\frac{\left(2\,bx^2+a\right)\cosh\left(dx+c\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(a*d^3*e^(-c)*gamma(-3, d*x) + a*d^3*e^c*gamma(-3, -d*x) + 2*b*d*e^(-c)
*gamma(-1, d*x) + 2*b*d*e^c*gamma(-1, -d*x))*d - 1/4*(2*b*x^2 + a)*cosh(d*x
 + c)/x^4
```

Fricas [A] time = 2.00653, size = 352, normalized size = 2.36

$$-\frac{2\left(\left(ad^2+12\,b\right)x^2+6\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^4+12\,bd^2\right)x^4\mathrm{Ei}\left(dx\right)+\left(ad^4+12\,bd^2\right)x^4\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)+2\left(\left(ad^3+1\right.\right.}{48\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a*d^2 + 12*b)*x^2 + 6*a)*cosh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4
*Ei(d*x) + (a*d^4 + 12*b*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*((a*d^3 + 12*b*d)*x
^3 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*Ei(d*x) - (a*d^4 + 12
*b*d^2)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*cosh(d*x+c)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19852, size = 320, normalized size = 2.15

$$\frac{ad^4x^4\mathrm{Ei}\left(-dx\right)e^{(-c)}+ad^4x^4\mathrm{Ei}\left(dx\right)e^c+12\,bd^2x^4\mathrm{Ei}\left(-dx\right)e^{(-c)}+12\,bd^2x^4\mathrm{Ei}\left(dx\right)e^c-ad^3x^3e^{(dx+c)}+ad^3x^3e^{(-dx-c)}-ad^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c + 12*b*d^2*x^4*Ei(-
d*x)*e^(-c) + 12*b*d^2*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*
```

```
e^(-d*x - c) - a*d^2*x^2*e^(d*x + c) - 12*b*d*x^3*e^(d*x + c) - a*d^2*x^2*e
^(-d*x - c) + 12*b*d*x^3*e^(-d*x - c) - 2*a*d*x*e^(d*x + c) - 12*b*x^2*e^(d
*x + c) + 2*a*d*x*e^(-d*x - c) - 12*b*x^2*e^(-d*x - c) - 6*a*e^(d*x + c) -
6*a*e^(-d*x - c))/x^4
```

3.49 $\int x^2 \left(a+bx^2\right)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=234

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{48ab\sinh}{d^5}$$

```
[Out] (-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*
x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48
*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c +
 d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (
30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh
[c + d*x])/d
```

Rubi [A] time = 0.388255, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5287, 3296, 2637}

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{48ab\sinh}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```


```
[Out] (-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*
x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48
*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c +
 d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (
30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh
[c + d*x])/d
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2 \left(a + bx^2\right)^2 \cosh(c + dx)\, dx = \int \left(a^2x^2 \cosh(c + dx) + 2abx^4 \cosh(c + dx) + b^2x^6 \cosh(c + dx)\right) dx$$
$$= a^2 \int x^2 \cosh(c + dx)\, dx + (2ab) \int x^4 \cosh(c + dx)\, dx + b^2 \int x^6 \cosh(c + dx)\, dx$$
$$= \frac{a^2x^2 \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2x^6 \sinh(c + dx)}{d} - \frac{\left(2a^2\right) \int x \sinh(c + dx}{d}$$
$$= -\frac{2a^2x \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} + \frac{a^2x^2 \sinh(c + dx)}{d}$$
$$= -\frac{2a^2x \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} + \frac{2a^2 \sinh(c + dx)}{d^3} -$$
$$= -\frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2x \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c +}{d^2}$$
$$= -\frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2x \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c +}{d^2}$$
$$= -\frac{720b^2x \cosh(c + dx)}{d^6} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2x \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c -}{d^4}$$
$$= -\frac{720b^2x \cosh(c + dx)}{d^6} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2x \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c -}{d^4}$$

Mathematica [A] time = 0.320592, size = 138, normalized size = 0.59

$$\frac{\left(a^2d^4 \left(d^2x^2 + 2\right) + 2abd^2 \left(d^4x^4 + 12d^2x^2 + 24\right) + b^2 \left(d^6x^6 + 30d^4x^4 + 360d^2x^2 + 720\right)\right) \sinh(c + dx) - 2dx \left(a^2d^4 + 4abd^2\right.}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*x*(a^2*d^4 + 4*a*b*d^2*(6 + d^2*x^2) + 3*b^2*(120 + 20*d^2*x^2 + d^4*
x^4))*Cosh[c + d*x] + (a^2*d^4*(2 + d^2*x^2) + 2*a*b*d^2*(24 + 12*d^2*x^2 +
 d^4*x^4) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/
d^7
```

Maple [B] time = 0.009, size = 738, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^4*b^2*((d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^
4*sinh(d*x+c)-120*(d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+
c)*cosh(d*x+c)+720*sinh(d*x+c))+1/d^4*b^2*c^6*sinh(d*x+c)-2*a^2*c*((d*x+c)*
sinh(d*x+c)-cosh(d*x+c))+a^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2
*sinh(d*x+c))-8/d^2*b*c^3*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-8/d^2*b*a*c*(
(d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(
d*x+c))+12/d^2*b*c^2*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(
d*x+c))+2/d^2*b*c^4*a*sinh(d*x+c)-6/d^4*b^2*c^5*((d*x+c)*sinh(d*x+c)-cosh(d
*x+c))-20/d^4*b^2*c^3*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x
+c)*sinh(d*x+c)-6*cosh(d*x+c))+15/d^4*b^2*c^4*((d*x+c)^2*sinh(d*x+c)-2*(d*x
+c)*cosh(d*x+c)+2*sinh(d*x+c))+2/d^2*b*a*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3
```

```
*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c)
)-6/d^4*b^2*c*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*s
inh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c)
)+15/d^4*b^2*c^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^
2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+a^2*c^2*sinh(d*x+c))
```

Maxima [A] time = 1.05534, size = 517, normalized size = 2.21

$$-\frac{1}{210}\,d\left(\frac{35\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)a^2e^{(dx)}}{d^4}+\frac{35\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)a^2e^{(-dx-c)}}{d^4}+\frac{42\left(d^5x^5e^c-5\,d^4x^4e^c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/210*d*(35*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/
d^4 + 35*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 42*(d^5*x
^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 12
0*e^c)*a*b*e^(d*x)/d^6 + 42*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2
+ 120*d*x + 120)*a*b*e^(-d*x - c)/d^6 + 15*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 4
2*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040
*d*x*e^c - 5040*e^c)*b^2*e^(d*x)/d^8 + 15*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5
 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b^2*e^(-d*x
- c)/d^8) + 1/105*(15*b^2*x^7 + 42*a*b*x^5 + 35*a^2*x^3)*cosh(d*x + c)
```

Fricas [A] time = 2.05895, size = 336, normalized size = 1.44

$$-\frac{2\left(3\,b^2d^5x^5+4\left(abd^5+15\,b^2d^3\right)x^3+\left(a^2d^5+24\,abd^3+360\,b^2d\right)x\right)\cosh\left(dx+c\right)-\left(b^2d^6x^6+2\,a^2d^4+2\left(abd^6+15\,b\right.\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*(3*b^2*d^5*x^5 + 4*(a*b*d^5 + 15*b^2*d^3)*x^3 + (a^2*d^5 + 24*a*b*d^3 +
 360*b^2*d)*x)*cosh(d*x + c) - (b^2*d^6*x^6 + 2*a^2*d^4 + 2*(a*b*d^6 + 15*b
^2*d^4)*x^4 + 48*a*b*d^2 + (a^2*d^6 + 24*a*b*d^4 + 360*b^2*d^2)*x^2 + 720*b
^2)*sinh(d*x + c))/d^7
```

Sympy [A] time = 10.1603, size = 286, normalized size = 1.22

$$\begin{cases}\frac{a^2x^2\sinh\left(c+dx\right)}{d}-\frac{2a^2x\cosh\left(c+dx\right)}{d^2}+\frac{2a^2\sinh\left(c+dx\right)}{d^3}+\frac{2abx^4\sinh\left(c+dx\right)}{d}-\frac{8abx^3\cosh\left(c+dx\right)}{d^2}+\frac{24abx^2\sinh\left(c+dx\right)}{d^3}-\frac{48abx\cosh\left(c+dx\right)}{d^4}+ \\ \left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right)\cosh\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**2
*sinh(c + d*x)/d**3 + 2*a*b*x**4*sinh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)
```

```
/d**2 + 24*a*b*x**2*sinh(c + d*x)/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a
*b*sinh(c + d*x)/d**5 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*
x)/d**2 + 30*b**2*x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**
4 + 360*b**2*x**2*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*
b**2*sinh(c + d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**
7/7)*cosh(c), True))
```

Giac [A] time = 1.21762, size = 410, normalized size = 1.75

$$\frac{\left(b^2d^6x^6 + 2\,abd^6x^4 - 6\,b^2d^5x^5 + a^2d^6x^2 - 8\,abd^5x^3 + 30\,b^2d^4x^4 - 2\,a^2d^5x + 24\,abd^4x^2 - 120\,b^2d^3x^3 + 2\,a^2d^4 - 48\,abd^3x +\right.}{2\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 - 6*b^2*d^5*x^5 + a^2*d^6*x^2 - 8*a*b*d^5*
x^3 + 30*b^2*d^4*x^4 - 2*a^2*d^5*x + 24*a*b*d^4*x^2 - 120*b^2*d^3*x^3 + 2*a
^2*d^4 - 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2*d*x + 720*b^
2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + 6*b^2*d^5*x^5 + a^2
*d^6*x^2 + 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 + 2*a^2*d^5*x + 24*a*b*d^4*x^2 +
120*b^2*d^3*x^3 + 2*a^2*d^4 + 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 +
 720*b^2*d*x + 720*b^2)*e^(-d*x - c)/d^7
```

3.50 $\int x\left(a+bx^2\right)^2\cosh(c+dx)\,dx$

Optimal. Leaf size=184

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{2abx^3\sinh(}{d}$$

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d
*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 -
 (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b*x*
Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d
^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d
```

Rubi [A] time = 0.27713, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5287, 3296, 2638}

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{2abx^3\sinh(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d
*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 -
 (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b*x*
Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d
^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^2\right)^2\cosh(c+dx)\,dx = \int \left(a^2x\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^5\cosh(c+dx)\right)dx$$
$$= a^2\int x\cosh(c+dx)\,dx + (2ab)\int x^3\cosh(c+dx)\,dx + b^2\int x^5\cosh(c+dx)\,dx$$
$$= \frac{a^2x\sinh(c+dx)}{d} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{b^2x^5\sinh(c+dx)}{d} - \frac{a^2\int\sinh(c+dx)\,dx}{d} - \ldots$$
$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{a^2x\sinh(c+dx)}{d} + \ldots$$
$$= -\frac{a^2\cosh(c+dx)}{d^2} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} + \ldots$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{6abx^2\cosh(c+dx)}{d^2} - \ldots$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \frac{6abx^2\cosh(c+dx)}{d^2} - \ldots$$
$$= -\frac{120b^2\cosh(c+dx)}{d^6} - \frac{12ab\cosh(c+dx)}{d^4} - \frac{a^2\cosh(c+dx)}{d^2} - \frac{60b^2x^2\cosh(c+dx)}{d^4} - \ldots$$

Mathematica [A] time = 0.230315, size = 113, normalized size = 0.61

$$\frac{dx\left(a^2d^4+2abd^2\left(d^2x^2+6\right)+b^2\left(d^4x^4+20d^2x^2+120\right)\right)\sinh(c+dx)-\left(a^2d^4+6abd^2\left(d^2x^2+2\right)+5b^2\left(d^4x^4+12d^2x^2\ldots\right.\right.}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^4 + 6*a*b*d^2*(2 + d^2*x^2) + 5*b^2*(24 + 12*d^2*x^2 + d^4*x^4))*
Cosh[c + d*x]) + d*x*(a^2*d^4 + 2*a*b*d^2*(6 + d^2*x^2) + b^2*(120 + 20*d^2
*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Maple [B] time = 0.008, size = 513, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^4*b^2*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^
3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x
+c))-5/d^4*b^2*c*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^
2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+10/d^4*b^2*c^2*((d*x+c
)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c)
)+2/d^2*b*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d
*x+c)-6*cosh(d*x+c))-10/d^4*b^2*c^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d
*x+c)+2*sinh(d*x+c))-6/d^2*b*c*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+
c)+2*sinh(d*x+c))+5/d^4*b^2*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+6/d^2*b*c
^2*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c)
)-1/d^4*b^2*c^5*sinh(d*x+c)-2/d^2*b*c^3*a*sinh(d*x+c)-c*a^2*sinh(d*x+c))
```

Maxima [A] time = 1.07846, size = 477, normalized size = 2.59

$$\frac{(bx^2+a)^3\cosh(dx+c)}{6b} - \frac{\left(\frac{a^3e^{(dx+c)}}{d} + \frac{a^3e^{(-dx-c)}}{d} + \frac{3\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2be^{(dx)}}{d^3} + \frac{3\,(d^2x^2+2\,dx+2)a^2be^{(-dx-c)}}{d^3} + \frac{3\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^3*cosh(d*x + c)/b - 1/12*(a^3*e^(d*x + c)/d + a^3*e^(-d*x -
 c)/d + 3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*b*e^(d*x)/d^3 + 3*(d^2*x^2
+ 2*d*x + 2)*a^2*b*e^(-d*x - c)/d^3 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d
^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^
3 + 12*d^2*x^2 + 24*d*x + 24)*a*b^2*e^(-d*x - c)/d^5 + (d^6*x^6*e^c - 6*d^5
*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c
 + 720*e^c)*b^3*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x
^3 + 360*d^2*x^2 + 720*d*x + 720)*b^3*e^(-d*x - c)/d^7)*d/b
```

Fricas [A] time = 2.08417, size = 273, normalized size = 1.48

$$-\frac{\left(5\,b^2d^4x^4 + a^2d^4 + 12\,abd^2 + 6\,(abd^4 + 10\,b^2d^2)x^2 + 120\,b^2\right)\cosh(dx+c) - \left(b^2d^5x^5 + 2\,(abd^5 + 10\,b^2d^3)x^3 + (a^2d^5\right.}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b^2*d^4*x^4 + a^2*d^4 + 12*a*b*d^2 + 6*(a*b*d^4 + 10*b^2*d^2)*x^2 + 12
0*b^2)*cosh(d*x + c) - (b^2*d^5*x^5 + 2*(a*b*d^5 + 10*b^2*d^3)*x^3 + (a^2*d
^5 + 12*a*b*d^3 + 120*b^2*d)*x)*sinh(d*x + c))/d^6
```

Sympy [A] time = 5.25899, size = 226, normalized size = 1.23

$$\begin{cases} \frac{a^2x\sinh(c+dx)}{d} - \frac{a^2\cosh(c+dx)}{d^2} + \frac{2abx^3\sinh(c+dx)}{d} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{12ab\cosh(c+dx)}{d^4} + \frac{b^2x^5\sinh(c+dx)}{d} - \frac{5b^2x^4}{} \\ \left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right)\cosh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**3*si
nh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3
 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**5*sinh(c + d*x)/d - 5*b**2*x**4*cosh
(c + d*x)/d**2 + 20*b**2*x**3*sinh(c + d*x)/d**3 - 60*b**2*x**2*cosh(c + d*
x)/d**4 + 120*b**2*x*sinh(c + d*x)/d**5 - 120*b**2*cosh(c + d*x)/d**6, Ne(d
, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*cosh(c), True))
```

Giac [A] time = 1.17159, size = 323, normalized size = 1.76

$$\frac{\left(b^2d^5x^5 + 2\,abd^5x^3 - 5\,b^2d^4x^4 + a^2d^5x - 6\,abd^4x^2 + 20\,b^2d^3x^3 - a^2d^4 + 12\,abd^3x - 60\,b^2d^2x^2 - 12\,abd^2 + 120\,b^2dx - 1\right.}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3 - 5*b^2*d^4*x^4 + a^2*d^5*x - 6*a*b*d^4*x^
2 + 20*b^2*d^3*x^3 - a^2*d^4 + 12*a*b*d^3*x - 60*b^2*d^2*x^2 - 12*a*b*d^2 +
 120*b^2*d*x - 120*b^2)*e^(d*x + c)/d^6 - 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3
+ 5*b^2*d^4*x^4 + a^2*d^5*x + 6*a*b*d^4*x^2 + 20*b^2*d^3*x^3 + a^2*d^4 + 12
*a*b*d^3*x + 60*b^2*d^2*x^2 + 12*a*b*d^2 + 120*b^2*d*x + 120*b^2)*e^(-d*x -
 c)/d^6
```

3.51 $\int (a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=136

$$\frac{a^2\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c}{d^2}$$

```
[Out] (-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Co
sh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3 +
 (a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sinh[c
 + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rubi [A] time = 0.18139, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5277, 2637, 3296}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Co
sh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3 +
 (a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sinh[c
 + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Rule 5277

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx = \int \left(a^2\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)\,dx$$
$$= a^2\int \cosh(c+dx)\,dx + (2ab)\int x^2\cosh(c+dx)\,dx + b^2\int x^4\cosh(c+dx)\,dx$$
$$= \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d} - \frac{(4ab)\int x\sinh(c+dx)\,dx}{d} -$$
$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^2\sinh(c+dx)}{d} + b^2$$
$$= -\frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{a^2\sinh(c+dx)}{d} + 12b^2$$
$$= -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{4ab\sinh(c+dx)}{d^3} + $$
$$= -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{24b^2\sinh(c+dx)}{d^5} +$$

Mathematica [A] time = 0.17899, size = 85, normalized size = 0.62

$$\frac{\left(a^2d^4+2abd^2\left(d^2x^2+2\right)+b^2\left(d^4x^4+12d^2x^2+24\right)\right)\sinh(c+dx)-4bdx\left(ad^2+b\left(d^2x^2+6\right)\right)\cosh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-4*b*d*x*(a*d^2 + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^4 + 2*a*b*d^2*(2
 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [B] time = 0.008, size = 332, normalized size = 2.4

$$\frac{1}{d}\left(\frac{b^2\left((dx+c)^4\sinh(dx+c)-4\,(dx+c)^3\cosh(dx+c)+12\,(dx+c)^2\sinh(dx+c)-24\,(dx+c)\cosh(dx+c)+24\,\mathrm{si}\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d*(1/d^4*b^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*
sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-4/d^4*b^2*c*((d*x+c)^3*s
inh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+6/d
^4*b^2*c^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+2/d^
2*b*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4/d^4*b^2
*c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-4/d^2*b*c*a*((d*x+c)*sinh(d*x+c)-cos
h(d*x+c))+1/d^4*b^2*c^4*sinh(d*x+c)+2/d^2*b*c^2*a*sinh(d*x+c)+a^2*sinh(d*x+
c))
```

Maxima [A] time = 1.03875, size = 255, normalized size = 1.88

$$\frac{a^2e^{(dx+c)}}{2\,d} - \frac{a^2e^{(-dx-c)}}{2\,d} + \frac{\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)abe^{(dx)}}{d^3} - \frac{\left(d^2x^2+2\,dx+2\right)abe^{(-dx-c)}}{d^3} + \frac{\left(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-2\right.}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^2*x^2*e^c - 2*d*x*e^c +
 2*e^c)*a*b*e^(d*x)/d^3 - (d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3 + 1/2*
(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^
(d*x)/d^5 - 1/2*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*
x - c)/d^5
```

Fricas [A] time = 2.18069, size = 207, normalized size = 1.52

$$-\frac{4\left(b^2d^3x^3+\left(abd^3+6\,b^2d\right)x\right)\cosh\left(dx+c\right)-\left(b^2d^4x^4+a^2d^4+4\,abd^2+2\left(abd^4+6\,b^2d^2\right)x^2+24\,b^2\right)\sinh\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(4*(b^2*d^3*x^3 + (a*b*d^3 + 6*b^2*d)*x)*cosh(d*x + c) - (b^2*d^4*x^4 + a^
2*d^4 + 4*a*b*d^2 + 2*(a*b*d^4 + 6*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^
5
```

Sympy [A] time = 2.95396, size = 172, normalized size = 1.26

$$\begin{cases}\frac{a^2\sinh(c+dx)}{d}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}+\frac{b^2x^4\sinh(c+dx)}{d}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{24b^2 \\ \left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right)\cosh(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**2*sinh(c + d*x)/d - 4*a*b*x*cosh
(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**2*x**4*sinh(c + d*x)/d - 4*b
**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d*x)/d**3 - 24*b**2*x*c
osh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2*a*b
*x**3/3 + b**2*x**5/5)*cosh(c), True))
```

Giac [A] time = 1.17539, size = 243, normalized size = 1.79

$$\frac{\left(b^2d^4x^4+2\,abd^4x^2-4\,b^2d^3x^3+a^2d^4-4\,abd^3x+12\,b^2d^2x^2+4\,abd^2-24\,b^2dx+24\,b^2\right)e^{(dx+c)}}{2\,d^5}-\frac{\left(b^2d^4x^4+2\,abd^4x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^2 - 4*b^2*d^3*x^3 + a^2*d^4 - 4*a*b*d^3*x +
12*b^2*d^2*x^2 + 4*a*b*d^2 - 24*b^2*d*x + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^
2*d^4*x^4 + 2*a*b*d^4*x^2 + 4*b^2*d^3*x^3 + a^2*d^4 + 4*a*b*d^3*x + 12*b^2*
d^2*x^2 + 4*a*b*d^2 + 24*b^2*d*x + 24*b^2)*e^(-d*x - c)/d^5
```

3.52 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} dx$

Optimal. Leaf size=110

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + \frac{6b^2x\sinh(c+d}{d^3}$$

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3 +
 (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIn
tegral[d*x]
```

Rubi [A] time = 0.188808, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5287, 3303, 3298, 3301, 3296, 2638}

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + \frac{6b^2x\sinh(c+d}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3 +
 (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIn
tegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \cosh(c+dx)}{x}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x} + 2abx\cosh(c+dx) + b^2x^3\cosh(c+dx)\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x}\,dx + (2ab)\int x\cosh(c+dx)\,dx + b^2\int x^3\cosh(c+dx)\,dx$$
$$= \frac{2abx\sinh(c+dx)}{d} + \frac{b^2x^3\sinh(c+dx)}{d} - \frac{(2ab)\int \sinh(c+dx)\,dx}{d} - \frac{\left(3b^2\right)\int x^2\sinh(}{d}$$
$$= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{2abx\sinh(c+dx)}{d}$$
$$= -\frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) + \frac{6b^2x\sinh(c+dx)}{d^3}$$
$$= -\frac{6b^2\cosh(c+dx)}{d^4} - \frac{2ab\cosh(c+dx)}{d^2} - \frac{3b^2x^2\cosh(c+dx)}{d^2} + a^2\cosh(c)\mathrm{Chi}(dx) +$$

Mathematica [A] time = 0.360715, size = 82, normalized size = 0.75

$$a^2\cosh(c)\mathrm{Chi}(dx) + a^2\sinh(c)\mathrm{Shi}(dx) + \frac{bx\left(2ad^2+b\left(d^2x^2+6\right)\right)\sinh(c+dx)}{d^3} - \frac{b\left(2ad^2+3b\left(d^2x^2+2\right)\right)\cosh(c+}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*(2*a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x])/d^4) + a^2*Cosh[c]*CoshIn
tegral[d*x] + (b*x*(2*a*d^2 + b*(6 + d^2*x^2))*Sinh[c + d*x])/d^3 + a^2*Sin
h[c]*SinhIntegral[d*x]
```

Maple [B] time = 0.05, size = 226, normalized size = 2.1

$$-\frac{a^2e^{-c}\mathrm{Ei}\,(1,dx)}{2} - \frac{abe^{-dx-c}}{d^2} - 3\,\frac{e^{dx+c}b^2}{d^4} - \frac{a^2e^{c}\mathrm{Ei}\,(1,-dx)}{2} - 3\,\frac{b^2e^{-dx-c}}{d^4} + \frac{abe^{dx+c}x}{d} - \frac{abe^{dx+c}}{d^2} + \frac{e^{dx+c}b^2x^3}{2\,d} - \frac{3\,e^{dx+c}b^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x,x)
```

```
[Out] -1/2*a^2*exp(-c)*Ei(1,d*x)-a/d^2*b*exp(-d*x-c)-3/d^4*b^2*exp(d*x+c)-1/2*a^2
*exp(c)*Ei(1,-d*x)-3/d^4*b^2*exp(-d*x-c)+a/d*b*exp(d*x+c)*x-a/d^2*b*exp(d*x
+c)+1/2/d*b^2*exp(d*x+c)*x^3-3/2/d^2*b^2*exp(d*x+c)*x^2+3/d^3*b^2*exp(d*x+c
)*x-a/d*b*exp(-d*x-c)*x-1/2/d*b^2*exp(-d*x-c)*x^3-3/2/d^2*b^2*exp(-d*x-c)*x
^2-3/d^3*b^2*exp(-d*x-c)*x
```

Maxima [B] time = 1.18744, size = 317, normalized size = 2.88

$$-\frac{1}{8}\left(4\,ab\left(\frac{\left(d^2x^2e^c - 2\,dxe^c + 2\,e^c\right)e^{(dx)}}{d^3} + \frac{\left(d^2x^2 + 2\,dx + 2\right)e^{(-dx-c)}}{d^3}\right) + b^2\left(\frac{\left(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c\right)}{d^5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d
*x + 2)*e^(-d*x - c)/d^3) + b^2*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*
e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2
+ 24*d*x + 24)*e^(-d*x - c)/d^5) + 4*a^2*cosh(d*x + c)*log(x^2)/d - 4*(Ei(-
d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/4*(b^2*x^4 + 4*a*b*x^2 + 2*a^2*log(
x^2))*cosh(d*x + c)
```

Fricas [A] time = 2.47665, size = 292, normalized size = 2.65

$$-\frac{2\left(3\,b^2d^2x^2 + 2\,abd^2 + 6\,b^2\right)\cosh\left(dx+c\right) - \left(a^2d^4\mathrm{Ei}\left(dx\right) + a^2d^4\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right) - 2\left(b^2d^3x^3 + 2\left(abd^3 + 3\,b^2d\right)x\right)\sinh}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(3*b^2*d^2*x^2 + 2*a*b*d^2 + 6*b^2)*cosh(d*x + c) - (a^2*d^4*Ei(d*x
) + a^2*d^4*Ei(-d*x))*cosh(c) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 + 3*b^2*d)*x)*s
inh(d*x + c) - (a^2*d^4*Ei(d*x) - a^2*d^4*Ei(-d*x))*sinh(c))/d^4
```

Sympy [A] time = 5.73347, size = 121, normalized size = 1.1

$$a^2\sinh\left(c\right)\mathrm{Shi}\left(dx\right) + a^2\cosh\left(c\right)\mathrm{Chi}\left(dx\right) + 2ab\left(\begin{cases}\frac{x\sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0\\ \frac{x^2\cosh(c)}{2} & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases}\frac{x^3\sinh(c+dx)}{d} - \frac{3x^2\cosh(c+}{d^2}\\ \frac{x^4\cosh(c)}{4}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*sinh(c +
 d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True)) + b**2*Pie
cewise((x**3*sinh(c + d*x)/d - 3*x**2*cosh(c + d*x)/d**2 + 6*x*sinh(c + d*x
)/d**3 - 6*cosh(c + d*x)/d**4, Ne(d, 0)), (x**4*cosh(c)/4, True))
```

Giac [B] time = 1.20143, size = 300, normalized size = 2.73

$$\frac{b^2d^3x^3e^{(dx+c)} - b^2d^3x^3e^{(-dx-c)} + a^2d^4\mathrm{Ei}\left(-dx\right)e^{(-c)} + a^2d^4\mathrm{Ei}\left(dx\right)e^c + 2\,abd^3xe^{(dx+c)} - 3\,b^2d^2x^2e^{(dx+c)} - 2\,abd^3xe^{(-dx-c)} - 3}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^3*x^3*e^(d*x + c) - b^2*d^3*x^3*e^(-d*x - c) + a^2*d^4*Ei(-d*x)*
e^(-c) + a^2*d^4*Ei(d*x)*e^c + 2*a*b*d^3*x*e^(d*x + c) - 3*b^2*d^2*x^2*e^(d
*x + c) - 2*a*b*d^3*x*e^(-d*x - c) - 3*b^2*d^2*x^2*e^(-d*x - c) - 2*a*b*d^2
*e^(d*x + c) + 6*b^2*d*x*e^(d*x + c) - 2*a*b*d^2*e^(-d*x - c) - 6*b^2*d*x*e
^(-d*x - c) - 6*b^2*e^(d*x + c) - 6*b^2*e^(-d*x - c))/d^4
```

3.53 $\int \frac{\left(a+bx^2\right)^2 \cosh(c+dx)}{x^2} \, dx$

Optimal. Leaf size=95

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2}$$

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.176575, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5287, 2637, 3297, 3303, 3298, 3301, 3296}

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \cosh(c+dx)}{x^2} dx = \int \left(2ab\cosh(c+dx) + \frac{a^2\cosh(c+dx)}{x^2} + b^2x^2\cosh(c+dx)\right) dx$$
$$= a^2 \int \frac{\cosh(c+dx)}{x^2} dx + (2ab) \int \cosh(c+dx)\,dx + b^2 \int x^2\cosh(c+dx)\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{x} + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} - \frac{\left(2b^2\right)\int x\sinh(c+dx)\,d}{d}$$
$$= -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} + \frac{(2}{}$$
$$= -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) + \frac{2b^2\sinh(c+dx)}{d^3} + 2$$

Mathematica [A] time = 0.262884, size = 95, normalized size = 1.

$$a^2d\sinh(c)\text{Chi}(dx) + a^2d\cosh(c)\text{Shi}(dx) - \frac{a^2\cosh(c+dx)}{x} + \frac{2ab\sinh(c+dx)}{d} + \frac{2b^2\sinh(c+dx)}{d^3} - \frac{2b^2x\cosh(c+}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral
[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^2*
x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.069, size = 190, normalized size = 2.

$$\frac{da^2e^{-c}\text{Ei}(1,dx)}{2} - \frac{abe^{-dx-c}}{d} - \frac{a^2e^{-dx-c}}{2x} - \frac{b^2e^{-dx-c}}{d^3} - \frac{b^2e^{-dx-c}x^2}{2d} - \frac{b^2e^{-dx-c}x}{d^2} + \frac{e^{dx+c}b^2}{d^3} - \frac{da^2e^c\text{Ei}(1,-dx)}{2} + \frac{abe^{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^2,x)
```

```
[Out] 1/2*d*a^2*exp(-c)*Ei(1,d*x)-a*b/d*exp(-d*x-c)-1/2*a^2*exp(-d*x-c)/x-1/d^3*b
^2*exp(-d*x-c)-1/2/d*b^2*exp(-d*x-c)*x^2-1/d^2*b^2*exp(-d*x-c)*x+1/d^3*b^2*
exp(d*x+c)-1/2*d*a^2*exp(c)*Ei(1,-d*x)+a*b/d*exp(d*x+c)+1/2/d*b^2*exp(d*x+c
```

)*x^2-1/d^2*b^2*exp(d*x+c)*x-1/2*a^2/x*exp(d*x+c)

Maxima [A] time = 1.20407, size = 242, normalized size = 2.55

$$-\frac{1}{6}\left(3\,a^2\mathrm{Ei}\left(-dx\right)e^{(-c)} - 3\,a^2\mathrm{Ei}\left(dx\right)e^c + \frac{6\,(dxe^c - e^c)abe^{(dx)}}{d^2} + \frac{6\,(dx+1)abe^{(-dx-c)}}{d^2} + \frac{\left(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c\right)b^2}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a^2*Ei(-d*x)*e^(-c) - 3*a^2*Ei(d*x)*e^c + 6*(d*x*e^c - e^c)*a*b*e^(
d*x)/d^2 + 6*(d*x + 1)*a*b*e^(-d*x - c)/d^2 + (d^3*x^3*e^c - 3*d^2*x^2*e^c
+ 6*d*x*e^c - 6*e^c)*b^2*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b^
2*e^(-d*x - c)/d^4)*d + 1/3*(b^2*x^3 + 6*a*b*x - 3*a^2/x)*cosh(d*x + c)
```

Fricas [A] time = 2.0144, size = 286, normalized size = 3.01

$$-\frac{2\left(a^2d^3 + 2\,b^2dx^2\right)\cosh\left(dx+c\right) - \left(a^2d^4x\mathrm{Ei}\left(dx\right) - a^2d^4x\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right) - 2\left(b^2d^2x^3 + 2\left(abd^2 + b^2\right)x\right)\sinh\left(dx+c\right)}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2*d^3 + 2*b^2*d*x^2)*cosh(d*x + c) - (a^2*d^4*x*Ei(d*x) - a^2*d^
4*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^2*x^3 + 2*(a*b*d^2 + b^2)*x)*sinh(d*x + c)
 - (a^2*d^4*x*Ei(d*x) + a^2*d^4*x*Ei(-d*x))*sinh(c))/(d^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)^2 \cosh\left(c + dx\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**2, x)
```

Giac [B] time = 1.19945, size = 266, normalized size = 2.8

$$-\frac{a^2d^4x\mathrm{Ei}\left(-dx\right)e^{(-c)} - a^2d^4x\mathrm{Ei}\left(dx\right)e^c - b^2d^2x^3e^{(dx+c)} + b^2d^2x^3e^{(-dx-c)} + a^2d^3e^{(dx+c)} - 2\,abd^2xe^{(dx+c)} + 2\,b^2dx^2e^{(dx+c)} + a^2}{2\,d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*d^4*x*Ei(-d*x)*e^(-c) - a^2*d^4*x*Ei(d*x)*e^c - b^2*d^2*x^3*e^(d*
x + c) + b^2*d^2*x^3*e^(-d*x - c) + a^2*d^3*e^(d*x + c) - 2*a*b*d^2*x*e^(d*
x + c) + 2*b^2*d*x^2*e^(d*x + c) + a^2*d^3*e^(-d*x - c) + 2*a*b*d^2*x*e^(-d
*x - c) + 2*b^2*d*x^2*e^(-d*x - c) - 2*b^2*x*e^(d*x + c) + 2*b^2*x*e^(-d*x
- c))/(d^3*x)
```

3.54 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx$

Optimal. Leaf size=114

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2ab\cosh(c)\text{Chi}(dx)+2ab\sinh(c$$

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(2*x^2) + 2*a*b*Cosh[c]*Co
shIntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a^2*d*Sinh[c + d
*x])/(2*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*Sinh[c]*SinhIntegral[d*x] + (a
^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] time = 0.225079, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2ab\cosh(c)\text{Chi}(dx)+2ab\sinh(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(2*x^2) + 2*a*b*Cosh[c]*Co
shIntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a^2*d*Sinh[c + d
*x])/(2*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*Sinh[c]*SinhIntegral[d*x] + (a
^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2 \cosh(c+dx)}{x^3}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^3}+\frac{2ab\cosh(c+dx)}{x}+b^2x\cosh(c+dx)\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x}\,dx+b^2\int x\cosh(c+dx)\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{2x^2}+\frac{b^2x\sinh(c+dx)}{d}-\frac{b^2\int \sinh(c+dx)\,dx}{d}+\frac{1}{2}\left(a^2d\right)\int \frac{\sinh(c+\ldots}{x^2}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\mathrm{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{2x}+\frac{b^2x\ldots}{\ldots}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\mathrm{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{2x}+\frac{b^2x\ldots}{\ldots}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{2x^2}+2ab\cosh(c)\mathrm{Chi}(dx)+\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)\ldots$$

Mathematica [A] time = 0.348361, size = 97, normalized size = 0.85

$$\frac{1}{2}\left(-\frac{a^2\cosh(c+dx)}{x^2}-\frac{a^2d\sinh(c+dx)}{x}+a\cosh(c)\left(ad^2+4b\right)\mathrm{Chi}(dx)+a\sinh(c)\left(ad^2+4b\right)\mathrm{Shi}(dx)-\frac{2b^2\cosh(c\ldots}{d^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] ((-2*b^2*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x^2 + a*(4*b + a*d^2)*Cos
h[c]*CoshIntegral[d*x] - (a^2*d*Sinh[c + d*x])/x + (2*b^2*x*Sinh[c + d*x])/
d + a*(4*b + a*d^2)*Sinh[c]*SinhIntegral[d*x])/2
```

Maple [A] time = 0.084, size = 188, normalized size = 1.7

$$-\frac{d^2a^2\mathrm{e}^{-c}\mathrm{Ei}\left(1,dx\right)}{4}+\frac{da^2\mathrm{e}^{-dx-c}}{4x}-\frac{a^2\mathrm{e}^{-dx-c}}{4x^2}-\frac{b^2\mathrm{e}^{-dx-c}}{2d^2}-ab\mathrm{e}^{-c}\mathrm{Ei}\left(1,dx\right)-\frac{b^2\mathrm{e}^{-dx-c}x}{2d}-\frac{\mathrm{e}^{dx+c}b^2}{2d^2}-\frac{d^2a^2\mathrm{e}^{c}\mathrm{Ei}\left(1,-dx\right)}{4}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^3,x)
```

```
[Out] -1/4*d^2*a^2*exp(-c)*Ei(1,d*x)+1/4*d*a^2*exp(-d*x-c)/x-1/4*a^2*exp(-d*x-c)/
x^2-1/2*b^2/d^2*exp(-d*x-c)-a*b*exp(-c)*Ei(1,d*x)-1/2*b^2/d*exp(-d*x-c)*x-1
/2*b^2/d^2*exp(d*x+c)-1/4*d^2*a^2*exp(c)*Ei(1,-d*x)+1/2*b^2/d*exp(d*x+c)*x-
1/4*a^2/x^2*exp(d*x+c)-1/4*d*a^2/x*exp(d*x+c)-a*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.3475, size = 223, normalized size = 1.96

$$\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a^2-b^2\left(\frac{\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)e^{(dx)}}{d^3}+\frac{\left(d^2x^2+2\,dx+2\right)e^{(-dx-c)}}{d^3}\right)-\frac{4\,ab\cosh\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - b^2*((d^2*x^2*
e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d
^3) - 4*a*b*cosh(d*x + c)*log(x^2)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*
b/d)*d + 1/2*(b^2*x^2 + 2*a*b*log(x^2) - a^2/x^2)*cosh(d*x + c)
```

Fricas [A] time = 1.99347, size = 359, normalized size = 3.15

$$-\frac{2\left(a^2d^2+2\,b^2x^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^4+4\,abd^2\right)x^2\mathrm{Ei}\left(dx\right)+\left(a^2d^4+4\,abd^2\right)x^2\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)+2\left(a^2d^3x-2\,b^2dx^3\right.}{4\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*d^2 + 2*b^2*x^2)*cosh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei
(d*x) + (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*cosh(c) + 2*(a^2*d^3*x - 2*b^2*
d*x^3)*sinh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) - (a^2*d^4 + 4*a*
b*d^2)*x^2*Ei(-d*x))*sinh(c))/(d^2*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**3, x)
```

Giac [A] time = 1.21957, size = 278, normalized size = 2.44

$$\frac{a^2d^4x^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+a^2d^4x^2\mathrm{Ei}\left(dx\right)e^{c}+4\,abd^2x^2\mathrm{Ei}\left(-dx\right)e^{(-c)}+4\,abd^2x^2\mathrm{Ei}\left(dx\right)e^{c}-a^2d^3xe^{(dx+c)}+2\,b^2dx^3e^{(dx+c)}+a^2d}{4\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^4*x^2*Ei(-d*x)*e^(-c) + a^2*d^4*x^2*Ei(d*x)*e^c + 4*a*b*d^2*x^2*
Ei(-d*x)*e^(-c) + 4*a*b*d^2*x^2*Ei(d*x)*e^c - a^2*d^3*x*e^(d*x + c) + 2*b^2
*d*x^3*e^(d*x + c) + a^2*d^3*x*e^(-d*x - c) - 2*b^2*d*x^3*e^(-d*x - c) - a^
2*d^2*e^(d*x + c) - 2*b^2*x^2*e^(d*x + c) - a^2*d^2*e^(-d*x - c) - 2*b^2*x^
2*e^(-d*x - c))/(d^2*x^2)
```

3.55 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} dx$

Optimal. Leaf size=133

$$\frac{1}{6}a^2d^3\sinh(c)\mathrm{Chi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\mathrm{Shi}(dx) - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{a^2\cosh(c+dx)}{3x^3} + 2abd\sinh(c$$

```
[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (2*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/(6*x) + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x
]*Sinh[c])/6 + (b^2*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/(6*x^2) + 2*a*
b*d*Cosh[c]*SinhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6
```

Rubi [A] time = 0.258916, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$\frac{1}{6}a^2d^3\sinh(c)\mathrm{Chi}(dx) + \frac{1}{6}a^2d^3\cosh(c)\mathrm{Shi}(dx) - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{a^2d\sinh(c+dx)}{6x^2} - \frac{a^2\cosh(c+dx)}{3x^3} + 2abd\sinh(c$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (2*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/(6*x) + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x
]*Sinh[c])/6 + (b^2*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/(6*x^2) + 2*a*
b*d*Cosh[c]*SinhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\cosh(c+dx)}{x^4}\,dx = \int \left(b^2\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x^2}\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx+(2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx+b^2\int \cosh(c+dx)\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}+\frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^3}\,d$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{6x^2}+\frac{1}{6}\left(a^2\right.$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\,\mathbf{Chi}(dx)\sinh(c)+$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\,\mathbf{Chi}(dx)\sinh(c)+$$
$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}+2abd\,\mathbf{Chi}(dx)\sinh(c)+$$

Mathematica [A] time = 0.401937, size = 114, normalized size = 0.86

$$\frac{1}{6}\left(-\frac{a^2d^2\cosh(c+dx)}{x}-\frac{a^2d\sinh(c+dx)}{x^2}-\frac{2a^2\cosh(c+dx)}{x^3}+ad\sinh(c)\left(ad^2+12b\right)\mathbf{Chi}(dx)+ad\cosh(c)\left(ad^2+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (12*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
d*x])/x + a*d*(12*b + a*d^2)*CoshIntegral[d*x]*Sinh[c] + (6*b^2*Sinh[c + d*
x])/d - (a^2*d*Sinh[c + d*x])/x^2 + a*d*(12*b + a*d^2)*Cosh[c]*SinhIntegral
[d*x])/6
```

Maple [A] time = 0.092, size = 222, normalized size = 1.7

$$\frac{d^3a^2e^{-c}\mathrm{Ei}(1,dx)}{12}-\frac{b^2e^{-dx-c}}{2d}-\frac{abe^{-dx-c}}{x}+dabe^{-c}\mathrm{Ei}(1,dx)-\frac{a^2d^2e^{-dx-c}}{12x}+\frac{da^2e^{-dx-c}}{12x^2}-\frac{a^2e^{-dx-c}}{6x^3}-\frac{e^{dx+c}a^2}{6x^3}-\frac{da^2e^{dx+}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^4,x)
```

```
[Out] 1/12*d^3*a^2*exp(-c)*Ei(1,d*x)-1/2*b^2/d*exp(-d*x-c)-a*b*exp(-d*x-c)/x+d*a*
b*exp(-c)*Ei(1,d*x)-1/12*d^2*a^2*exp(-d*x-c)/x+1/12*d*a^2*exp(-d*x-c)/x^2-1
/6*a^2*exp(-d*x-c)/x^3-1/6*a^2/x^3*exp(d*x+c)-1/12*d*a^2/x^2*exp(d*x+c)-1/1
2*d^2*a^2/x*exp(d*x+c)+1/2*b^2/d*exp(d*x+c)-1/12*d^3*a^2*exp(c)*Ei(1,-d*x)-
```

```
a*b/x*exp(d*x+c)-d*a*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.23669, size = 182, normalized size = 1.37

$$\frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma\left(-2,dx\right)-a^2d^2e^c\Gamma\left(-2,-dx\right)-6\,ab\text{Ei}\left(-dx\right)e^{(-c)}+6\,ab\text{Ei}\left(dx\right)e^c-\frac{3\left(dxe^c-e^c\right)b^2e^{(dx)}}{d^2}-\frac{3\left(dx+1\right)b^2e^{(-dx-c)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) - 6*a*b*Ei
(-d*x)*e^(-c) + 6*a*b*Ei(d*x)*e^c - 3*(d*x*e^c - e^c)*b^2*e^(d*x)/d^2 - 3*(
d*x + 1)*b^2*e^(-d*x - c)/d^2)*d + 1/3*(3*b^2*x - (6*a*b*x^2 + a^2)/x^3)*co
sh(d*x + c)
```

Fricas [A] time = 2.04377, size = 381, normalized size = 2.86

$$-\frac{2\left(2\,a^2d+\left(a^2d^3+12\,abd\right)x^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^4+12\,abd^2\right)x^3\text{Ei}\left(dx\right)-\left(a^2d^4+12\,abd^2\right)x^3\text{Ei}\left(-dx\right)\right)\cosh\left(c\right)+2\left(a^2\right.}{12\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(2*a^2*d + (a^2*d^3 + 12*a*b*d)*x^2)*cosh(d*x + c) - ((a^2*d^4 + 1
2*a*b*d^2)*x^3*Ei(d*x) - (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(
a^2*d^2*x - 6*b^2*x^3)*sinh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*x)
+ (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*sinh(c))/(d*x^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2\cosh\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**4, x)
```

Giac [A] time = 1.1575, size = 319, normalized size = 2.4

$$-\frac{a^2d^4x^3\text{Ei}\left(-dx\right)e^{(-c)}-a^2d^4x^3\text{Ei}\left(dx\right)e^c+12\,abd^2x^3\text{Ei}\left(-dx\right)e^{(-c)}-12\,abd^2x^3\text{Ei}\left(dx\right)e^c+a^2d^3x^2e^{(dx+c)}+a^2d^3x^2e^{(-dx-c)}+}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^4*x^3*Ei(-d*x)*e^(-c) - a^2*d^4*x^3*Ei(d*x)*e^c + 12*a*b*d^2*x
^3*Ei(-d*x)*e^(-c) - 12*a*b*d^2*x^3*Ei(d*x)*e^c + a^2*d^3*x^2*e^(d*x + c) +
 a^2*d^3*x^2*e^(-d*x - c) + a^2*d^2*x*e^(d*x + c) + 12*a*b*d*x^2*e^(d*x + c
) - 6*b^2*x^3*e^(d*x + c) - a^2*d^2*x*e^(-d*x - c) + 12*a*b*d*x^2*e^(-d*x -
 c) + 6*b^2*x^3*e^(-d*x - c) + 2*a^2*d*e^(d*x + c) + 2*a^2*d*e^(-d*x - c))/
(d*x^3)
```

3.56 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$

Optimal. Leaf size=175

$$\frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2\cosh}{4}$$

```
[Out] -(a^2*Cosh[c + d*x])/(4*x^4) - (a*b*Cosh[c + d*x])/x^2 - (a^2*d^2*Cosh[c +
d*x])/(24*x^2) + b^2*Cosh[c]*CoshIntegral[d*x] + a*b*d^2*Cosh[c]*CoshIntegr
al[d*x] + (a^2*d^4*Cosh[c]*CoshIntegral[d*x])/24 - (a^2*d*Sinh[c + d*x])/(1
2*x^3) - (a*b*d*Sinh[c + d*x])/x - (a^2*d^3*Sinh[c + d*x])/(24*x) + b^2*Sin
h[c]*SinhIntegral[d*x] + a*b*d^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[
c]*SinhIntegral[d*x])/24
```

Rubi [A] time = 0.35956, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2\cosh}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -(a^2*Cosh[c + d*x])/(4*x^4) - (a*b*Cosh[c + d*x])/x^2 - (a^2*d^2*Cosh[c +
d*x])/(24*x^2) + b^2*Cosh[c]*CoshIntegral[d*x] + a*b*d^2*Cosh[c]*CoshIntegr
al[d*x] + (a^2*d^4*Cosh[c]*CoshIntegral[d*x])/24 - (a^2*d*Sinh[c + d*x])/(1
2*x^3) - (a*b*d*Sinh[c + d*x])/x - (a^2*d^3*Sinh[c + d*x])/(24*x) + b^2*Sin
h[c]*SinhIntegral[d*x] + a*b*d^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[
c]*SinhIntegral[d*x])/24
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2\cosh(c+dx)}{x^5}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^3}+\frac{b^2\cosh(c+dx)}{x}\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx + (2ab)\int \frac{\cosh(c+dx)}{x^3}\,dx + b^2\int \frac{\cosh(c+dx)}{x}\,dx$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} + \frac{1}{4}\left(a^2d\right)\int \frac{\sinh(c+dx)}{x^4}\,dx + (abd)\int \frac{\sinh(c}{x^2}$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} + b^2\cosh(c)\text{Chi}(dx) - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{abd\,s}{}$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} + b^2\cosh(c)\text{Chi}(dx) - \frac{a^2d}{}$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} + b^2\cosh(c)\text{Chi}(dx) + abd$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} + b^2\cosh(c)\text{Chi}(dx) + abd$$

$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} + b^2\cosh(c)\text{Chi}(dx) + abd$$

Mathematica [A] time = 0.438824, size = 124, normalized size = 0.71

$$\frac{x^4\cosh(c)\left(a^2d^4+24abd^2+24b^2\right)\text{Chi}(dx)+x^4\sinh(c)\left(a^2d^4+24abd^2+24b^2\right)\text{Shi}(dx)-a\left(dx\left(ad^2x^2+2a+24bx^2\right)\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] ((24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Cosh[c]*CoshIntegral[d*x] - a*((6*a +
24*b*x^2 + a*d^2*x^2)*Cosh[c + d*x] + d*x*(2*a + 24*b*x^2 + a*d^2*x^2)*Sinh
[c + d*x]) + (24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Sinh[c]*SinhIntegral[d*x])
/(24*x^4)
```

Maple [A] time = 0.116, size = 291, normalized size = 1.7

$$-\frac{d^4a^2e^{-c}\text{Ei}(1,dx)}{48}+\frac{bdae^{-dx-c}}{2x}-\frac{abe^{-dx-c}}{2x^2}-\frac{d^2abe^{-c}\text{Ei}(1,dx)}{2}-\frac{b^2e^{-c}\text{Ei}(1,dx)}{2}+\frac{a^2d^3e^{-dx-c}}{48x}-\frac{a^2d^2e^{-dx-c}}{48x^2}+\frac{da^2e^{-}}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*cosh(d*x+c)/x^5,x)
```

```
[Out] -1/48*d^4*a^2*exp(-c)*Ei(1,d*x)+1/2*d*a*b*exp(-d*x-c)/x-1/2*a*b*exp(-d*x-c)
/x^2-1/2*d^2*a*b*exp(-c)*Ei(1,d*x)-1/2*b^2*exp(-c)*Ei(1,d*x)+1/48*d^3*a^2*e
xp(-d*x-c)/x-1/48*d^2*a^2*exp(-d*x-c)/x^2+1/24*d*a^2*exp(-d*x-c)/x^3-1/8*a^
```

```
2*exp(-d*x-c)/x^4-1/8*a^2/x^4*exp(d*x+c)-1/24*d*a^2/x^3*exp(d*x+c)-1/48*d^2
*a^2/x^2*exp(d*x+c)-1/48*d^3*a^2/x*exp(d*x+c)-1/48*d^4*a^2*exp(c)*Ei(1,-d*x
)-1/2*a*b/x^2*exp(d*x+c)-1/2*d*a*b/x*exp(d*x+c)-1/2*d^2*a*b*exp(c)*Ei(1,-d*
x)-1/2*b^2*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.27655, size = 188, normalized size = 1.07

$$\frac{1}{8}\left(\left(d^3e^{(-c)}\Gamma(-3,dx)+d^3e^c\Gamma(-3,-dx)\right)a^2+4\left(de^{(-c)}\Gamma(-1,dx)+de^c\Gamma(-1,-dx)\right)ab-\frac{4\,b^2\cosh(dx+c)\log\left(x^2\right)}{d}+\frac{4\left(\mathrm{Ei}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*((d^3*e^(-c)*gamma(-3, d*x) + d^3*e^c*gamma(-3, -d*x))*a^2 + 4*(d*e^(-c
)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a*b - 4*b^2*cosh(d*x + c)*log(x^2
)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/d)*d + 1/4*(2*b^2*log(x^2) - (4
*a*b*x^2 + a^2)/x^4)*cosh(d*x + c)
```

Fricas [A] time = 1.9739, size = 439, normalized size = 2.51

$$-\frac{2\left(\left(a^2d^2+24\,ab\right)x^2+6\,a^2\right)\cosh(dx+c)-\left(\left(a^2d^4+24\,abd^2+24\,b^2\right)x^4\mathrm{Ei}(dx)+\left(a^2d^4+24\,abd^2+24\,b^2\right)x^4\mathrm{Ei}(-dx)\right)\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a^2*d^2 + 24*a*b)*x^2 + 6*a^2)*cosh(d*x + c) - ((a^2*d^4 + 24*a*
b*d^2 + 24*b^2)*x^4*Ei(d*x) + (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x))
*cosh(c) + 2*(2*a^2*d*x + (a^2*d^3 + 24*a*b*d)*x^3)*sinh(d*x + c) - ((a^2*d
^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) - (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4
*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)^2\cosh(c+dx)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**5, x)
```

Giac [A] time = 1.17539, size = 397, normalized size = 2.27

$$\frac{a^2d^4x^4\mathrm{Ei}(-dx)e^{(-c)}+a^2d^4x^4\mathrm{Ei}(dx)e^c+24\,abd^2x^4\mathrm{Ei}(-dx)e^{(-c)}+24\,abd^2x^4\mathrm{Ei}(dx)e^c-a^2d^3x^3e^{(dx+c)}+a^2d^3x^3e^{(-dx-c)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c + 24*a*b*d^2*x^
4*Ei(-d*x)*e^(-c) + 24*a*b*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) +
a^2*d^3*x^3*e^(-d*x - c) + 24*b^2*x^4*Ei(-d*x)*e^(-c) + 24*b^2*x^4*Ei(d*x)*
e^c - a^2*d^2*x^2*e^(d*x + c) - 24*a*b*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-
d*x - c) + 24*a*b*d*x^3*e^(-d*x - c) - 2*a^2*d*x*e^(d*x + c) - 24*a*b*x^2*e
^(d*x + c) + 2*a^2*d*x*e^(-d*x - c) - 24*a*b*x^2*e^(-d*x - c) - 6*a^2*e^(d*
x + c) - 6*a^2*e^(-d*x - c))/x^4
```

3.57 $\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=273

$$\frac{(-a)^{3/2}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2b^{5/2}}$$

```
[Out] (-2*x*Cosh[c + d*x])/(b*d^2) + ((-a)^(3/2)*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2)
) + (2*Sinh[c + d*x])/(b*d^3) - (a*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d
*x])/(b*d) - ((-a)^(3/2)*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Sinh[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Rubi [A] time = 0.728719, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5293, 2637, 3296, 5281, 3303, 3298, 3301}

$$\frac{(-a)^{3/2}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```


```
[Out] (-2*x*Cosh[c + d*x])/(b*d^2) + ((-a)^(3/2)*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2)
) + (2*Sinh[c + d*x])/(b*d^3) - (a*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d
*x])/(b*d) - ((-a)^(3/2)*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Sinh[c - (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

```
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{a+bx^2}\,dx = \int\left(-\frac{a\cosh(c+dx)}{b^2}+\frac{x^2\cosh(c+dx)}{b}+\frac{a^2\cosh(c+dx)}{b^2\left(a+bx^2\right)}\right)dx$$

$$= -\frac{a\int\cosh(c+dx)\,dx}{b^2}+\frac{a^2\int\frac{\cosh(c+dx)}{a+bx^2}\,dx}{b^2}+\frac{\int x^2\cosh(c+dx)\,dx}{b}$$

$$= -\frac{a\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}+\frac{a^2\int\left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b^2}-\frac{2\int x\sinh(c}{bd}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}-\frac{(-a)^{3/2}\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^2}-\frac{(-a)^{3/2}\int\frac{\text{co}}{\sqrt{}}}{2b}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2}+\frac{2\sinh(c+dx)}{bd^3}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}-\frac{\left((-a)^{3/2}\cosh\left(c-\right.\right.}{}$$

$$= -\frac{2x\cosh(c+dx)}{bd^2}+\frac{(-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}}{2b^{5/2}}$$

Mathematica [C] time = 0.418781, size = 274, normalized size = 1.

$$\frac{ia^{3/2}d^3\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-ia^{3/2}d^3\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-a^{3/2}d^3\sinh\left(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (-4*b^(3/2)*d*x*Cosh[c + d*x] + I*a^(3/2)*d^3*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b
]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - I*a^(3/2)*d^3*Cosh[c + (I*
Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 4*b^(3/2)*Si
nh[c + d*x] - 2*a*Sqrt[b]*d^2*Sinh[c + d*x] + 2*b^(3/2)*d^2*x^2*Sinh[c + d*
x] - a^(3/2)*d^3*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] - I*d*x] - a^(3/2)*d^3*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(S
```

```
qrt[a]*d)/Sqrt[b] + I*d*x])/(2*b^(5/2)*d^3)
```

Maple [A] time = 0.189, size = 369, normalized size = 1.4

$$-\frac{e^{-dx-c}x^2}{2bd}+\frac{e^{-dx-c}a}{2db^2}-\frac{e^{-dx-c}x}{bd^2}-\frac{e^{-dx-c}}{d^3b}-\frac{a^2}{4b^2}e^{-\frac{1}{b}(d\sqrt{-ab}+cb)}\text{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b+cb\right)\right)\frac{1}{\sqrt{-ab}}+\frac{a^2}{4b^2}e^{\frac{1}{b}(d\sqrt{-ab-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^2+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x^2+1/2/d*exp(-d*x-c)/b^2*a-1/d^2*exp(-d*x-c)/b*x-1/d^
3*exp(-d*x-c)/b-1/4/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2+1/4/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)
-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a^2-1/4/b^2/(-a*b)^(1/2)*ex
p((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a^2+1/4/b^
2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b
-c*b)/b)*a^2+1/2/d/b*exp(d*x+c)*x^2-1/2/d/b^2*a*exp(d*x+c)-1/d^2/b*exp(d*x+
c)*x+1/d^3/b*exp(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19521, size = 1296, normalized size = 4.75

$$-\frac{8\,bdx\cosh(dx+c)+\left(\left(ad^2\cosh(dx+c)^2-ad^2\sinh(dx+c)^2\right)\sqrt{-\frac{ad^2}{b}}\text{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\left(ad^2\cosh(dx+c)^2-ad^2\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(8*b*d*x*cosh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)
^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^
2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b
)*Ei(d*x + sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2
)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(
b*d^2*x^2 - a*d^2 + 2*b)*sinh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*si
nh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x +
c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sin
h(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh
```

```
(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^
2/b)))/(b^2*d^3*cosh(d*x + c)^2 - b^2*d^3*sinh(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**4*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx + c)}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^2 + a), x)
```

3.58 $\int \frac{x^3 \cosh(c+dx)}{a+bx^2} \, dx$

Optimal. Leaf size=209

$$-\frac{a\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}+\frac{a\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\sinh}{}$$

```
[Out] -(Cosh[c + d*x]/(b*d^2)) - (a*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2) + (x*Sinh[c + d*x])/(b*d) +
(a*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(2*b^2) - (a*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b^2)
```

Rubi [A] time = 0.35826, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3296, 2638, 3303, 3298, 3301}

$$-\frac{a\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2b^2}+\frac{a\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(Cosh[c + d*x]/(b*d^2)) - (a*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2) + (x*Sinh[c + d*x])/(b*d) +
(a*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(2*b^2) - (a*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(2*b^2)
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx = \int \left(\frac{x\cosh(c+dx)}{b} - \frac{ax\cosh(c+dx)}{b\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int x\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{b}$$

$$= \frac{x\sinh(c+dx)}{bd} - \frac{a\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b} - \frac{\int \sinh(c+dx)\,dx}{bd}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} + \frac{a\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} - \frac{a\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} - \frac{\left(a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\left(a\cosh\left(c+\frac{\sqrt{-}}{\sqrt{}}\right.\right.}{}$$

$$= -\frac{\cosh(c+dx)}{bd^2} - \frac{a\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} - \frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} +$$

Mathematica [C] time = 0.348637, size = 210, normalized size = 1.

$$-\frac{ad^2\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right) + ad^2\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right) + iad^2\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right.}{2b^2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] -(2*b*Cosh[c + d*x] + a*d^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((
Sqrt[a]*d)/Sqrt[b]) + I*d*x] + a*d^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosInt
egral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - 2*b*d*x*Sinh[c + d*x] + I*a*d^2*Sinh[c
 - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*a*d^
2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])
/(2*b^2*d^2)
```

Maple [A] time = 0.06, size = 268, normalized size = 1.3

$$-\frac{e^{-dx-c}x}{2\,bd} - \frac{e^{-dx-c}}{2\,bd^2} + \frac{a}{4\,b^2}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)\,b-cb\right)\right) + \frac{a}{4\,b^2}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x-1/2/d^2*exp(-d*x-c)/b+1/4/b^2*exp((d*(-a*b)^(1/2)-c*
b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a+1/4/b^2*exp(-(d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a+1/2/d/b*exp(d*x+c)*x-1/2
/d^2/b*exp(d*x+c)+1/4/b^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)*a+1/4/b^2*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1
/2)+(d*x+c)*b-c*b)/b)*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(dx^3e^{(2c)} - x^2e^{(2c)}\right)e^{(dx)} - \left(dx^3 + x^2\right)e^{(-dx)}}{2\left(bd^2x^2e^c + ad^2e^c\right)} - \frac{1}{2}\int \frac{2\left(adx^2e^c - axe^c\right)e^{(dx)}}{b^2d^2x^4 + 2\,abd^2x^2 + a^2d^2}\,dx + \frac{1}{2}\int \frac{2\left(adx^2 + ax\right)e^{(-dx)}}{b^2d^2x^4e^c + 2\,abd^2x^2e^c + a^2d^2e^c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^3*e^(2*c) - x^2*e^(2*c))*e^(d*x) - (d*x^3 + x^2)*e^(-d*x))/(b*d^2
*x^2*e^c + a*d^2*e^c) - 1/2*integrate(2*(a*d*x^2*e^c - a*x*e^c)*e^(d*x)/(b^
2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2), x) + 1/2*integrate(2*(a*d*x^2 + a*x)*
e^(-d*x)/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c), x)
```

Fricas [B] time = 2.10293, size = 1100, normalized size = 5.26

$$\frac{4\,bdx\sinh\left(dx + c\right) - 4\,b\cosh\left(dx + c\right) - \left(\left(ad^2\cosh\left(dx + c\right)^2 - ad^2\sinh\left(dx + c\right)^2\right)\mathrm{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \left(ad^2\cosh\left(dx + c\right)^2 \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x*sinh(d*x + c) - 4*b*cosh(d*x + c) - ((a*d^2*cosh(d*x + c)^2 -
a*d^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 -
a*d^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b))
 + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)
))*cosh(-c + sqrt(-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x
 + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x
 + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(
-a*d^2/b)))/(b^2*d^2*cosh(d*x + c)^2 - b^2*d^2*sinh(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\cosh\left(c + dx\right)}{a + bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a), x)
```

3.59 $\int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx$

Optimal. Leaf size=226

$$\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}$$

```
[Out] (Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sinh[c + d*x]/(b*d) - (Sqrt[-a]
*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b^(3/2)) - (Sqrt[-a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*b^(3/2))
```

Rubi [A] time = 0.375852, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 2637, 5281, 3303, 3298, 3301}

$$\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sinh[c + d*x]/(b*d) - (Sqrt[-a]
*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b^(3/2)) - (Sqrt[-a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*b^(3/2))
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx &= \int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b\left(a+bx^2\right)}\right)dx \\
&= \frac{\int \cosh(c+dx)\,dx}{b} - \frac{a\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{b} \\
&= \frac{\sinh(c+dx)}{bd} - \frac{a\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b} \\
&= \frac{\sinh(c+dx)}{bd} - \frac{\sqrt{-a}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b} - \frac{\sqrt{-a}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} \\
&= \frac{\sinh(c+dx)}{bd} - \frac{\left(\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b} - \frac{\left(\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\sqrt{\ldots}\right.}{\sqrt{-a}\ldots}}{2b} \\
&= \frac{\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}} + \frac{\sinh(c+\ldots}{bd}
\end{aligned}$$

Mathematica [C] time = 0.251724, size = 213, normalized size = 0.94

$$\frac{-i\sqrt{a}d\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+i\sqrt{a}d\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\sqrt{a}d\sinh\left(c-\ldots\right.}{2b^{3/2}d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] ((-I)*Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/S
qrt[b]) + I*d*x] + I*Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[
(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 2*Sqrt[b]*Sinh[c + d*x] + Sqrt[a]*d*Sinh[c -
 (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + Sqrt[a]*
d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])
/(2*b^(3/2)*d)
```

Maple [A] time = 0.045, size = 259, normalized size = 1.2

$$-\frac{e^{-dx-c}}{2bd} - \frac{a}{4b}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\operatorname{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b-cb\right)\right)\frac{1}{\sqrt{-ab}} + \frac{a}{4b}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\operatorname{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b-1/4/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a+1/4/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a+1/2/d/b*exp(d*x+c)-1/4/b/(
-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*
b)/b)*a+1/4/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)
-(d*x+c)*b+c*b)/b)*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16906, size = 1087, normalized size = 4.81

$$\frac{\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh(dx+c)^2-\sinh(dx+c)^2\right)\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\left(\cosh(dx+c)^2-\sinh(dx+c)^2\right)\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d
^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(
-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - s
inh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2
- sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (
sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b))
 - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2
/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*
x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh
(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)) + 4*sinh(
d*x + c))/(b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a), x)
```

3.60 $\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=177

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\sinh\left(c-\frac{\sqrt{-}}{\sqrt{}}\right.}{}$$

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]
*d)/Sqrt[b] + d*x])/(2*b)
```

Rubi [A] time = 0.254037, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\sinh\left(c-\frac{\sqrt{-}}{\sqrt{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]
*d)/Sqrt[b] + d*x])/(2*b)
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}}$$

$$= \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{b}}-\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{b}}+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int}{2\sqrt{}}$$

$$= \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\right.}{2b}$$

Mathematica [C] time = 0.174738, size = 171, normalized size = 0.97

$$\frac{\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+i\left(\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-\right.\right.}{2b}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

```
[Out] (Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x
] + Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x
] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] - Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]))/(2*b)
```

Maple [A] time = 0.032, size = 200, normalized size = 1.1

$$-\frac{1}{4b}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\text{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b-cb\right)\right)-\frac{1}{4b}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\text{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b+cb\right)\right)-\frac{1}{4b}e^{\frac{1}{b}(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a),x)
```

```
[Out] -1/4/b*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1
/4/b*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1
/4/b*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4
/b*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{xe^{(dx+2c)}-xe^{(-dx)}}{2\left(bdx^2e^c+ade^c\right)}+\frac{1}{2}\int\frac{\left(bx^2e^c-ae^c\right)e^{(dx)}}{b^2dx^4+2abdx^2+a^2d}\,dx-\frac{1}{2}\int\frac{\left(bx^2-a\right)e^{(-dx)}}{b^2dx^4e^c+2abdx^2e^c+a^2de^c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c) + 1/2*integrate(
(b*x^2*e^c - a*e^c)*e^(d*x)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 1/2*int
egrate((b*x^2 - a)*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c),
x)
```

Fricas [A] time = 2.09171, size = 455, normalized size = 2.57

$$\frac{\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx-\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(-c+\sqrt{-\frac{ad^2}{b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-
a*d^2/b)) + (Ei(d*x + sqrt(-a*d^2/b)) + Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c
+ sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*
sinh(c + sqrt(-a*d^2/b)) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^
2/b)))*sinh(-c + sqrt(-a*d^2/b)))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(c+dx\right)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a), x)
```

3.61 $\int \frac{\cosh(c+dx)}{a+bx^2}\,dx$

Optimal. Leaf size=213

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\right.}{}$$

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

Rubi [A] time = 0.258696, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5281, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2),x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}}$$

$$= -\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2\sqrt{-a}} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2\sqrt{-a}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\sinh}{\ldots}}{2\sqrt{-a}}$$

$$= \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\ldots\right)}{2\sqrt{-a}\sqrt{b}}$$

Mathematica [C] time = 0.200531, size = 180, normalized size = 0.85

$$\frac{i\left(\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+i\left(\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-i\ldots\right.\right.\right.}{2\sqrt{a}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2),x]
```

```
[Out] ((I/2)*(Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
+ I*d*x] - Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x] + Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] + I*d*x])))/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.021, size = 212, normalized size = 1.

$$\frac{1}{4}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\text{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b-cb\right)\right)\frac{1}{\sqrt{-ab}}-\frac{1}{4}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\text{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b+cb\right)\right)\frac{1}{\sqrt{-ab}}+\frac{1}{4}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a),x)
```

```
[Out] 1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b
-c*b)/b)-1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)+1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15703, size = 624, normalized size = 2.93

$$-\frac{\left(\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-\left(\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/
b)) + (sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/
b)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

3.62 $\int \frac{\cosh(c+dx)}{x(a+bx^2)} \, dx$

Optimal. Leaf size=197

$$-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a}-\frac{\sinh\left(c-\right.}{}$$

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a) + (Sinh[c]*SinhIntegral[d*x])/a
 + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(2*a) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(2*a)
```

Rubi [A] time = 0.372347, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5293, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a}-\frac{\sinh\left(c-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a) + (Sinh[c]*SinhIntegral[d*x])/a
 + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]
)/(2*a) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 + d*x])/(2*a)
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a}$$

$$= -\frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a} + \frac{\sinh(c)\int \frac{\sinh(dx)}{x}\,dx}{a}$$

$$= \frac{\cosh(c)\mathbf{Chi}(dx)}{a} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a} - \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a}$$

$$= \frac{\cosh(c)\mathbf{Chi}(dx)}{a} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a} - \frac{\left(\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a} + \frac{\left(\sqrt{b}\cosh\left(c+\right.\right.}{}$$

$$= \frac{\cosh(c)\mathbf{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a} + \frac{\sinh}{}$$

Mathematica [C] time = 0.310288, size = 187, normalized size = 0.95

$$-\frac{\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+i\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

```
[Out] -(-2*Cosh[c]*CoshIntegral[d*x] + Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegra
l[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosInte
gral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - 2*Sinh[c]*SinhIntegral[d*x] + I*Sinh[c
- (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*Sinh[
c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*a)
```

Maple [A] time = 0.043, size = 227, normalized size = 1.2

$$\frac{1}{4a}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\text{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b+cb\right)\right)+\frac{1}{4a}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\text{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b-cb\right)\right)-\frac{e^{-c}\text{Ei}(1}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^2+a),x)
```

```
[Out] 1/4/a*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+
1/4/a*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/
2/a*exp(-c)*Ei(1,d*x)+1/4/a*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1
/2)+(d*x+c)*b-c*b)/b)+1/4/a*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)-1/2/a*exp(c)*Ei(1,-d*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Fricas [A] time = 2.20263, size = 545, normalized size = 2.77

$$-\frac{\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-2\left(\mathrm{Ei}(dx)+\mathrm{Ei}(-dx)\right)\cosh(c)+\left(\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) - 2*(Ei(d*x) + Ei(-d*x))*cosh(c) + (Ei(d*x + sqrt(-a*d^2/b)) + E
i(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2
/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 2*(Ei(d*x) - E
i(-d*x))*sinh(c) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^2/b)))*s
inh(-c + sqrt(-a*d^2/b)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

3.63 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=249

$$\frac{\sqrt{b}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-$$

```
[Out] -(Cosh[c + d*x]/(a*x)) + (Sqrt[b]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) - (Sqrt[b]*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2)) + (d
*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - (Sqrt[b]*
Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2
*(-a)^(3/2)) - (Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2))
```

Rubi [A] time = 0.495823, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\sqrt{b}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}}-\frac{\sqrt{b}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```


```
[Out] -(Cosh[c + d*x]/(a*x)) + (Sqrt[b]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) - (Sqrt[b]*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2)) + (d
*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - (Sqrt[b]*
Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2
*(-a)^(3/2)) - (Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2))
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b\cosh(c+dx)}{a\left(a+bx^2\right)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{3/2}} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} + \frac{(d\cosh(c))\int \frac{\sinh(dx)}{x}\,dx}{a} + \frac{(d\sinh(c))\int \frac{\text{co}}{}}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\,\mathbf{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\mathbf{Shi}(dx)}{a} - \frac{\left(b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{3/2}} -$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}} + \frac{d}{}$$

Mathematica [C] time = 0.357135, size = 243, normalized size = 0.98

$$\frac{-i\sqrt{b}x\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+i\sqrt{b}x\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\sqrt{b}x\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*Cosh[c + d*x] - I*Sqrt[b]*x*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*Cos
Integral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + I*Sqrt[b]*x*Cosh[c + (I*Sqrt[a]*
d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 2*Sqrt[a]*d*x*CoshIn
tegral[d*x]*Sinh[c] + 2*Sqrt[a]*d*x*Cosh[c]*SinhIntegral[d*x] + Sqrt[b]*x*S
inh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + S
qrt[b]*x*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] +
I*d*x])/(2*a^(3/2)*x)
```

Maple [A] time = 0.055, size = 288, normalized size = 1.2

$$-\frac{e^{-dx-c}}{2ax}+\frac{b}{4a}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b+cb\right)\right)\frac{1}{\sqrt{-ab}}-\frac{b}{4a}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a),x)
```

```
[Out] -1/2*exp(-d*x-c)/a/x+1/4*b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2
)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/2*d/a*exp(-c)*Ei(1,d*x)-
1/2/a/x*exp(d*x+c)+1/4*b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2*d/a*exp(c)*Ei(1,-d*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49935, size = 1323, normalized size = 5.31

$$-\frac{4\,ad\cosh\left(dx+c\right)-\left(\left(bx\cosh\left(dx+c\right)^{2}-bx\sinh\left(dx+c\right)^{2}\right)\sqrt{-\frac{ad^{2}}{b}}\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^{2}}{b}}\right)+\left(bx\cosh\left(dx+c\right)^{2}-bx\sinh\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*cosh(d*x + c) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 - 2*(a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*cosh(c) + ((b*x*cosh(d*x + c)^2 -
 b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x*cosh(d
*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*
cosh(-c + sqrt(-a*d^2/b)) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (b*x*cosh(d*x + c)^2 - b*x*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b))
 - 2*(a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c) - ((b*x*cosh(d*x + c)^2 -
 b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x*cosh(d
*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*
sinh(-c + sqrt(-a*d^2/b)))/(a^2*d*x*cosh(d*x + c)^2 - a^2*d*x*sinh(d*x + c)
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (c + dx)}{x^2 \left(a + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**2*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (dx + c)}{\left(bx^2 + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

3.64 $\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal. Leaf size=270

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}-$$

```
[Out] -Cosh[c + d*x]/(2*a*x^2) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (d^2*Cosh[c]
*CoshIntegral[d*x])/(2*a) + (b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*Sinh[c + d*x])/(2*a*x)
 - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a
) - (b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x])/(2*a^2) + (b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] + d*x])/(2*a^2)
```

Rubi [A] time = 0.504207, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5293, 3297, 3303, 3298, 3301}

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```


```
[Out] -Cosh[c + d*x]/(2*a*x^2) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (d^2*Cosh[c]
*CoshIntegral[d*x])/(2*a) + (b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*Sinh[c + d*x])/(2*a*x)
 - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a
) - (b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x])/(2*a^2) + (b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] + d*x])/(2*a^2)
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}dx = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2x\cosh(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{x}dx}{a^2} + \frac{b^2\int \frac{x\cosh(c+dx)}{a+bx^2}dx}{a^2}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b^2\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} + \frac{d\int \frac{\sinh(c+dx)}{x^2}dx}{2a} - \frac{(b\cosh(c))\int \frac{\cosh(}{x}}{a^2}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} - \frac{b^{3/2}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{2a^2} + \ldots$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{d\sinh(c+dx)}{2ax} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} + \frac{\left(d^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} + \frac{b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} + \ldots$$

Mathematica [C] time = 0.530193, size = 257, normalized size = 0.95

$$x^2\cosh(c)\left(-\left(2b-ad^2\right)\right)\mathrm{Chi}(dx) + bx^2\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right) + bx^2\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\Big(\ldots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

```
[Out] (-(a*Cosh[c + d*x]) - (2*b - a*d^2)*x^2*Cosh[c]*CoshIntegral[d*x] + b*x^2*C
osh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]
+ b*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x] - a*d*x*Sinh[c + d*x] - 2*b*x^2*Sinh[c]*SinhIntegral[d*x] + a*d^2*x^2
*Sinh[c]*SinhIntegral[d*x] + I*b*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinInt
egral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*b*x^2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*a^2*x^2)
```

Maple [A] time = 0.063, size = 330, normalized size = 1.2

$$\frac{d\mathrm{e}^{-dx-c}}{4\,ax} - \frac{\mathrm{e}^{-dx-c}}{4\,ax^2} - \frac{b}{4\,a^2}\mathrm{e}^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)\,b+cb\right)\right) - \frac{b}{4\,a^2}\mathrm{e}^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)\,b\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x^2+a),x)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2-1/4*b/a^2*exp(-(d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*b/a^2*exp((d*(-a*b)^(1/
2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4*d^2/a*exp(-c)*Ei(1,d*
x)+1/2/a^2*exp(-c)*Ei(1,d*x)*b-1/4*d/a/x*exp(d*x+c)-1/4/a/x^2*exp(d*x+c)-1/
4*b/a^2*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b
)-1/4*b/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)
/b)-1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/2*b/a^2*exp(c)*Ei(1,-d*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Fricas [B] time = 2.51016, size = 1283, normalized size = 4.75

$$-\frac{2\,adx\sinh(dx+c)+2\,a\cosh(dx+c)-\left(\left(bx^2\cosh(dx+c)^2-bx^2\sinh(dx+c)^2\right)\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\left(bx^2\cosh(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((b*x^2*cosh(d*x + c)^2 -
 b*x^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x^2*cosh(d*x + c)^2 -
 b*x^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 - ((a*d^2 - 2*b)*x^2*Ei(d*x) + (a*d^2 - 2*b)*x^2*Ei(-d*x))*cosh(c) - ((b*x
^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x
^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh
(-c + sqrt(-a*d^2/b)) - ((b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei
(d*x - sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei
(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((a*d^2 - 2*b)*x^2*Ei(d
*x) - (a*d^2 - 2*b)*x^2*Ei(-d*x))*sinh(c) + ((b*x^2*cosh(d*x + c)^2 - b*x^2
*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2
*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/(a^
2*x^2*cosh(d*x + c)^2 - a^2*x^2*sinh(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(x**3*(a + b*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

3.65 $\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} \, dx$

Optimal. Leaf size=449

$$-\frac{ad \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3}-\frac{ad \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}+\frac{3\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right.}{4b^{5/2}}$$

```
[Out] (x*Cosh[c + d*x])/(2*b^2) - (x^3*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (3*Sqrt
[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(4*b^(5/2)) - (3*Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) - (a*d*CoshIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) - (a*d*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) + Sinh[c +
 d*x]/(b^2*d) + (a*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-
a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(4*b^(5/2))
```

Rubi [A] time = 0.859464, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5291, 5293, 2637, 5281, 3303, 3298, 3301, 5292, 3296}

$$-\frac{ad \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3}-\frac{ad \sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3}+\frac{3\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right.}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```


```
[Out] (x*Cosh[c + d*x])/(2*b^2) - (x^3*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (3*Sqrt
[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(4*b^(5/2)) - (3*Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) - (a*d*CoshIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) - (a*d*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^3) + Sinh[c +
 d*x]/(b^2*d) + (a*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] - d*x])/(4*b^3) - (3*Sqrt[-a]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)) - (a*d*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^3) - (3*Sqrt[-
a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(4*b^(5/2))
```


Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4 \cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{3\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx}{2b} + \frac{d\int \frac{x^3\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{3\int\left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b\left(a+bx^2\right)}\right)dx}{2b} + \frac{d\int\left(\frac{x\sinh(c+dx)}{b} - \frac{ax\sinh(c+dx)}{b\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{3\int\cosh(c+dx)\,dx}{2b^2} - \frac{(3a)\int\frac{\cosh(c+dx)}{a+bx^2}\,dx}{2b^2} + \frac{d\int x\sinh(c+dx)\,dx}{2b^2} - \frac{(ad}{}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{3\sinh(c+dx)}{2b^2d} - \frac{\int\cosh(c+dx)\,dx}{2b^2} - \frac{(3a)\int\left(\frac{\sqrt{-a}\cosh(c}{2a\left(\sqrt{-a}-\sqrt{}\right.}\right.}{}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\sinh(c+dx)}{b^2d} - \frac{\left(3\sqrt{-a}\right)\int\frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^2} - \frac{\left(3\sqrt{-a}\right)\int\frac{\cosh}{\sqrt{-a}}}{4b^2}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\sinh(c+dx)}{b^2d} - \frac{\left(3\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}}{4b^2}$$

$$= \frac{x\cosh(c+dx)}{2b^2} - \frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{3\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} - \frac{3\sqrt{-a}\cosh\left(c\right.}{}$$

Mathematica [C] time = 1.50657, size = 621, normalized size = 1.38

$$-\frac{\frac{3\sqrt{a}\sinh(c)\left(\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\left(\text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)+\text{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right)\right)\right)}{\sqrt{b}} - \frac{ad\sinh(c)\left(\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{Cos}\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (2*Cosh[d*x]*((a*x*Cosh[c])/(a + b*x^2) + (2*Sinh[c])/d) + 2*((2*Cosh[c])/d
 + (a*x*Sinh[c])/(a + b*x^2))*Sinh[d*x] - ((3*I)*Sqrt[a]*Cosh[c]*(Cos[(Sqrt
[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*
d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt
[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sq
rt[b] + I*d*x])))/Sqrt[b] + (I*a*d*Cosh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[
b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sq
rt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b -
 (3*Sqrt[a]*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[
a]*d)/Sqrt[b]] + CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/S
qrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/Sqrt[b] - (a*d*Sinh[c]*(Cos
[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sq
rt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d
)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]
*d)/Sqrt[b] + I*d*x])))/b)/(4*b^2)
```

Maple [A] time = 0.319, size = 532, normalized size = 1.2

$$\frac{d^2 e^{-dx-c} ax}{4 b^2 \left(b d^2 x^2 + a d^2\right)} - \frac{3a}{8b^2} e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)} \mathrm{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) \frac{1}{\sqrt{-ab}} + \frac{3a}{8b^2} e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)} \mathrm{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^2*exp(-d*x-c)*a/b^2/(b*d^2*x^2+a*d^2)*x-3/8/b^2*a/(-a*b)^(1/2)*exp((d
*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+3/8/b^2*a/(-a*
b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/
b)-1/2/d*exp(-d*x-c)/b^2-1/8*d/b^3*a*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-
a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8*d/b^3*a*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/2/d/b^2*exp(d*x+c)+1/4*d^2*exp(d*x+c)*
a/b^2/(b*d^2*x^2+a*d^2)*x+3/8/b^2*a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-3/8/b^2*a/(-a*b)^(1/2)*exp(-(d*(-a
*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/8*d/b^3*a*exp((
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/b^3*a*e
xp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31252, size = 2488, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
(a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^
2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b
)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sin
h(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*
x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b
)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*si
nh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d
*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d
^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2
 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 8*(b^2*x^2 + a*b)*si
nh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^
2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*
b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^
```

```
2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3*
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x
^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + 3
*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-
a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)
^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x
+ c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a
*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((b^4*d*x^2 + a*b^3*d)*cosh(d*x + c)^2
 - (b^4*d*x^2 + a*b^3*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx+c)}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

3.66 $\int \frac{x^3 \cosh(c+dx)}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=431

$$-\frac{\sqrt{-a}d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}+\frac{\sqrt{-a}d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}+\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}$$

```
[Out] Cosh[c + d*x]/(2*b^2) - (x^2*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (C
osh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
b^2) - (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[
-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) - (Sqrt[-a]*d*Cosh[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)
) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(2*b^2) - (Sqrt[-a]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2)
```

Rubi [A] time = 0.700681, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5291, 5293, 3303, 3298, 3301, 5292, 2638, 5280}

$$-\frac{\sqrt{-a}d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}}+\frac{\sqrt{-a}d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}}+\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] Cosh[c + d*x]/(2*b^2) - (x^2*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) + (C
osh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*
b^2) - (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[
-a]*d)/Sqrt[b]])/(4*b^(5/2)) + (Sqrt[-a]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b
] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*b^(5/2)) - (Sqrt[-a]*d*Cosh[c +
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^(5/2)
) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(2*b^2) - (Sqrt[-a]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(4*b^(5/2)) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2)
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx = -\frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{b} + \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^2}\,dx}{2b}$$

$$= -\frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{b} + \frac{d\int \left(\frac{\sinh(c+dx)}{b} - \frac{a\sinh(c+dx)}{b\left(a+bx^2\right)}\right)dx}{2b}$$

$$= -\frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2b^{3/2}} + \frac{d\int \sinh(c+dx)\,dx}{2b^2} - \frac{(ad)\int \frac{\sinh(c+dx)}{a+bx^2}\,d}{2b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} - \frac{(ad)\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\cosh\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{2b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\cosh\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{2b^2}$$

$$= \frac{\cosh(c+dx)}{2b^2} - \frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{\cosh\left(c+\frac{\sqrt{-ad}}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2b^2} + \frac{\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{2b^2}$$

Mathematica [C] time = 0.740256, size = 582, normalized size = 1.35

$$\frac{a^{3/2}d\cosh\left(c-\frac{i\sqrt{ad}}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}}-idx\right) + a^{3/2}d\cosh\left(c+\frac{i\sqrt{ad}}{\sqrt{b}}\right)\mathrm{Si}\left(ixd+\frac{\sqrt{ad}}{\sqrt{b}}\right) + 2ib^{3/2}x^2\sinh\left(c-\frac{i\sqrt{ad}}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}}-idx\right) - 2ib^{3}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (2*a*Sqrt[b]*Cosh[c + d*x] + (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
 + I*d*x]*(2*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] - I*Sqrt[a]*d*Sinh[c -
 (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]*(2*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*Sqrt[a]*d*Sinh[c + (I*S
qrt[a]*d)/Sqrt[b]]) + a^(3/2)*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral
[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + Sqrt[a]*b*d*x^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt
[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (2*I)*a*Sqrt[b]*Sinh[c - (I
*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + (2*I)*b^(3/
2)*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] + a^(3/2)*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] + I*d*x] + Sqrt[a]*b*d*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegra
l[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (2*I)*a*Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqr
t[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (2*I)*b^(3/2)*x^2*Sinh[c +
 (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*b^(5/2
)*(a + b*x^2))
```

Maple [A] time = 0.095, size = 495, normalized size = 1.2

$$\frac{d^2\mathrm{e}^{-dx-c}a}{4\,b^2\left(bd^2x^2+ad^2\right)} - \frac{1}{4\,b^2}\mathrm{e}^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)\,b-cb\right)\right) - \frac{1}{4\,b^2}\mathrm{e}^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^2*exp(-d*x-c)*a/b^2/(b*d^2*x^2+a*d^2)-1/4/b^2*exp((d*(-a*b)^(1/2)-c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4/b^2*exp(-(d*(-a*b)^(1/2)+c*
b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/b^2*a/(-a*b)^(1/2)*exp(
(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8*d/b^2*a/
(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c
*b)/b)-1/4/b^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c
*b)/b)-1/4/b^2*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b
-c*b)/b)+1/4*d^2*exp(d*x+c)*a/b^2/(b*d^2*x^2+a*d^2)+1/8*d/b^2*a/(-a*b)^(1/2
)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d/
b^2*a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15299, size = 2099, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(d*x + sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)
) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d
*x - sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(
d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((2*(b*
x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cos
h(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2
+ ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((b^3*x^2 + a*b^2)
```

```
*cosh(d*x + c)^2 - (b^3*x^2 + a*b^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

3.67 $\int \frac{x^2 \cosh(c+dx)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=416

$$\frac{d \sinh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^2}+\frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4b^2}+\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\cosh($$

```
[Out] -(x*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) - (Cosh[c - (Sqr
t[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(
3/2)) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/S
qrt[b]])/(4*b^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sq
rt[-a]*d)/Sqrt[b]])/(4*b^2) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Cosh[c -
(Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2) - (
Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4
*Sqrt[-a]*b^(3/2))
```

Rubi [A] time = 0.611313, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5291, 5281, 3303, 3298, 3301, 5292}

$$\frac{d \sinh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^2}+\frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4b^2}+\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\cosh($$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```


```
[Out] -(x*Cosh[c + d*x])/(2*b*(a + b*x^2)) + (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) - (Cosh[c - (Sqr
t[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(
3/2)) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/S
qrt[b]])/(4*b^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sq
rt[-a]*d)/Sqrt[b]])/(4*b^2) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*b^2) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*Sqrt[-a]*b^(3/2)) + (d*Cosh[c -
(Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*b^2) - (
Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4
*Sqrt[-a]*b^(3/2))
```


Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx &= -\frac{x\cosh(c+dx)}{2b\left(a+bx^2\right)}+\frac{\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{2b}+\frac{d\int \frac{x\sinh(c+dx)}{a+bx^2}\,dx}{2b}\\
&= -\frac{x\cosh(c+dx)}{2b\left(a+bx^2\right)}+\frac{\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}+\frac{d\int \left(-\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)}{2b}\\
&= -\frac{x\cosh(c+dx)}{2b\left(a+bx^2\right)}-\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}b}-\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}-\frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4b^{3/2}}+\frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{3/2}}\\
&= -\frac{x\cosh(c+dx)}{2b\left(a+bx^2\right)}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\int \frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}b}+\frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4b^{3/2}}\\
&= -\frac{x\cosh(c+dx)}{2b\left(a+bx^2\right)}+\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\text{Cl}}{}
\end{aligned}$$

Mathematica [C] time = 0.991712, size = 364, normalized size = 0.88

$$\left(a+bx^2\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)\left(\sqrt{a}d\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)+i\sqrt{b}\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\right)+\left(a+bx^2\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)\Big($$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] (-2*Sqrt[a]*b*x*Cosh[c + d*x] + (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[
b]) + I*d*x]*(I*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c
- (I*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I
*d*x]*((-I)*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I
*Sqrt[a]*d)/Sqrt[b]]) + (a + b*x^2)*(I*Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqr
t[b]] - Sqrt[b]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]])*SinIntegral[(Sqrt[a]*d)/Sq
rt[b] - I*d*x] - (a + b*x^2)*(I*Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] +
 Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]])*SinIntegral[(Sqrt[a]*d)/Sqrt[b] +
 I*d*x])/(4*Sqrt[a]*b^2*(a + b*x^2))
```

Maple [A] time = 0.068, size = 491, normalized size = 1.2

$$-\frac{d^2 e^{-dx-c} x}{4b\left(bd^2x^2+ad^2\right)}+\frac{d}{8b^2}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b-cb\right)\right)+\frac{d}{8b^2}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/4*d^2*exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)*x+1/8*d/b^2*exp((d*(-a*b)^(1/2)-c*
b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/8*d/b^2*exp(-(d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/b/(-a*b)^(1/2)*exp((d*
(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/b/(-a*b)^(1
/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/
4*d^2*exp(d*x+c)/b/(b*d^2*x^2+a*d^2)*x-1/8*d/b^2*exp((d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d/b^2*exp(-(d*(-a*b)^(1/2)-c*b
)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/b/(-a*b)^(1/2)*exp((d*(-a*
b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/b/(-a*b)^(1/2)*
exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19618, size = 2433, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
 (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2
```

```
 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)
) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh
(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
 c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(
d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x
 - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
 a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*
x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 -
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^3*d*x^
2 + a^2*b^2*d)*cosh(d*x + c)^2 - (a*b^3*d*x^2 + a^2*b^2*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\cosh{(dx+c)}}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

3.68 $\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=239

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\,c\cdots}{\cdots}$$

```
[Out] -Cosh[c + d*x]/(2*b*(a + b*x^2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d
*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (d*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(
3/2)) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```

Rubi [A] time = 0.325431, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5289, 5280, 3303, 3298, 3301}

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}}+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}}-\frac{d\,c\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```


```
[Out] -Cosh[c + d*x]/(2*b*(a + b*x^2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d
*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(3/2)) + (d*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*Sqrt[-a]*b^(
3/2)) - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(4*Sqrt[-a]*b^(3/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*Sqrt[-a]*b^(3/2))
```


Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}dx = -\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}dx}{2b}$$

$$= -\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)} + \frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2b}$$

$$= -\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{4\sqrt{-a}b} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b}$$

$$= -\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{4\sqrt{-a}b} + \frac{\left(d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{b}x}dx}{4\sqrt{-a}b}$$

$$= -\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)} - \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cos}{}$$

Mathematica [C] time = 0.513809, size = 239, normalized size = 1.

$$\frac{i\left(d\left(a+bx^2\right)\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-d\left(a+bx^2\right)\sinh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+i\left(d\left(a\right.\right.}{4\sqrt{a}b^{3/2}\left(a+bx\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

```
[Out] ((I/4)*(d*(a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sinh[c -
(I*Sqrt[a]*d)/Sqrt[b]] - d*(a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*
d*x]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]] + I*(2*Sqrt[a]*Sqrt[b]*Cosh[c + d*x] +
 d*(a + b*x^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt
[b] - I*d*x] + d*(a + b*x^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(S
qrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*b^(3/2)*(a + b*x^2))
```

Maple [A] time = 0.044, size = 291, normalized size = 1.2

$$-\frac{d^2e^{-dx-c}}{4b\left(bd^2x^2+ad^2\right)} - \frac{d}{8b}e^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)b-cb\right)\right)\frac{1}{\sqrt{-ab}} + \frac{d}{8b}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/4*d^2*exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^(1/2)*exp((d*(-a*b)
^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/8*d/b/(-a*b)^(1/2)*
exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*d^
2*exp(d*x+c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/b/(-a*b)^(1/2)*exp(-(d*
(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.16685, size = 1393, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*cosh(d*x + c) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x
 + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2
/b)))*cosh(c + sqrt(-a*d^2/b)) - (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - ((b*x^2 + a)*c
osh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt
(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b
*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + ((b*x^
2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*
x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)
^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) +
 ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)
*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^2*x^2 + a^2*b)
*cosh(d*x + c)^2 - (a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(dx+c)}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

3.69 $\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^2} dx$

Optimal. Leaf size=476

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh}{}$$

```
[Out] -Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*Sq
rt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b
]) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt
[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Cosh[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) + (S
inh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*
(-a)^(3/2)*Sqrt[b])
```

Rubi [A] time = 0.817828, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5281, 3297, 3303, 3298, 3301}

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^2,x]
```


```
[Out] -Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*Sq
rt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b
]) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt
[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Cosh[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) + (S
inh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*
(-a)^(3/2)*Sqrt[b])
```


Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx = \int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cosh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx$$

$$= -\frac{b\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{4a} - \frac{b\int \frac{\cosh(c+dx)}{-ab-b^2x^2}\,dx}{2a}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{b\int \left(-\frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\cosh(c+dx)}{2ab\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{2a} + \frac{d\int \frac{\sinh(\ldots}{\sqrt{-a}\sqrt{\ldots}}}{4a\ldots}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-\ldots}}{\sqrt{\ldots}}\right.\right.}{\ldots}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-\ldots\right.}{\ldots}$$

$$= -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right.\ldots}{4(-\ldots}$$

Mathematica [C] time = 0.687951, size = 590, normalized size = 1.24

$$\frac{-ia^{3/2}d\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right) + ia^{3/2}d\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right) - b^{3/2}x^2\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\mathrm{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right) - b^{3/\ldots}}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^2,x]
```

```
[Out] (2*Sqrt[a]*b*x*Cosh[c + d*x] - (a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b
]) + I*d*x]*((-I)*Sqrt[b]*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[
c - (I*Sqrt[a]*d)/Sqrt[b]]) - (a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] +
 I*d*x]*(I*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*
Sqrt[a]*d)/Sqrt[b]]) - I*a^(3/2)*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinInteg
ral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*Sqrt[a]*b*d*x^2*Cosh[c - (I*Sqrt[a]*d)
/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - a*Sqrt[b]*Sinh[c - (I*
Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - b^(3/2)*x^2*
Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] +
I*a^(3/2)*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
 + I*d*x] + I*Sqrt[a]*b*d*x^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
Sqrt[a]*d)/Sqrt[b] + I*d*x] - a*Sqrt[b]*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*Sin
Integral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - b^(3/2)*x^2*Sinh[c + (I*Sqrt[a]*d)/
Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*a^(3/2)*b*(a + b*x^2)
)
```

Maple [A] time = 0.029, size = 503, normalized size = 1.1

$$\frac{d^2\mathrm{e}^{-dx-c}x}{4\,a\left(bd^2x^2+ad^2\right)}-\frac{d}{8\,ab}\mathrm{e}^{\frac{1}{b}\left(d\sqrt{-ab}-cb\right)}\mathrm{Ei}\left(1,\frac{1}{b}\left(d\sqrt{-ab}+(dx+c)\,b-cb\right)\right)-\frac{d}{8\,ab}\mathrm{e}^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\mathrm{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] 1/4*d^2*exp(-d*x-c)*x/a/(b*d^2*x^2+a*d^2)-1/8*d/b/a*exp((d*(-a*b)^(1/2)-c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8*d/b/a*exp(-(d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/a/(-a*b)^(1/2)*exp((d*(
-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/a/(-a*b)^(1/
2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4
*d^2*exp(d*x+c)*x/a/(b*d^2*x^2+a*d^2)+1/8*d/b/a*exp(-(d*(-a*b)^(1/2)-c*b)/b
)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/8*d/b/a*exp((d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/a/(-a*b)^(1/2)*exp(-(d*(-a*
b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/a/(-a*b)^(1/2)
*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27362, size = 2431, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
(a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2
- (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
 - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(
d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) -
 (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d
*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*co
sh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*
cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x
- sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*co
sh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*
cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x -
 sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + s
qrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 +
 a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a
*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*x
^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + (
(b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*
d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^2*d*x
^2 + a^3*b*d)*cosh(d*x + c)^2 - (a^2*b^2*d*x^2 + a^3*b*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

3.70 $\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=435

$$-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sinh\left(c\right.}{}$$

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c
 + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2)
- (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d
)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*Sin
hIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqr
t[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*a^2)
```

Rubi [A] time = 0.839029, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3303, 3298, 3301, 5289, 5280}

$$-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sinh\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```


```
[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c
 + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2)
- (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])
/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d
)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*Sin
hIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[
-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqr
t[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*a^2)
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx = \int \left(\frac{\cosh(c+dx)}{a^2x} - \frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)^2} - \frac{bx\cosh(c+dx)}{a^2\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} - \frac{b\int \left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{2a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a^2}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a^2} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^2} - \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2a^2} - \frac{d\int}{}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4(-a)^{3/2}} - \frac{\left(\sqrt{b}\,\mathrm{co}\right.}{}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2a^2}$$

$$= \frac{\cosh(c+dx)}{2a\left(a+bx^2\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathbf{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2a^2}$$

Mathematica [C] time = 4.80435, size = 363, normalized size = 0.83

$$\frac{4\cosh(c)\text{Chi}(dx) + \frac{i\left(\text{Chi}\left(d\left(x+\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)\left(2i\sqrt{b}\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)-\sqrt{a}d\sinh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\right)+\text{Chi}\left(d\left(x-\frac{i\sqrt{a}}{\sqrt{b}}\right)\right)\left(\sqrt{a}d\sinh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)+2i\sqrt{b}\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\right)-\text{Shi}\left(d\left(x\cdots\right.\right.\right.}{\cdots}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

```
[Out] (4*Cosh[c]*CoshIntegral[d*x] + (I*(((-2*I)*a*Sqrt[b]*Cosh[c + d*x])/(a + b*
x^2) + CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*((2*I)*Sqrt[b]*Cosh[c - (I
*Sqrt[a]*d)/Sqrt[b]] - Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) + CoshInt
egral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*((2*I)*Sqrt[b]*Cosh[c + (I*Sqrt[a]*d)
/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) - (4*I)*Sqrt[b]*Sinh
[c]*SinhIntegral[d*x] - (Sqrt[a]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]] - (2*I)*
Sqrt[b]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]])*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b
] + x)] - (Sqrt[a]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + (2*I)*Sqrt[b]*Sinh[c
 + (I*Sqrt[a]*d)/Sqrt[b]])*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/Sqrt
[b])/(4*a^2)
```

Maple [A] time = 0.074, size = 546, normalized size = 1.3

$$\frac{e^{-dx-c}d^2}{4a\left((dx+c)^2 b-2(dx+c)bc+ad^2+bc^2\right)}-\frac{d}{8a}e^{-\frac{1}{b}\left(d\sqrt{-ab}+cb\right)}\text{Ei}\left(1,-\frac{1}{b}\left(d\sqrt{-ab}-(dx+c)b+cb\right)\right)\frac{1}{\sqrt{-ab}}+\frac{d}{8a}e^{\frac{1}{b}(d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^2+a)^2,x)
```

```
[Out] 1/4*exp(-d*x-c)*d^2/a/((d*x+c)^2*b-2*(d*x+c)*b*c+a*d^2+b*c^2)-1/8/a/(-a*b)^
(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*
d+1/8/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)*d+1/4/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(
d*x+c)*b+c*b)/b)+1/4/a^2*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(
d*x+c)*b-c*b)/b)-1/2/a^2*exp(-c)*Ei(1,d*x)+1/4*exp(d*x+c)*d^2/a/((d*x+c)^2*
b-2*(d*x+c)*b*c+a*d^2+b*c^2)+1/8/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*d-1/8/a/(-a*b)^(1/2)*exp(-(d*(-a*b)
^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d+1/4/a^2*exp((d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a^2*exp(-(d*(-
a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2/a^2*exp(c)*E
i(1,-d*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29714, size = 2253, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a*cosh(d*x + c) - ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*si
nh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2
- 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) + (b*x^2 + a)*Ei(-d*x))*cosh(c) - ((2*
(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*
cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqr
t(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)
^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - ((2*(b*x^2 + a
)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x +
 c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b
)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x
^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-
d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) -
(b*x^2 + a)*Ei(-d*x))*sinh(c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2
+ a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x
+ c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*
x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-
c + sqrt(-a*d^2/b)))/((a^2*b*x^2 + a^3)*cosh(d*x + c)^2 - (a^2*b*x^2 + a^3)
*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^2 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x), x)
```

3.71 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$

Optimal. Leaf size=500

$$\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos}{}$$

```
[Out] -(Cosh[c + d*x]/(a^2*x)) + (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] - Sqrt[
b]*x)) - (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt[b]*x)) - (3*Sqrt[b
]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*(-a)^(5/2)) + (3*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2)) + (d*CoshIntegral[d*x]*Sinh[c])/a^
2 + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[
b]])/(4*a^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-
a]*d)/Sqrt[b]])/(4*a^2) + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (3
*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(4*(-a)^(5/2)) + (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2))
```

Rubi [A] time = 1.25317, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```


```
[Out] -(Cosh[c + d*x]/(a^2*x)) + (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] - Sqrt[
b]*x)) - (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt[b]*x)) - (3*Sqrt[b
]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(4*(-a)^(5/2)) + (3*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2)) + (d*CoshIntegral[d*x]*Sinh[c])/a^
2 + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[
b]])/(4*a^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-
a]*d)/Sqrt[b]])/(4*a^2) + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a^2) + (3
*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(4*(-a)^(5/2)) + (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2))
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}dx = \int \left(\frac{\cosh(c+dx)}{a^2x^2} - \frac{b\cosh(c+dx)}{a(a+bx^2)^2} - \frac{b\cosh(c+dx)}{a^2(a+bx^2)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{(a+bx^2)^2}dx}{a}$$

$$= -\frac{\cosh(c+dx)}{a^2x} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{b}x)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{b}x)}\right)dx}{a^2} - \frac{b\int \left(-\frac{b\cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b\cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2}\right.}{a}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{2(-a)^{5/2}} + \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{2(-a)^{5/2}} + \frac{b^2\int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2}dx}{4a^2} + \frac{b^2\int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2}}{4a^2}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} + \frac{d\text{Chi}(dx)\sinh(c)}{a^2} + \frac{d\cosh(c)\text{Shi}(d}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} - \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right.}{2(-a)^{5/2}}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} - \frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right.}{2(-a)^{5/2}}$$

$$= -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{b}x)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{b}x)} - \frac{3\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right.}{4(-a)^{5/2}}$$

Mathematica [C] time = 1.03631, size = 675, normalized size = 1.35

$$\frac{ia^{3/2}dx\cosh\left(c-\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right) - ia^{3/2}dx\cosh\left(c+\frac{i\sqrt{a}d}{\sqrt{b}}\right)\text{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right) + 4a^{3/2}dx\sinh(c)\text{Chi}(dx) + 4a^{3/2}dx\cos}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

```
[Out] (-4*a^(3/2)*Cosh[c + d*x] - 6*Sqrt[a]*b*x^2*Cosh[c + d*x] + 4*a^(3/2)*d*x*C
oshIntegral[d*x]*Sinh[c] + 4*Sqrt[a]*b*d*x^3*CoshIntegral[d*x]*Sinh[c] + x*
(a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((-3*I)*Sqrt[b]*Cos
h[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) +
 x*(a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((3*I)*Sqrt[b]*Cosh
[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) +
4*a^(3/2)*d*x*Cosh[c]*SinhIntegral[d*x] + 4*Sqrt[a]*b*d*x^3*Cosh[c]*SinhInt
egral[d*x] + I*a^(3/2)*d*x*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[a]*b*d*x^3*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + 3*a*Sqrt[b]*x*Sinh[c - (I*Sqrt
[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + 3*b^(3/2)*x^3*Si
nh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*
a^(3/2)*d*x*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
 + I*d*x] - I*Sqrt[a]*b*d*x^3*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
Sqrt[a]*d)/Sqrt[b] + I*d*x] + 3*a*Sqrt[b]*x*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 3*b^(3/2)*x^3*Sinh[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(4*a^(5/2)*x*(a +
```

```
b*x^2))
```

Maple [A] time = 0.086, size = 595, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -3/4*exp(-d*x-c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b*d
^2*x^2+a*d^2)+1/8*d/a^2*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d
*x+c)*b-c*b)/b)+1/8*d/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)-3/8/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+3/8/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b+1/2*d/a^2*exp(-c)*Ei(1,
d*x)-3/4*exp(d*x+c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*exp(d*x+c)/a/x*d^2/(b
*d^2*x^2+a*d^2)-1/8*d/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)-1/8*d/a^2*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1
/2)+(d*x+c)*b-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/8/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1
/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/2*d/a^2*exp(c)*Ei(1
,-d*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33658, size = 2786, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(3*a*b*d*x^2 + 2*a^2*d)*cosh(d*x + c) - (((a*b*d^2*x^3 + a^2*d^2*x)
*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3
+ a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 -
(a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x +
 c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-
a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x)
- (a*b*d^2*x^3 + a^2*d^2*x)*Ei(-d*x))*cosh(c) - (((a*b*d^2*x^3 + a^2*d^2*x)
*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3
+ a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
```

```
))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 -
(a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x +
 c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x
 + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*
cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
 - sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*
x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (
b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x) + (a*b*d^
2*x^3 + a^2*d^2*x)*Ei(-d*x))*sinh(c) + (((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x
 + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*
cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
 + sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*
x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (
b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*sinh(-c + sqrt(-a*d^2/b)))/((a^3*b*d*x^3 + a^4*d*x)*cosh(d*x + c)^2 - (a^
3*b*d*x^3 + a^4*d*x)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

3.72 $\int \frac{x^3 \cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=476

$$\frac{d^2 \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{d^2 \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3}-\frac{3d \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}+\frac{3d\,s\ldots}{}$$

```
[Out] -(x^2*Cosh[c + d*x])/(4*b*(a + b*x^2)^2) - Cosh[c + d*x]/(4*b^2*(a + b*x^2)
) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(16*b^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(16*b^3) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) + (3*d*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*
b^(5/2)) - (d*x*Sinh[c + d*x])/(8*b^2*(a + b*x^2)) - (3*d*Cosh[c + (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2
)) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(16*b^3) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sq
rt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*b^3)
```

Rubi [A] time = 1.06151, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5291, 5289, 5280, 3303, 3298, 3301, 5290, 5293}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3}+\frac{d^2 \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16b^3}-\frac{3d \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}}+\frac{3d\,s\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```


```
[Out] -(x^2*Cosh[c + d*x])/(4*b*(a + b*x^2)^2) - Cosh[c + d*x]/(4*b^2*(a + b*x^2)
) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(16*b^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a
]*d)/Sqrt[b] + d*x])/(16*b^3) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*b^(5/2)) + (3*d*CoshIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*Sqrt[-a]*
b^(5/2)) - (d*x*Sinh[c + d*x])/(8*b^2*(a + b*x^2)) - (3*d*Cosh[c + (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2
)) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(16*b^3) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2)) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sq
rt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*b^3)
```


Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3 \cosh(c+dx)}{\left(a+bx^2\right)^3} dx = -\frac{x^2 \cosh(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}dx}{2b} + \frac{d\int \frac{x^2\sinh(c+dx)}{\left(a+bx^2\right)^2}dx}{4b}$$

$$= -\frac{x^2 \cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} - \frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}dx}{8b^2} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}dx}{4b^2} + \frac{d^2}{\cdots}$$

$$= -\frac{x^2 \cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} - \frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)} + \frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{8b^2} + \cdots$$

$$= -\frac{x^2 \cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} - \frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}dx}{16\sqrt{-a}b^2} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{b}x}dx}{16\sqrt{-a}b^2} - \frac{d\int\cdots}{\cdots}$$

$$= -\frac{x^2 \cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} - \frac{dx\sinh(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\left(d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{b}x}dx}{16\sqrt{-a}b^2} - \cdots$$

$$= -\frac{x^2 \cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh(c+dx)}{4b^2\left(a+bx^2\right)} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} + \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\cdots}{16b^3}$$

Mathematica [C] time = 1.82448, size = 648, normalized size = 1.36

$$-\frac{\frac{id^2\sinh(c)\left(\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\left(\text{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right)-\text{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)\right)}{b}+\frac{d^2\cosh(c)\left(\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\text{CosInte}\cdots\right.}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*Cosh[d*x]*(2*(a + 2*b*x^2)*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*
x^2)^2 - (2*(d*x*(a + b*x^2)*Cosh[c] + 2*(a + 2*b*x^2)*Sinh[c])*Sinh[d*x])/
(a + b*x^2)^2 + ((3*I)*d*Sinh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((S
qrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]
*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sq
rt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*Sqrt[
b]) - (I*d^2*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt
[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/
Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d
*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b + (3*d*Cosh[c]*(CosInte
gral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral
[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sq
rt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/
Sqrt[b] + I*d*x])))/(Sqrt[a]*Sqrt[b]) + (d^2*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[
b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*
CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinInt
egral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*
x])))/b)/(16*b^2)
```

Maple [B] time = 0.376, size = 820, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] -1/32*d^2/b^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+
c*b)/b)-1/32*d^2/b^3*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)+3/32*d/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-3/32*d/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4*d^4*exp(-d*x-c)/b/(b^2*d
^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/8*d^4*exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*
a*b*d^4*x^2+a^2*d^4)+1/16*d^5*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)*x^3+1/16*d^5*exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1
/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(d
*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/4*d^4*exp(d*x+c)/b/(b^2
*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/8*d^4*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2
*a*b*d^4*x^2+a^2*d^4)-1/32*d^2/b^3*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-
a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32*d^2/b^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32*d/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1
/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32*d/b^2/(-a*b)^(1/2)
*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4081, size = 3344, normalized size = 7.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(2*a*b^2*x^2 + a^2*b)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*
d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 -
 (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x -
sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c
)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3
*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*
b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqr
t(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^
2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh
(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/
```

```
b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a*b^2*d*x^3 +
 a^2*b*d*x)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*c
osh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^
2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2
*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (
(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x^4 + 2*a*b^2*x^2
+ a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*s
inh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(
-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^
3)*cosh(d*x + c)^2 - (a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

3.73 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=746

$$\frac{d^2 \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2 \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{d \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d \cdots}{\cdots}$$

```
[Out] -Cosh[c + d*x]/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*a*
b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Cosh[c + d*x])/(4*b*(a + b*x^2)^2) - (
Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(1
6*(-a)^(3/2)*b^(3/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) + (Cosh[c - (Sqrt[-a]*d)/S
qrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) -
 (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*Sqrt[-a]*b^(5/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
h[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*Sinh[c + d*x])/(
8*b^2*(a + b*x^2)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c + (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]
*b^(5/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a*b^2) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2))
```

Rubi [A] time = 1.10007, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5291, 5281, 3297, 3303, 3298, 3301, 5288}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2 \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{d \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```


```
[Out] -Cosh[c + d*x]/(16*a*b^(3/2)*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*a*
b^(3/2)*(Sqrt[-a] + Sqrt[b]*x)) - (x*Cosh[c + d*x])/(4*b*(a + b*x^2)^2) - (
Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(1
6*(-a)^(3/2)*b^(3/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]*b^(5/2)) + (Cosh[c - (Sqrt[-a]*d)/S
qrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) -
 (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(16*Sqrt[-a]*b^(5/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
h[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*a*b^2) - (d*Sinh[c + d*x])/(
8*b^2*(a + b*x^2)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-
a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c + (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*Sqrt[-a]
*b^(5/2)) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a*b^2) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
```

```
t[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*Sqrt[-a]*b^(5/2))
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{x^2 \cosh(c+dx)}{\left(a+bx^2\right)^3} dx = -\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^2} dx}{4b} + \frac{d\int \frac{x\sinh(c+dx)}{\left(a+bx^2\right)^2} dx}{4b}$$

$$= -\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} + \frac{\int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\cosh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right) dx}{4b} + \cdots$$

$$= -\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} - \frac{\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2} dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2} dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{8a} + \cdots$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} - \cdots$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{8b^2\left(a+bx^2\right)} + \frac{\int \frac{\cosh(\cdots}{\sqrt{-a}\cdots}}{16(-\cdots}$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\cdots\right.}{16\sqrt{-a}b^{5/2}}$$

$$= -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-}\cdots}{\sqrt{}\cdots}\right.}{16(-a)^{3/2}b^{3/2}}$$

Mathematica [C] time = 2.50882, size = 932, normalized size = 1.25

$$\frac{\frac{2\sqrt{a}b^2\cosh(c)\cosh(dx)x^3}{\left(bx^2+a\right)^2} + \frac{2\sqrt{a}b^2\sinh(c)\sinh(dx)x^3}{\left(bx^2+a\right)^2} - \frac{2a^{3/2}bd\cosh(dx)\sinh(c)x^2}{\left(bx^2+a\right)^2} - \frac{2a^{3/2}bd\cosh(c)\sinh(dx)x^2}{\left(bx^2+a\right)^2} - \frac{2a^{3/2}b\cosh(c)\cosh(dx)x}{\left(bx^2+a\right)^2} - \frac{2a^{3/}\cdots}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^(3/2)*b*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 + (2*Sqrt[a]*b^2*x^3*Cosh
[c]*Cosh[d*x])/(a + b*x^2)^2 - (2*a^(5/2)*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^
2 - (2*a^(3/2)*b*d*x^2*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 + (I*CosIntegral[-(
(Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((b + a*d^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]
+ I*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (I*CosInt
egral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((b + a*d^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt
[b]] - I*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[b] - (2*a
^(5/2)*d*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*d*x^2*Cosh[c]*Sinh
[d*x])/(a + b*x^2)^2 - (2*a^(3/2)*b*x*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 + (2
*Sqrt[a]*b^2*x^3*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 - I*Sqrt[a]*d*Cos[(Sqrt[a
]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[b]*
Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] +
 (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
- I*d*x])/Sqrt[b] - Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(S
qrt[a]*d)/Sqrt[b] - I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinInt
egral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[b] - Sqrt[a]*d*Sin[(Sqrt[a]*d)/Sqr
t[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[a]*d*Cos[(S
qrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - I*Sqr
```

```
t[b]*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d
*x] - (I*a*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqr
t[b] + I*d*x])/Sqrt[b] - Sqrt[b]*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegr
al[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (a*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*S
inIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[b] - Sqrt[a]*d*Sin[(Sqrt[a]*d
)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(16*a^(3/2)*b^
2)
```

Maple [A] time = 0.108, size = 1064, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] 1/32/b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x
+c)*b-c*b)/b)-1/32/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(
-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d/b^2/a*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/b^2/(-a*b)^(1/2)*exp((d*(-a*b
)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32*d^2/b^2/(-a*b)^
(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-
1/32*d/b^2/a*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b
)/b)+1/16*d^5*exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+1/16*d^
4*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*exp(-d*x-c
)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/16*d^4*exp(-d*x-c)/b/(b^2*d^4
*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/32/b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32/b/a/(-a*b)^(1/2)*exp(-(d*(
-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32*d^2/b^2/(-
a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/
b)+1/32*d^2/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(
1/2)+(d*x+c)*b-c*b)/b)+1/32*d/b^2/a*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a
*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d/b^2/a*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,
-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b
*d^4*x^2+a^2*d^4)*x^2+1/16*d^4*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)*x^3-1/16*d^5*exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/16
*d^4*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39734, size = 4169, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(a*b^2*d*x^3 - a^2*b*d*x)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*
b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 +
a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x +
 c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3
*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x
+ c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x
^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sq
rt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)
^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^
2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c
)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)
*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2
*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b
^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a^2*b*d^2*x^2
+ a^3*d^2)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*co
sh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2
 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*c
osh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 +
a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4
+ 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3
)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*
b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b^
2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2
*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x
^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2
*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(
d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b
^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^
2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b
^2*d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^4 + 2*a^3*b^3*d*x^2 + a^4*b^2*d)*sinh(
d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh (dx + c)}{\left(bx^2 + a\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

3.74 $\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=512

$$-\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}+\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-$$

```
[Out] -Cosh[c + d*x]/(4*b*(a + b*x^2)^2) - (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d^2*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (d*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(
-a)^(3/2)*b^(3/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (
Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d*Sinh[c + d*x])/(16*a*b^(
3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/(16*a*b^(3/2)*(Sqrt[-a] +
Sqrt[b]*x)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (d*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^
(3/2)) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a*b^2)
```

Rubi [A] time = 0.831389, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5289, 5280, 3297, 3303, 3298, 3301}

$$-\frac{d^2\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{16ab^2}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16ab^2}+\frac{d\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] -Cosh[c + d*x]/(4*b*(a + b*x^2)^2) - (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) - (d^2*Cosh[c - (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a*b^2) + (d*Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(
-a)^(3/2)*b^(3/2)) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (
Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(3/2)*b^(3/2)) - (d*Sinh[c + d*x])/(16*a*b^(
3/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/(16*a*b^(3/2)*(Sqrt[-a] +
Sqrt[b]*x)) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) + (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a*b^2) + (d*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^
(3/2)) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a*b^2)
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx = -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d\int \frac{\sinh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{4b}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} + \frac{d\int \left(-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b\sinh(c+dx)}{2a\left(-ab-b^2x^2\right)}\right)dx}{4b}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\sinh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\,dx}{16a} - \frac{d\int \frac{\sinh(c+dx)}{-ab-b^2x^2}\,dx}{8a}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{d\int \left(-\frac{\sqrt{-a}\sinh(c+dx)}{2ab\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{-a}\,\mathrm{si}}{2ab(\sqrt{}}\right.}{8a}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{16(-a)^{3/2}b} + \frac{d\int \frac{\sinh}{\sqrt{-a}}}{16(-}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$

$$= -\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2} - \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}$$

Mathematica [C] time = 1.71206, size = 637, normalized size = 1.24

$$\frac{\frac{id^2\sinh(c)\left(\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(-\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)-\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIntegral}\left(\frac{\sqrt{a}d}{\sqrt{b}}+idx\right)+\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\left(\mathrm{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right)-\mathrm{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}}-idx\right)\right)\right)}{b} - \frac{d^2\cosh(c)\left(\cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\mathrm{CosIn}\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

```
[Out] ((2*Cosh[d*x]*(-2*a*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (2*
(d*x*(a + b*x^2)*Cosh[c] - 2*a*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2 + (I*d*Sin
h[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]
- Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(
Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral
[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*Sqrt[b]) + (I*d^2*Sinh[c]*(CosInt
egral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegra
l[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/S
qrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x])))/b + (d*Cosh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) +
I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*
Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d
)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/(Sqrt[a]*S
qrt[b]) - (d^2*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/
Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b
] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*
d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])))/b)/(16*a*b)
```

Maple [A] time = 0.069, size = 743, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] -1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*ex
p(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*exp(-d*x-c)/b/(b^
2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*exp((d*(-a*b)^(1/2)-c*b)/b)
*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/32*d^2/b^2/a*exp(-(d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d/b/a/(-a*b)^(1/2)*ex
p((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/32*d/b/a
/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+
c*b)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*
d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*exp(d*x+c)/b
/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*exp((d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/b^2/a*exp(-(d*(-a*b)^(
1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32*d/b/a/(-a*b)^(1/2
)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d
/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c
)*b-c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35751, size = 3306, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*a^2*b*cosh(d*x + c) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2
)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x +
c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^
2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) +
((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x
^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + (((a*
b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 +
 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2
```

```
*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt
(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh
(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 +
 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^
2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x + c
) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*
d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*
x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*
d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (
b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x
^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)
*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 -
(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^
2/b)))/((a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*cosh(d*x + c)^2 - (a^2*b^4*
x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{(bx^2 + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

3.75 $\int \frac{\cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal. Leaf size=856

result too large to display

```
[Out] -Cosh[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cosh[c
 + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-a)^(
3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/(16*a^2*Sqrt[b]*
(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3
/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*CoshInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2*b
) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqr
t[b]])/(16*a^2*b) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*
x)) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) + (3*d*Cos
h[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a
^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sinh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*Cosh[c
 - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*
b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2))
```

Rubi [A] time = 1.25523, antiderivative size = 856, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5281, 3297, 3303, 3298, 3301}

$$\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] -Cosh[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cosh[c
 + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-a)^(
3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/(16*a^2*Sqrt[b]*
(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (Sqrt[-a
]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3
/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshI
ntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*CoshInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^2*b
) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqr
t[b]])/(16*a^2*b) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*
x)) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) + (3*d*Cos
h[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a
^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
```

```
 - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sinh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(3/2)) - (3*d*Cosh[c
 - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*
b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2))
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx = \int \left(-\frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3} - \frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}+bx\right)^3} - \frac{3b\cosh(c+}{16a^2\left(\sqrt{-a}\sqrt{b}\right.}\right.$$

$$= -\frac{(3b)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}dx}{16a^2} - \frac{(3b)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}+bx\right)^2}dx}{16a^2} - \frac{(3b)\int \frac{\cosh(c+dx)}{-ab-b^2x^2}dx}{8a^2} - \frac{b^{3/2}\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^3}dx}{8(-a)^{3/2}} - \frac{b^{3/2}}{}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\cosh(}{16a^2\sqrt{b}\left(\sqrt{\right.}}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\cosh(}{16a^2\sqrt{b}\left(\sqrt{\right.}}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\cosh(}{16a^2\sqrt{b}\left(\sqrt{\right.}}$$

$$= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2} - \frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)^2} + \frac{3\cosh(}{16a^2\sqrt{b}\left(\sqrt{\right.}}$$

Mathematica [C] time = 2.38417, size = 933, normalized size = 1.09

$$\frac{\frac{6b^{5/2}\cosh(c)\cosh(dx)x^3}{\left(bx^2+a\right)^2} + \frac{6b^{5/2}\sinh(c)\sinh(dx)x^3}{\left(bx^2+a\right)^2} + \frac{2ab^{3/2}d\cosh(dx)\sinh(c)x^2}{\left(bx^2+a\right)^2} + \frac{2ab^{3/2}d\cosh(c)\sinh(dx)x^2}{\left(bx^2+a\right)^2} + \frac{10ab^{3/2}\cosh(c)\cosh(dx)x}{\left(bx^2+a\right)^2} + \frac{10ab^{3/2}\text{s}}{(}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

```
[Out] ((10*a*b^(3/2)*x*Cosh[c]*Cosh[d*x])/(a + b*x^2)^2 + (6*b^(5/2)*x^3*Cosh[c]*
Cosh[d*x])/(a + b*x^2)^2 + (2*a^2*Sqrt[b]*d*Cosh[d*x]*Sinh[c])/(a + b*x^2)^
2 + (2*a*b^(3/2)*d*x^2*Cosh[d*x]*Sinh[c])/(a + b*x^2)^2 + (CosIntegral[-((S
qrt[a]*d)/Sqrt[b]) + I*d*x]*(I*(3*b - a*d^2)*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]
] - 3*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[a] + (I*CosI
ntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((-3*b + a*d^2)*Cosh[c + (I*Sqrt[a]*d)
/Sqrt[b]] + (3*I)*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]))/Sqrt[
a] + (2*a^2*Sqrt[b]*d*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 + (2*a*b^(3/2)*d*x^2
*Cosh[c]*Sinh[d*x])/(a + b*x^2)^2 + (10*a*b^(3/2)*x*Sinh[c]*Sinh[d*x])/(a +
 b*x^2)^2 + (6*b^(5/2)*x^3*Sinh[c]*Sinh[d*x])/(a + b*x^2)^2 - (3*I)*Sqrt[b]
*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + ((3*I)*b*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x])/Sqrt[a] - I*Sqrt[a]*d^2*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIn
tegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - (3*b*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]
*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x])/Sqrt[a] + Sqrt[a]*d^2*Cos[(Sqrt[
a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - 3*Sqrt[b]
*d*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x
] + (3*I)*Sqrt[b]*d*Cos[(Sqrt[a]*d)/Sqrt[b]]*Cosh[c]*SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x] - ((3*I)*b*Cosh[c]*Sin[(Sqrt[a]*d)/Sqrt[b]]*SinIntegral[
(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[a] + I*Sqrt[a]*d^2*Cosh[c]*Sin[(Sqrt[a]*
d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - (3*b*Cos[(Sqrt[a]*d)
/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/Sqrt[a] + Sqrt[
a]*d^2*Cos[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I
```

```
*d*x] - 3*Sqrt[b]*d*Sin[(Sqrt[a]*d)/Sqrt[b]]*Sinh[c]*SinIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x])/(16*a^2*b^(3/2))
```

Maple [A] time = 0.037, size = 1064, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a)^3,x)
```

```
[Out] -1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*ex
p(-d*x-c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(-d*x-c
)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*exp(-d*x-c)/a/(b^2*d^4*x^4
+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*
b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d^2/b/a/(-a*b)^(1/2)*exp
((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32*d/b/a^
2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-3/32
*d/b/a^2*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
-3/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(
d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*
(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4
*x^2+a^2*d^4)*x^2+3/16*d^4*exp(d*x+c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)*x^3+1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4
*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^(1/
2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*
d^2/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*
x+c)*b-c*b)/b)+3/32*d/b/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)+3/32*d/b/a^2*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*
b)^(1/2)+(d*x+c)*b-c*b)/b)-3/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*
b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.33339, size = 4342, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*cosh(d*x + c) - ((3*(a*b^2*d^2*x^4 +
2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2
```

```
*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a
^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2
 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqr
t(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2
 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)
*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b
*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 -
 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei
(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - ((3*(a*b^2*d^2*x^4 + 2*
a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x
^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2
*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 -
 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
 a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*s
inh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d
^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3
*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-
d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^2 + a^3*d
^2)*sinh(d*x + c) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*
x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 -
((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2
)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b
*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/
b)) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a
*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*
b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)
^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)
*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x
+ c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((
a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*
cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d
^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)
) + (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b
^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^
2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2
 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x
^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + s
qrt(-a*d^2/b)))/((a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*cosh(d*x + c)^
2 - (a^3*b^3*d*x^4 + 2*a^4*b^2*d*x^2 + a^5*b*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{\left(bx^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

3.76 $\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=730

$$\frac{d^2 \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2 \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}-\cosh($$

```
[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Co
sh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]
*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[
c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2
*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/S
qrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d*Sinh[c +
 d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x])/(16*a^2*
Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 + (5*d*Co
sh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c - (Sqrt[-a]*d)/S
qrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) -
 (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^2*b)
```

Rubi [A] time = 1.68183, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5293, 3303, 3298, 3301, 5289, 5280, 3297}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2 \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}-\cosh($$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```


```
[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Co
sh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]
*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[
c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2
*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/S
qrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d*Sinh[c +
 d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x])/(16*a^2*
Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 + (5*d*Co
sh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*
(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a
]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c - (Sqrt[-a]*d)/S
```

```
qrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) -
 (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] + d*x])/(16*a^2*b)
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x} - \frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)^3} - \frac{bx\cosh(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{bx\cosh(c+dx)}{a^3\left(a+bx^2\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^3} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} - \frac{b\int\left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^3} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^2}\,dx}{2a^2} - \frac{d\int \cdots}{\cdots}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{\sqrt{b}\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2a^3} - \frac{\sqrt{b}\int \cdots}{\cdots}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d\int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4(-a)^{5/2}} + \frac{d\int \frac{\sinh(\cdots}{\sqrt{-a}+\cdots}}{4(-a\cdots}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\cosh\left(c-\cdots\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} - \frac{\cosh\left(c-\cdots\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cosh\left(\cdots\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2} + \frac{\cosh(c+dx)}{2a^2\left(a+bx^2\right)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3} + \frac{d^2\cosh\left(\cdots\right.}{\cdots}$$

Mathematica [C] time = 3.262, size = 1558, normalized size = 2.13

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

```
[Out] (12*a^2*b*Cosh[c + d*x] + 8*a*b^2*x^2*Cosh[c + d*x] + 16*b*(a + b*x^2)^2*Co
sh[c]*CoshIntegral[d*x] - 8*a^2*b*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshInteg
ral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a^3*d^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*
CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 16*a*b^2*x^2*Cosh[c - (I*Sqrt[a
]*d)/Sqrt[b]]*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*a^2*b*d^2*x^2*C
osh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
8*b^3*x^4*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshIntegral[d*((I*Sqrt[a])/Sqrt[
b] + x)] + a*b^2*d^2*x^4*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] - (5*I)*a^(5/2)*Sqrt[b]*d*CoshIntegral[d*((I*Sqrt[a
])/Sqrt[b] + x)]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]] - (10*I)*a^(3/2)*b^(3/2)*d
*x^2*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[
b]] - (5*I)*Sqrt[a]*b^(5/2)*d*x^4*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]] + (a + b*x^2)^2*CoshIntegral[d*(((-I)*Sqrt
[a])/Sqrt[b] + x)]*((-8*b + a*d^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + (5*I)*
```

```
Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 2*a^2*b*d*x*Sinh[c + d
*x] - 2*a*b^2*d*x^3*Sinh[c + d*x] + 16*a^2*b*Sinh[c]*SinhIntegral[d*x] + 32
*a*b^2*x^2*Sinh[c]*SinhIntegral[d*x] + 16*b^3*x^4*Sinh[c]*SinhIntegral[d*x]
 - (5*I)*a^(5/2)*Sqrt[b]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*(
(I*Sqrt[a])/Sqrt[b] + x)] - (10*I)*a^(3/2)*b^(3/2)*d*x^2*Cosh[c - (I*Sqrt[a
]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (5*I)*Sqrt[a]*b^(
5/2)*d*x^4*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt
[b] + x)] - 8*a^2*b*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt
[a])/Sqrt[b] + x)] + a^3*d^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d
*((I*Sqrt[a])/Sqrt[b] + x)] - 16*a*b^2*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*
SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*a^2*b*d^2*x^2*Sinh[c - (I*Sqr
t[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 8*b^3*x^4*Sinh
[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a*b
^2*d^2*x^4*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt
[b] + x)] - (5*I)*a^(5/2)*Sqrt[b]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhInt
egral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (10*I)*a^(3/2)*b^(3/2)*d*x^2*Cosh[c +
(I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (5*I)*Sq
rt[a]*b^(5/2)*d*x^4*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x] + 8*a^2*b*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(
I*Sqrt[a]*d)/Sqrt[b] - d*x] - a^3*d^2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhI
ntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 16*a*b^2*x^2*Sinh[c + (I*Sqrt[a]*d)/
Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - 2*a^2*b*d^2*x^2*Sinh[c
 + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 8*b^3
*x^4*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d
*x] - a*b^2*d^2*x^4*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x])/(16*a^3*b*(a + b*x^2)^2)
```

Maple [A] time = 0.105, size = 1090, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^2+a)^3,x)
```

```
[Out] 1/16*exp(-d*x-c)*d^2*((d*x+c)^3*b-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*b
*c^2-a*c*d^2-b*c^3+4*(d*x+c)^2*b-8*(d*x+c)*b*c+6*a*d^2+4*b*c^2)/a^2/((d*x+c
)^4*b^2-4*(d*x+c)^3*b^2*c+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*(d*x+c)
*a*b*c*d^2-4*(d*x+c)*b^2*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/32/b/a^2*exp(
-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*d^2-1/32/b
/a^2*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d^2
-5/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(
d*x+c)*b+c*b)/b)*d+5/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d+1/4/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/4/a^3*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei
(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/2/a^3*exp(-c)*Ei(1,d*x)-1/16*exp(d*x
+c)*d^2*((d*x+c)^3*b-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*b*c^2-a*c*d^2-
b*c^3-4*(d*x+c)^2*b+8*(d*x+c)*b*c-6*a*d^2-4*b*c^2)/a^2/((d*x+c)^4*b^2-4*(d*
x+c)^3*b^2*c+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*(d*x+c)*a*b*c*d^2-4*
(d*x+c)*b^2*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/32/b/a^2*exp((d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*d^2-1/32/b/a^2*exp(-(d*(-
a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*d^2+5/32/a^2/(-a
*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b
)*d-5/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2
)+(d*x+c)*b-c*b)/b)*d+1/4/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1
/2)-(d*x+c)*b+c*b)/b)+1/4/a^3*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^
(1/2)+(d*x+c)*b-c*b)/b)-1/2/a^3*exp(c)*Ei(1,-d*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15226, size = 4301, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(2*a*b^2*x^2 + 3*a^2*b)*cosh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 - 8
*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d
^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(
d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4
+ 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^
2/b)) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*
b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + 16*((b
^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) + (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(
-d*x))*cosh(c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*
d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 -
8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a^3*d^2 + (a*b^2*d^2
- 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^
3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*si
nh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x
 + c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a
*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*
x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (
a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x +
c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) -
(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(-d*x))*sinh(c) - (((a^3*d^2 + (a*b^2*d^2
 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a
^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*s
inh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-
a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 -
```

```
8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*
b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^
2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^
2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((
a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*cosh(d*x + c)^2 - (a^3*b^3*x^4 + 2*a^4
*b^2*x^2 + a^5*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

3.77 $\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=874

result too large to display

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a]
 - Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)
) + (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*S
qrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^
(7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b
])
```

Rubi [A] time = 2.68917, antiderivative size = 874, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```


```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a]
 - Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)
) + (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*S
qrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^
(7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
```

```
[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b
])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \int \left(\frac{\cosh(c+dx)}{a^3x^2} - \frac{b\cosh(c+dx)}{a\left(a+bx^2\right)^3} - \frac{b\cosh(c+dx)}{a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)}\right) dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^2}\,dx}{a^3} - \frac{b\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx}{a^2} - \frac{b\int \frac{\cosh(c+dx)}{\left(a+bx^2\right)^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right) dx}{a^3} - \frac{b\int \left(-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2} - \frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2} - \frac{b}{2a}\cdots\right.}{a^2}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{b}x}\,dx}{2(-a)^{7/2}} - \frac{b\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{b}x}\,dx}{2(-a)^{7/2}} + \frac{\left(3b^2\right)\int \frac{\cosh(c+dx)}{\left(\sqrt{-a}\sqrt{b}-bx\right)^2}\,dx}{16a^3} + \frac{\left(3b^2\right)\int \frac{\cosh(c+d\cdots}{\left(\sqrt{-a}\sqrt{b}+b\cdots\right.}}{16a^3}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{\cdots}}{16a^3\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{\cdots}}{16a^3\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{\cdots}}{16a^3\cdots}$$

$$= -\frac{\cosh(c+dx)}{a^3x} - \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2} + \frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}+\sqrt{b}x\right)^2} - \frac{7\sqrt{\cdots}}{16a^3\cdots}$$

Mathematica [C] time = 3.37809, size = 1359, normalized size = 1.55

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

```
[Out] (-16*a^(5/2)*Sqrt[b]*Cosh[c + d*x] - 50*a^(3/2)*b^(3/2)*x^2*Cosh[c + d*x] -
 30*Sqrt[a]*b^(5/2)*x^4*Cosh[c + d*x] + 16*a^(5/2)*Sqrt[b]*d*x*CoshIntegral
[d*x]*Sinh[c] + 32*a^(3/2)*b^(3/2)*d*x^3*CoshIntegral[d*x]*Sinh[c] + 16*Sqr
t[a]*b^(5/2)*d*x^5*CoshIntegral[d*x]*Sinh[c] + x*(a + b*x^2)^2*CoshIntegral
[d*((I*Sqrt[a])/Sqrt[b] + x)]*((-I)*(15*b - a*d^2)*Cosh[c - (I*Sqrt[a]*d)/S
qrt[b]] + 7*Sqrt[a]*Sqrt[b]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) + x*(a + b*x
^2)^2*CoshIntegral[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]*(I*(15*b - a*d^2)*Cosh[c
 + (I*Sqrt[a]*d)/Sqrt[b]] + 7*Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt
[b]]) - 2*a^(5/2)*Sqrt[b]*d*x*Sinh[c + d*x] - 2*a^(3/2)*b^(3/2)*d*x^3*Sinh[
c + d*x] + 16*a^(5/2)*Sqrt[b]*d*x*Cosh[c]*SinhIntegral[d*x] + 32*a^(3/2)*b^
(3/2)*d*x^3*Cosh[c]*SinhIntegral[d*x] + 16*Sqrt[a]*b^(5/2)*d*x^5*Cosh[c]*Si
nhIntegral[d*x] + 7*a^(5/2)*Sqrt[b]*d*x*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*Sin
hIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 14*a^(3/2)*b^(3/2)*d*x^3*Cosh[c -
(I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 7*Sqrt[a
]*b^(5/2)*d*x^5*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])
/Sqrt[b] + x)] - (15*I)*a^2*b*x*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegra
l[d*((I*Sqrt[a])/Sqrt[b] + x)] + I*a^3*d^2*x*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (30*I)*a*b^2*x^3*Sinh[c - (I*
Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + (2*I)*a^2*b
```

```
*d^2*x^3*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b
] + x)] - (15*I)*b^3*x^5*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] + I*a*b^2*d^2*x^5*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*S
inhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 7*a^(5/2)*Sqrt[b]*d*x*Cosh[c + (
I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - 14*a^(3/2
)*b^(3/2)*d*x^3*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/
Sqrt[b] - d*x] - 7*Sqrt[a]*b^(5/2)*d*x^5*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*Si
nhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (15*I)*a^2*b*x*Sinh[c + (I*Sqrt[a
]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*a^3*d^2*x*Sinh[
c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (30*
I)*a*b^2*x^3*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqr
t[b] - d*x] + (2*I)*a^2*b*d^2*x^3*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhInteg
ral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (15*I)*b^3*x^5*Sinh[c + (I*Sqrt[a]*d)/Sq
rt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + I*a*b^2*d^2*x^5*Sinh[c +
 (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x])/(16*a^(7
/2)*Sqrt[b]*x*(a + b*x^2)^2)
```

Maple [A] time = 0.128, size = 1178, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x)
```

```
[Out] 1/16*exp(-d*x-c)/a^2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*ex
p(-d*x-c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+1/16*exp(-d*x
-c)/a*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(-d*x-c)/a^2*x*d^4/(
b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/2*exp(-d*x-c)/a/x*d^4/(b^2*d^4*x^4+2
*a*b*d^4*x^2+a^2*d^4)+1/32/a^2*d^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)
*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32/a^2*d^2/(-a*b)^(1/2)*exp(-(d*(
-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+7/32*d/a^3*exp(
(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+7/32*d/a^3*e
xp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-15/32/a
^3/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-
c*b)/b)*b+15/32/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*
b)^(1/2)-(d*x+c)*b+c*b)/b)*b+1/2*d/a^3*exp(-c)*Ei(1,d*x)-1/16*exp(d*x+c)/a^
2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*exp(d*x+c)/a^3*x^3*d^
4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/16*exp(d*x+c)/a*d^5/(b^2*d^4*x^
4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(d*x+c)/a^2*x*d^4/(b^2*d^4*x^4+2*a*b*d^4*
x^2+a^2*d^4)*b-1/2*exp(d*x+c)/a/x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1
/32/a^2*d^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(
d*x+c)*b+c*b)/b)+1/32/a^2*d^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-7/32*d/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-7/32*d/a^3*exp(-(d*(-a*b)^(1/2)-c*b
)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+15/32/a^3/(-a*b)^(1/2)*exp((d*
(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-15/32/a^3/(-a
*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)
/b)*b-1/2*d/a^3*exp(c)*Ei(1,-d*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17239, size = 4894, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*cosh(d*x + c) - ((7*(a
*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d^2*
x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*d^2 - 15*b^3)
*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x + c)
^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (7
*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d
^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2 - 15*b
^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x +
 c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))
*cosh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*
x)*Ei(d*x) - (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(-d*x))*cosh(c
) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(
a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^2
 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cos
h(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (
a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^
2/b)) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 -
7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*
d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*
cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3
+ (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d
*x + c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2
 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b
^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*
x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x
^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt
(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c
)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((
a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*
b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2
)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x +
sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^
2*x^3 + a^3*d^2*x)*Ei(d*x) + (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*
Ei(-d*x))*sinh(c) + ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(
d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^
2 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 1
5*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(
d*x + sqrt(-a*d^2/b)) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*co
sh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x +
c)^2 - (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2
 - 15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2
```

```
- 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*
Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*d*x^5 + 2*a
^5*b*d*x^3 + a^6*d*x)*cosh(d*x + c)^2 - (a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^
6*d*x)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{\left(bx^2+a\right)^3 x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

3.78 $\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=791

result too large to display

```
[Out] -Cosh[c + d*x]/(2*a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b*C
osh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (d^
2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3*b*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4
) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
h[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) -
 (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a]
- Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) -
 (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a
^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sinh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh[c + (Sqrt[-a]*d)/
Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (9*Sqrt[b]*d*
Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(1
6*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rubi [A] time = 1.86841, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5293, 3297, 3303, 3298, 3301, 5289, 5280}

$$-\frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^3}-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^3}-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```


```
[Out] -Cosh[c + d*x]/(2*a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b*C
osh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (d^
2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3*b*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4
) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sin
h[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) -
 (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a]
- Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) -
 (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a
^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sinh
```

```
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh[c + (Sqrt[-a]*d)/
Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (9*Sqrt[b]*d*
Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(1
6*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}dx = \int\left(\frac{\cosh(c+dx)}{a^3x^3} - \frac{3b\cosh(c+dx)}{a^4x} + \frac{b^2x\cosh(c+dx)}{a^2\left(a+bx^2\right)^3} + \frac{2b^2x\cosh(c+dx)}{a^3\left(a+bx^2\right)^2} + \frac{3b^2x\cosh(c+dx)}{a^4\left(a+bx^2\right)}\right.$$

$$= \frac{\int\frac{\cosh(c+dx)}{x^3}dx}{a^3} - \frac{(3b)\int\frac{\cosh(c+dx)}{x}dx}{a^4} + \frac{\left(3b^2\right)\int\frac{x\cosh(c+dx)}{a+bx^2}dx}{a^4} + \frac{\left(2b^2\right)\int\frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}dx}{a^3} + \frac{b^2\int\frac{x\cos}{(a+}}{a}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} + \frac{\left(3b^2\right)\int\left(-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} + \frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx}{a^4} +$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{3b\sin}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} - \frac{d\sinh(c+dx)}{2a^3x} - \frac{3b\sin}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{3}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{3}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{3}{}$$

$$= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b\cosh(c+dx)}{4a^2\left(a+bx^2\right)^2} - \frac{b\cosh(c+dx)}{a^3\left(a+bx^2\right)} - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3} + \frac{3}{}$$

Mathematica [C] time = 3.71688, size = 998, normalized size = 1.26

$$-\frac{-ia\sinh(c)\left(\text{CosIntegral}\left(idx-\frac{\sqrt{a}d}{\sqrt{b}}\right)\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) - \text{CosIntegral}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right)\sin\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + \cos\left(\frac{\sqrt{a}d}{\sqrt{b}}\right)\left(\text{Si}\left(ixd+\frac{\sqrt{a}d}{\sqrt{b}}\right) - \text{Si}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -((2*a*Cosh[d*x]*(2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Cosh[c] + d*x*(4*a^2 +
7*a*b*x^2 + 3*b^2*x^4)*Sinh[c]))/(x^2*(a + b*x^2)^2) + (2*a*(d*x*(4*a^2 + 7
*a*b*x^2 + 3*b^2*x^4)*Cosh[c] + 2*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)*Sinh[c])*
Sinh[d*x])/(x^2*(a + b*x^2)^2) + 8*(6*b - a*d^2)*(Cosh[c]*CoshIntegral[d*x]
 + Sinh[c]*SinhIntegral[d*x]) - (9*I)*Sqrt[a]*Sqrt[b]*d*Sinh[c]*(Cos[(Sqrt[
a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - Cos[(Sqrt[a]*d
)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[
b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - SinIntegral[(Sqrt[a]*d)/Sqr
t[b] + I*d*x])) + (24*I)*b*Sinh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*
d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Si
n[(Sqrt[a]*d)/Sqrt[b]] + Cos[(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)
/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - I*a*d^2*Si
```

```
nh[c]*(CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]]
 - CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + Cos[
(Sqrt[a]*d)/Sqrt[b]]*(-SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegr
al[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) - 9*Sqrt[a]*Sqrt[b]*d*Cosh[c]*(CosIntegra
l[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] + CosIntegral[(S
qrt[a]*d)/Sqrt[b] + I*d*x]*Sin[(Sqrt[a]*d)/Sqrt[b]] - Cos[(Sqrt[a]*d)/Sqrt[
b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqr
t[b] + I*d*x])) - 24*b*Cosh[c]*(Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqr
t[a]*d)/Sqrt[b]) + I*d*x] + Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d
)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt
[b] - I*d*x] + SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])) + a*d^2*Cosh[c]*(
Cos[(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + Cos[
(Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + Sin[(Sqrt[a
]*d)/Sqrt[b]]*(SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + SinIntegral[(Sqrt
[a]*d)/Sqrt[b] + I*d*x])))/(16*a^4)
```

Maple [B] time = 0.152, size = 1294, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x^2+a)^3,x)
```

```
[Out] 3/16*d^5*exp(-d*x-c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+7/16*d
^5*exp(-d*x-c)/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-3/4*d^4*exp(-d*x
-c)/a^3*x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+9/32*d/a^3/(-a*b)^(1/2)
*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-9/3
2*d/a^3/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+
c)*b-c*b)/b)*b+1/32*d^2/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*
b)^(1/2)+(d*x+c)*b-c*b)/b)-3/4/a^4*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*
b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/4/a^4*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/4*d^2/a^3*exp(-c)*Ei(1,d*x)-3/16*d^5*exp
(d*x+c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-7/16*d^5*exp(d*x+c)
/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-3/4*d^4*exp(d*x+c)/a^3*x^2/(b^
2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-9/32*d/a^3/(-a*b)^(1/2)*exp((d*(-a*b)^
(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b+3/2/a^4*b*exp(c)*Ei(
1,-d*x)+9/32*d/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b
)^(1/2)+(d*x+c)*b-c*b)/b)*b+1/32*d^2/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp((d*(-a*b)^(1/2)-c*b)/b)
*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-3/4/a^4*exp(-(d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/4/a^4*exp((d*(-a*b)^(1/2)-c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+1/4*d^5*exp(-d*x-c)/a/x/(b^2*
d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d
^4*x^2+a^2*d^4)*b-1/4*d^4*exp(-d*x-c)/a/x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*
d^4)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)-1/4*d^5*exp(
d*x+c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(d*x+c)/a^2/(b^2*
d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/4*d^4*exp(d*x+c)/a/x^2/(b^2*d^4*x^4+2*a*
b*d^4*x^2+a^2*d^4)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20699, size = 4891, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*cosh(d*x + c) + ((((a*b^2*d^2
- 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*co
sh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
(a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2
*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)
^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 +
 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 -
 ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a
^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*
x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2
/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 8*(((a*b^2*d^2 -
 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x
) + ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a
^2*b)*x^2)*Ei(-d*x))*cosh(c) + ((((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 -
24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh
(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3
*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + s
qrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
+ (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2
*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9
*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*cosh(-c + sqrt(-a*d^2/b)) + 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)
*sinh(d*x + c) + ((((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 +
2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 +
9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
 - (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 2
4*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^
2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqr
t(-a*d^2/b)) - 8*(((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 +
(a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x) - ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2
- 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(-d*x))*sinh(c) - ((((a*b^2*d^2
 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*c
osh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
 (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^
2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (((a*b^2*d^2 - 24*b^3)*x^6
+ 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2
- ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*
a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d
```

```
*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a^4*b^2*x^6 +
 2*a^5*b*x^4 + a^6*x^2)*cosh(d*x + c)^2 - (a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^3 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

3.79 $\int x^3 \left(a+bx^3\right) \cosh(c+dx) \, dx$

Optimal. Leaf size=154

$$-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{ax^3\sinh(c+dx)}{d}+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{360bx^2\sinh(c+}{d^5}$$

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c +
d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 + (
720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c +
d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b*x^6
*Sinh[c + d*x])/d
```

Rubi [A] time = 0.297123, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5287, 3296, 2638, 2637}

$$-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{6a\cosh(c+dx)}{d^4}+\frac{ax^3\sinh(c+dx)}{d}+\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{360bx^2\sinh(c+}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c +
d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 + (
720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c +
d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b*x^6
*Sinh[c + d*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3 \left(a + bx^3\right) \cosh(c + dx) \, dx = \int \left(ax^3 \cosh(c + dx) + bx^6 \cosh(c + dx)\right) dx$$
$$= a \int x^3 \cosh(c + dx) \, dx + b \int x^6 \cosh(c + dx) \, dx$$
$$= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^6 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) \, dx}{d} - \frac{(6b) \int x^5 \sinh(c}{d}$$
$$= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^6 \sinh(c + dx)}{d} +$$
$$= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{ax^3 \sinh(c + dx)}{d} +$$
$$= -\frac{6a \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} - \frac{6bx^5 \cosh(c + dx)}{d^2}$$
$$= -\frac{6a \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} - \frac{6bx^5 \cosh(c + dx)}{d^2}$$
$$= -\frac{6a \cosh(c + dx)}{d^4} - \frac{720bx \cosh(c + dx)}{d^6} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx}{d^4}$$
$$= -\frac{6a \cosh(c + dx)}{d^4} - \frac{720bx \cosh(c + dx)}{d^6} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx}{d^4}$$

Mathematica [A] time = 0.160777, size = 100, normalized size = 0.65

$$\frac{\left(ad^4x \left(d^2x^2 + 6\right) + b \left(d^6x^6 + 30d^4x^4 + 360d^2x^2 + 720\right)\right) \sinh(c + dx) - 3d \left(ad^2 \left(d^2x^2 + 2\right) + 2bx \left(d^4x^4 + 20d^2x^2 + 12\right.\right.}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-3*d*(a*d^2*(2 + d^2*x^2) + 2*b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a*d^4*x*(6 + d^2*x^2) + b*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6)
)*Sinh[c + d*x])/d^7
```

Maple [B] time = 0.008, size = 551, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^4*(1/d^3*b*((d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*
sinh(d*x+c)-120*(d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)
*cosh(d*x+c)+720*sinh(d*x+c))-6/d^3*b*c*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*
cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*s
inh(d*x+c)-120*cosh(d*x+c))+15/d^3*b*c^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3
*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c)
)-20/d^3*b*c^3*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sin
h(d*x+c)-6*cosh(d*x+c))+15/d^3*b*c^4*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(
d*x+c)+2*sinh(d*x+c))-6/d^3*b*c^5*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^3*b
*c^6*sinh(d*x+c)+a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)
*sinh(d*x+c)-6*cosh(d*x+c))-3*a*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x
+c)+2*sinh(d*x+c))+3*a*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-a*c^3*sinh(d*x
```

```
+c))
```

Maxima [A] time = 1.04636, size = 362, normalized size = 2.35

$$-\frac{1}{56}\,d\left(\frac{7\left(d^4x^4e^c - 4\,d^3x^3e^c + 12\,d^2x^2e^c - 24\,dxe^c + 24\,e^c\right)ae^{(dx)}}{d^5} + \frac{7\left(d^4x^4 + 4\,d^3x^3 + 12\,d^2x^2 + 24\,dx + 24\right)ae^{(-dx-c)}}{d^5} + \frac{4\,(\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/56*d*(7*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*
e^c)*a*e^(d*x)/d^5 + 7*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e
^(-d*x - c)/d^5 + 4*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 42*d^5*x^5*e^c - 210*d^4
*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040*d*x*e^c - 5040*e^c)*b*
e^(d*x)/d^8 + 4*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5 + 210*d^4*x^4 + 840*d^3*x
^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b*e^(-d*x - c)/d^8) + 1/28*(4*b*x^7 +
7*a*x^4)*cosh(d*x + c)
```

Fricas [A] time = 1.76403, size = 240, normalized size = 1.56

$$-\frac{3\left(2\,bd^5x^5 + ad^5x^2 + 40\,bd^3x^3 + 2\,ad^3 + 240\,bdx\right)\cosh\left(dx + c\right) - \left(bd^6x^6 + ad^6x^3 + 30\,bd^4x^4 + 6\,ad^4x + 360\,bd^2x^2 + 72\cdots\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(3*(2*b*d^5*x^5 + a*d^5*x^2 + 40*b*d^3*x^3 + 2*a*d^3 + 240*b*d*x)*cosh(d*x
 + c) - (b*d^6*x^6 + a*d^6*x^3 + 30*b*d^4*x^4 + 6*a*d^4*x + 360*b*d^2*x^2 +
 720*b)*sinh(d*x + c))/d^7
```

Sympy [A] time = 9.67527, size = 185, normalized size = 1.2

$$\begin{cases} \frac{ax^3\sinh(c+dx)}{d} - \frac{3ax^2\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} - \frac{6a\cosh(c+dx)}{d^4} + \frac{bx^6\sinh(c+dx)}{d} - \frac{6bx^5\cosh(c+dx)}{d^2} + \frac{30bx^4\sinh(c+dx)}{d^3} - \frac{120bx^3\cosh(\cdots}{d^4} \cdots \\ \left(\frac{ax^4}{4} + \frac{bx^7}{7}\right)\cosh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sin
h(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**6*sinh(c + d*x)/d - 6*b*x**
5*cosh(c + d*x)/d**2 + 30*b*x**4*sinh(c + d*x)/d**3 - 120*b*x**3*cosh(c + d
*x)/d**4 + 360*b*x**2*sinh(c + d*x)/d**5 - 720*b*x*cosh(c + d*x)/d**6 + 720
*b*sinh(c + d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*cosh(c), True))
```

Giac [A] time = 1.14007, size = 259, normalized size = 1.68

$$\frac{\left(bd^6x^6 - 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 - 3\,ad^5x^2 - 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 - 6\,ad^3 - 720\,bdx + 720\,b\right)e^{(dx+c)}}{2\,d^7} - \frac{(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^6*x^6 - 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 - 3*a*d^5*x^2 - 120
*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 - 6*a*d^3 - 720*b*d*x + 720*b)*e^(d*
x + c)/d^7 - 1/2*(b*d^6*x^6 + 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 + 3*a*
d^5*x^2 + 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 + 6*a*d^3 + 720*b*d*x +
 720*b)*e^(-d*x - c)/d^7
```

3.80 $\int x^2 \left(a+bx^3\right) \cosh(c+dx)\, dx$

Optimal. Leaf size=124

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}-\frac{60bx^2\cosh(c+dx)}{d^4}$$

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c +
 d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120*b
*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x])/
d^3 + (b*x^5*Sinh[c + d*x])/d
```

Rubi [A] time = 0.228484, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5287, 3296, 2637, 2638}

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}-\frac{60bx^2\cosh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c +
 d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120*b
*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x])/
d^3 + (b*x^5*Sinh[c + d*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int x^2 \left(a + bx^3\right) \cosh(c + dx)\, dx = \int \left(ax^2 \cosh(c + dx) + bx^5 \cosh(c + dx)\right) dx$$
$$= a \int x^2 \cosh(c + dx)\, dx + b \int x^5 \cosh(c + dx)\, dx$$
$$= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx)\, dx}{d} - \frac{(5b) \int x^4 \sinh(c +}{d}$$
$$= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} + $$
$$= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{ax^2 \sinh(c + dx)}{d} + $$
$$= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + $$
$$= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + $$
$$= -\frac{120b \cosh(c + dx)}{d^6} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2}$$

Mathematica [A] time = 0.133114, size = 84, normalized size = 0.68

$$\frac{d\left(ad^2\left(d^2x^2+2\right)+bx\left(d^4x^4+20d^2x^2+120\right)\right)\sinh(c+dx)-\left(2ad^4x+5b\left(d^4x^4+12d^2x^2+24\right)\right)\cosh(c+dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-((2*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*(a*d^2*
(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Maple [B] time = 0.007, size = 389, normalized size = 3.1

$$\frac{1}{d^3}\left(\frac{b\left((dx+c)^5 \sinh(dx+c) - 5\,(dx+c)^4 \cosh(dx+c) + 20\,(dx+c)^3 \sinh(dx+c) - 60\,(dx+c)^2 \cosh(dx+c) + 1\right.}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^3*(1/d^3*b*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*
sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c
))-5/d^3*b*c*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*si
nh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+10/d^3*b*c^2*((d*x+c)^3*si
nh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-10/d
^3*b*c^3*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))+5/d^3*
b*c^4*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-1/d^3*b*c^5*sinh(d*x+c)+a*((d*x+c)^
2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*a*c*((d*x+c)*sinh(d*x+
c)-cosh(d*x+c))+a*c^2*sinh(d*x+c))
```

Maxima [B] time = 1.04248, size = 360, normalized size = 2.9

$$\frac{\left(bx^3+a\right)^2 \cosh(dx+c)}{6\,b} - \frac{\left(\frac{a^2 e^{(dx+c)}}{d} + \frac{a^2 e^{(-dx-c)}}{d} + \frac{2\left(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c\right)abe^{(dx)}}{d^4} + \frac{2\left(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6\right)abe^{(-dx-c)}}{d^4} + \frac{\left(d^6x^6e^c -\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^3 + a)^2*cosh(d*x + c)/b - 1/12*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
 c)/d + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4
 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d*x - c)/d^4 + (d^6*x^6*e^c
- 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*
d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 12
0*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b^2*e^(-d*x - c)/d^7)*d/b
```

Fricas [A] time = 1.73341, size = 200, normalized size = 1.61

$$-\frac{\left(5\,bd^4x^4+2\,ad^4x+60\,bd^2x^2+120\,b\right)\cosh\left(dx+c\right)-\left(bd^5x^5+ad^5x^2+20\,bd^3x^3+2\,ad^3+120\,bdx\right)\sinh\left(dx+c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((5*b*d^4*x^4 + 2*a*d^4*x + 60*b*d^2*x^2 + 120*b)*cosh(d*x + c) - (b*d^5*x
^5 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^3 + 120*b*d*x)*sinh(d*x + c))/d^6
```

Sympy [A] time = 6.04663, size = 151, normalized size = 1.22

$$\begin{cases} \frac{ax^2\sinh(c+dx)}{d}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{bx^5\sinh(c+dx)}{d}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{60bx^2\cosh(c+dx)}{d^4}+\frac{120bx\sinh(c}{d^5} \\ \left(\frac{ax^3}{3}+\frac{bx^6}{6}\right)\cosh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c +
 d*x)/d**3 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*x*
*3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c + d*x
)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*cosh(c
), True))
```

Giac [A] time = 1.18935, size = 211, normalized size = 1.7

$$\frac{\left(bd^5x^5-5\,bd^4x^4+ad^5x^2+20\,bd^3x^3-2\,ad^4x-60\,bd^2x^2+2\,ad^3+120\,bdx-120\,b\right)e^{(dx+c)}}{2\,d^6}-\frac{\left(bd^5x^5+5\,bd^4x^4+ad^5x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^5*x^5 - 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 - 2*a*d^4*x - 60*b*
d^2*x^2 + 2*a*d^3 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*(b*d^5*x^5 + 5
*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^4*x + 60*b*d^2*x^2 + 2*a*d^3
+ 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

3.81 $\int x\left(a+bx^3\right)\cosh(c+dx)\,dx$

Optimal. Leaf size=94

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+d}{d^4}$$

```
[Out] -((a*Cosh[c + d*x])/d^2) - (24*b*x*Cosh[c + d*x])/d^4 - (4*b*x^3*Cosh[c + d
*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (a*x*Sinh[c + d*x])/d + (12*b*x^2*Sin
h[c + d*x])/d^3 + (b*x^4*Sinh[c + d*x])/d
```

Rubi [A] time = 0.159169, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5287, 3296, 2638, 2637}

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+d}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] -((a*Cosh[c + d*x])/d^2) - (24*b*x*Cosh[c + d*x])/d^4 - (4*b*x^3*Cosh[c + d
*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (a*x*Sinh[c + d*x])/d + (12*b*x^2*Sin
h[c + d*x])/d^3 + (b*x^4*Sinh[c + d*x])/d
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^3\right)\cosh(c+dx)\,dx = \int \left(ax\cosh(c+dx)+bx^4\cosh(c+dx)\right)dx$$
$$= a\int x\cosh(c+dx)\,dx + b\int x^4\cosh(c+dx)\,dx$$
$$= \frac{ax\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d} - \frac{a\int\sinh(c+dx)\,dx}{d} - \frac{(4b)\int x^3\sinh(c+dx)\,dx}{d}$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{bx^4\sinh(c+dx)}{d} + \frac{(12b)\int x}{}$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{12bx^2\sinh(c+dx)}{d^3} + \frac{bx^4\sin}{}$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{24bx\cosh(c+dx)}{d^4} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{ax\sinh(c+dx)}{d} + \frac{12bx^2 s}{}$$
$$= -\frac{a\cosh(c+dx)}{d^2} - \frac{24bx\cosh(c+dx)}{d^4} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c+dx)}{d^5} + \frac{ax\sin}{}$$

Mathematica [A] time = 0.110483, size = 66, normalized size = 0.7

$$\frac{\left(ad^4x+b\left(d^4x^4+12d^2x^2+24\right)\right)\sinh(c+dx)-d\left(ad^2+4bx\left(d^2x^2+6\right)\right)\cosh(c+dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-(d*(a*d^2 + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^4*x + b*(24 + 12*d
^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Maple [B] time = 0.007, size = 257, normalized size = 2.7

$$\frac{1}{d^2}\left(\frac{b\left((dx+c)^4\sinh(dx+c)-4\,(dx+c)^3\cosh(dx+c)+12\,(dx+c)^2\sinh(dx+c)-24\,(dx+c)\cosh(dx+c)+24\sin\right.}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*cosh(d*x+c),x)
```

```
[Out] 1/d^2*(1/d^3*b*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*
sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-4/d^3*b*c*((d*x+c)^3*sin
h(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))+6/d^3
*b*c^2*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-4/d^3*b*
c^3*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1
/d^3*b*c^4*sinh(d*x+c)-c*a*sinh(d*x+c))
```

Maxima [B] time = 1.04164, size = 265, normalized size = 2.82

$$-\frac{1}{20}d\left(\frac{5\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)ae^{(dx)}}{d^3}+\frac{5\left(d^2x^2+2\,dx+2\right)ae^{(-dx-c)}}{d^3}+\frac{2\left(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x^2e^c+12\right.}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/20*d*(5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 5*(d^2*x^2 + 2
*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*
e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 2*(d^5*x^5 +
5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) +
1/10*(2*b*x^5 + 5*a*x^2)*cosh(d*x + c)
```

Fricas [A] time = 1.71885, size = 155, normalized size = 1.65

$$-\frac{\left(4\,bd^3x^3 + ad^3 + 24\,bdx\right)\cosh\left(dx + c\right) - \left(bd^4x^4 + ad^4x + 12\,bd^2x^2 + 24\,b\right)\sinh\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((4*b*d^3*x^3 + a*d^3 + 24*b*d*x)*cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x + 1
2*b*d^2*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] time = 3.26191, size = 116, normalized size = 1.23

$$\begin{cases} \frac{ax\sinh\left(c+dx\right)}{d} - \frac{a\cosh\left(c+dx\right)}{d^2} + \frac{bx^4\sinh\left(c+dx\right)}{d} - \frac{4bx^3\cosh\left(c+dx\right)}{d^2} + \frac{12bx^2\sinh\left(c+dx\right)}{d^3} - \frac{24bx\cosh\left(c+dx\right)}{d^4} + \frac{24b\sinh\left(c+dx\right)}{d^5} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^5}{5}\right)\cosh\left(c\right) & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**4*sinh(c + d*x
)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*c
osh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x**5
/5)*cosh(c), True))
```

Giac [A] time = 1.36826, size = 161, normalized size = 1.71

$$\frac{\left(bd^4x^4 - 4\,bd^3x^3 + ad^4x + 12\,bd^2x^2 - ad^3 - 24\,bdx + 24\,b\right)e^{\left(dx+c\right)}}{2\,d^5} - \frac{\left(bd^4x^4 + 4\,bd^3x^3 + ad^4x + 12\,bd^2x^2 + ad^3 + 24\,bd}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 - 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x^2 - a*d^3 - 24*b*d*x +
24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x
^2 + a*d^3 + 24*b*d*x + 24*b)*e^(-d*x - c)/d^5
```

3.82 $\int \left(a+bx^3\right)\cosh(c+dx)\,dx$

Optimal. Leaf size=66

$$\frac{a\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4}+\frac{bx^3\sinh(c+dx)}{d}$$

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/
d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d
```

Rubi [A] time = 0.103047, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5277, 2637, 3296, 2638}

$$\frac{a\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4}+\frac{bx^3\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/
d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d
```

Rule 5277

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)\cosh(c+dx)\,dx = \int \left(a\cosh(c+dx)+bx^3\cosh(c+dx)\right)dx$$
$$= a\int \cosh(c+dx)\,dx + b\int x^3\cosh(c+dx)\,dx$$
$$= \frac{a\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{(3b)\int x^2\sinh(c+dx)\,dx}{d}$$
$$= -\frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} + \frac{(6b)\int x\cosh(c+dx)\,dx}{d^2}$$
$$= -\frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{bx^3\sinh(c+dx)}{d} - \frac{(6b)\int}{}$$
$$= -\frac{6b\cosh(c+dx)}{d^4} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{bx^3\sin}{}$$

Mathematica [A] time = 0.0808538, size = 49, normalized size = 0.74

$$\frac{d\left(ad^2+bx\left(d^2x^2+6\right)\right)\sinh(c+dx)-3b\left(d^2x^2+2\right)\cosh(c+dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-3*b*(2 + d^2*x^2)*Cosh[c + d*x] + d*(a*d^2 + b*x*(6 + d^2*x^2))*Sinh[c +
d*x])/d^4
```

Maple [B] time = 0.007, size = 158, normalized size = 2.4

$$\frac{1}{d}\left(\frac{b\left((dx+c)^3\sinh(dx+c)-3\,(dx+c)^2\cosh(dx+c)+6\,(dx+c)\sinh(dx+c)-6\,\cosh(dx+c)\right)}{d^3}-3\,\frac{cb\left((dx+c)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c),x)
```

```
[Out] 1/d*(1/d^3*b*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(
d*x+c)-6*cosh(d*x+c))-3/d^3*b*c*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c
)+2*sinh(d*x+c))+3/d^3*b*c^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-1/d^3*b*c^3*
sinh(d*x+c)+a*sinh(d*x+c))
```

Maxima [A] time = 1.02864, size = 140, normalized size = 2.12

$$\frac{ae^{(dx+c)}}{2\,d} - \frac{ae^{(-dx-c)}}{2\,d} + \frac{\left(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c\right)be^{(dx)}}{2\,d^4} - \frac{\left(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6\right)be^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^3*x^3*e^c - 3*d^2*x^2*e
^c + 6*d*x*e^c - 6*e^c)*b*e^(d*x)/d^4 - 1/2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x +
```

```
6)*b*e^(-d*x - c)/d^4
```

Fricas [A] time = 1.81258, size = 119, normalized size = 1.8

$$-\frac{3\left(bd^2x^2+2\,b\right)\cosh\left(dx+c\right)-\left(bd^3x^3+ad^3+6\,bdx\right)\sinh\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(3*(b*d^2*x^2 + 2*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3 + 6*b*d*x)*sinh(d*
x + c))/d^4
```

Sympy [A] time = 1.40829, size = 82, normalized size = 1.24

$$\begin{cases}\frac{a\sinh\left(c+dx\right)}{d}+\frac{bx^3\sinh\left(c+dx\right)}{d}-\frac{3bx^2\cosh\left(c+dx\right)}{d^2}+\frac{6bx\sinh\left(c+dx\right)}{d^3}-\frac{6b\cosh\left(c+dx\right)}{d^4} & \text{for } d\neq 0\\ \left(ax+\frac{bx^4}{4}\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d
*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((
a*x + b*x**4/4)*cosh(c), True))
```

Giac [A] time = 1.22369, size = 119, normalized size = 1.8

$$\frac{\left(bd^3x^3-3\,bd^2x^2+ad^3+6\,bdx-6\,b\right)e^{(dx+c)}}{2\,d^4}-\frac{\left(bd^3x^3+3\,bd^2x^2+ad^3+6\,bdx+6\,b\right)e^{(-dx-c)}}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 - 3*b*d^2*x^2 + a*d^3 + 6*b*d*x - 6*b)*e^(d*x + c)/d^4 - 1/2
*(b*d^3*x^3 + 3*b*d^2*x^2 + a*d^3 + 6*b*d*x + 6*b)*e^(-d*x - c)/d^4
```

3.83 $\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$

Optimal. Leaf size=56

$$a\cosh(c)\mathrm{Chi}(dx)+a\sinh(c)\mathrm{Shi}(dx)+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*
x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.128231, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5287, 3303, 3298, 3301, 3296, 2637}

$$a\cosh(c)\mathrm{Chi}(dx)+a\sinh(c)\mathrm{Shi}(dx)+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

```
[Out] (-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*
x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x}+bx^2\cosh(c+dx)\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x}\,dx+b\int x^2\cosh(c+dx)\,dx\\ &= \frac{bx^2\sinh(c+dx)}{d}-\frac{(2b)\int x\sinh(c+dx)\,dx}{d}+(a\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(a\sinh(c))\int\\ &= -\frac{2bx\cosh(c+dx)}{d^2}+a\cosh(c)\mathbf{Chi}(dx)+\frac{bx^2\sinh(c+dx)}{d}+a\sinh(c)\mathbf{Shi}(dx)+\frac{(2b)\int}{}\\ &= -\frac{2bx\cosh(c+dx)}{d^2}+a\cosh(c)\mathbf{Chi}(dx)+\frac{2b\sinh(c+dx)}{d^3}+\frac{bx^2\sinh(c+dx)}{d}+a\sinh(c\end{aligned}$$

Mathematica [A] time = 0.164428, size = 49, normalized size = 0.88

$$a\cosh(c)\mathbf{Chi}(dx)+a\sinh(c)\mathbf{Shi}(dx)+\frac{b\left(\left(d^2x^2+2\right)\sinh(c+dx)-2dx\cosh(c+dx)\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*(-2*d*x*Cosh[c + d*x] + (2 + d^2*x^2)*Sinh
[c + d*x]))/d^3 + a*Sinh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.032, size = 113, normalized size = 2.

$$\frac{b\mathrm{e}^{dx+c}x^2}{2\,d}-\frac{b\mathrm{e}^{dx+c}x}{d^2}-\frac{b\mathrm{e}^{-dx-c}x^2}{2\,d}-\frac{b\mathrm{e}^{-dx-c}x}{d^2}-\frac{a\mathrm{e}^{c}\mathrm{Ei}\,(1,-dx)}{2}-\frac{a\mathrm{e}^{-c}\mathrm{Ei}\,(1,dx)}{2}+\frac{b\mathrm{e}^{dx+c}}{d^3}-\frac{b\mathrm{e}^{-dx-c}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x,x)
```

```
[Out] 1/2/d*b*exp(d*x+c)*x^2-1/d^2*b*exp(d*x+c)*x-1/2/d*b*exp(-d*x-c)*x^2-1/d^2*b
*exp(-d*x-c)*x-1/2*a*exp(c)*Ei(1,-d*x)-1/2*a*exp(-c)*Ei(1,d*x)+1/d^3*b*exp(
d*x+c)-1/d^3*b*exp(-d*x-c)
```

Maxima [B] time = 1.23097, size = 189, normalized size = 3.38

$$-\frac{1}{6}\left(b\left(\frac{\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)e^{(dx)}}{d^4}+\frac{\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)e^{(-dx-c)}}{d^4}\right)+\frac{2\,a\cosh\,(dx+c)\log\left(x^3\right)}{d}-\frac{3\left(\mathrm{Ei}\,(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/6*(b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d
^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + 2*a*cosh(d*x + c)*log(x
^3)/d - 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + 1/3*(b*x^3 + a*log(x^3))
*cosh(d*x + c)
```

Fricas [A] time = 1.69971, size = 211, normalized size = 3.77

$$-\frac{4\,bdx\cosh\left(dx+c\right)-\left(ad^3\mathrm{Ei}\left(dx\right)+ad^3\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-2\left(bd^2x^2+2\,b\right)\sinh\left(dx+c\right)-\left(ad^3\mathrm{Ei}\left(dx\right)-ad^3\mathrm{Ei}\left(-dx\right.\right.}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x*cosh(d*x + c) - (a*d^3*Ei(d*x) + a*d^3*Ei(-d*x))*cosh(c) - 2*
(b*d^2*x^2 + 2*b)*sinh(d*x + c) - (a*d^3*Ei(d*x) - a*d^3*Ei(-d*x))*sinh(c))
/d^3
```

Sympy [A] time = 4.8626, size = 66, normalized size = 1.18

$$a\sinh\left(c\right)\mathrm{Shi}\left(dx\right)+a\cosh\left(c\right)\mathrm{Chi}\left(dx\right)+b\left(\begin{cases}\frac{x^2\sinh\left(c+dx\right)}{d}-\frac{2x\cosh\left(c+dx\right)}{d^2}+\frac{2\sinh\left(c+dx\right)}{d^3} & \text{for } d\neq 0\\ \frac{x^3\cosh\left(c\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x**2*sinh(c + d*x)/d
 - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**3*cosh(c)/
3, True))
```

Giac [A] time = 1.27269, size = 147, normalized size = 2.62

$$\frac{bd^2x^2e^{(dx+c)}-bd^2x^2e^{(-dx-c)}+ad^3\mathrm{Ei}\left(-dx\right)e^{(-c)}+ad^3\mathrm{Ei}\left(dx\right)e^c-2\,bdxe^{(dx+c)}-2\,bdxe^{(-dx-c)}+2\,be^{(dx+c)}-2\,be^{(-dx-c)}}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2*e^(d*x + c) - b*d^2*x^2*e^(-d*x - c) + a*d^3*Ei(-d*x)*e^(-c)
 + a*d^3*Ei(d*x)*e^c - 2*b*d*x*e^(d*x + c) - 2*b*d*x*e^(-d*x - c) + 2*b*e^(
d*x + c) - 2*b*e^(-d*x - c))/d^3
```

3.84 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx$

Optimal. Leaf size=55

$$ad\sinh(c)\text{Chi}(dx) + ad\cosh(c)\text{Shi}(dx) - \frac{a\cosh(c+dx)}{x} - \frac{b\cosh(c+dx)}{d^2} + \frac{bx\sinh(c+dx)}{d}$$

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.129105, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$ad\sinh(c)\text{Chi}(dx) + ad\cosh(c)\text{Shi}(dx) - \frac{a\cosh(c+dx)}{x} - \frac{b\cosh(c+dx)}{d^2} + \frac{bx\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)\cosh(c+dx)}{x^2}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^2}+bx\cosh(c+dx)\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^2}\,dx+b\int x\cosh(c+dx)\,dx\\ &= -\frac{a\cosh(c+dx)}{x}+\frac{bx\sinh(c+dx)}{d}-\frac{b\int \sinh(c+dx)\,dx}{d}+(ad)\int \frac{\sinh(c+dx)}{x}\,dx\\ &= -\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+\frac{bx\sinh(c+dx)}{d}+(ad\cosh(c))\int \frac{\sinh(dx)}{x}\,dx+\\ &= -\frac{b\cosh(c+dx)}{d^2}-\frac{a\cosh(c+dx)}{x}+ad\text{Chi}(dx)\sinh(c)+\frac{bx\sinh(c+dx)}{d}+ad\cosh(\end{aligned}$$

Mathematica [A] time = 0.131198, size = 55, normalized size = 1.

$$ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

```
[Out] -((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sinh
[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [A] time = 0.054, size = 110, normalized size = 2.

$$-\frac{a\mathrm{e}^{-dx-c}}{2x}+\frac{da\mathrm{e}^{-c}\mathrm{Ei}\,(1,dx)}{2}-\frac{b\mathrm{e}^{-dx-c}x}{2d}-\frac{b\mathrm{e}^{-dx-c}}{2d^2}-\frac{a\mathrm{e}^{dx+c}}{2x}-\frac{da\mathrm{e}^{c}\mathrm{Ei}\,(1,-dx)}{2}+\frac{b\mathrm{e}^{dx+c}x}{2d}-\frac{b\mathrm{e}^{dx+c}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^2,x)
```

```
[Out] -1/2*a*exp(-d*x-c)/x+1/2*d*a*exp(-c)*Ei(1,d*x)-1/2/d*b*exp(-d*x-c)*x-1/2/d^
2*b*exp(-d*x-c)-1/2*a/x*exp(d*x+c)-1/2*d*a*exp(c)*Ei(1,-d*x)+1/2/d*b*exp(d*
x+c)*x-1/2/d^2*b*exp(d*x+c)
```

Maxima [A] time = 1.15927, size = 138, normalized size = 2.51

$$-\frac{1}{4}\left(2\,a\mathrm{Ei}\,(-dx)\,e^{(-c)}-2\,a\mathrm{Ei}\,(dx)\,e^{c}+\frac{\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)be^{(dx)}}{d^3}+\frac{\left(d^2x^2+2\,dx+2\right)be^{(-dx-c)}}{d^3}\right)d+\frac{1}{2}\left(bx^2-\frac{2\,a}{x}\right)\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*Ei(-d*x)*e^(-c) - 2*a*Ei(d*x)*e^c + (d^2*x^2*e^c - 2*d*x*e^c + 2*
e^c)*b*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3)*d + 1/2*(b*x
^2 - 2*a/x)*cosh(d*x + c)
```

Fricas [A] time = 1.82228, size = 223, normalized size = 4.05

$$\frac{2\,bdx^2\sinh\left(dx+c\right)-2\left(ad^2+bx\right)\cosh\left(dx+c\right)+\left(ad^3x\text{Ei}\left(dx\right)-ad^3x\text{Ei}\left(-dx\right)\right)\cosh\left(c\right)+\left(ad^3x\text{Ei}\left(dx\right)+ad^3x\text{Ei}\left(-dx\right)}{2\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x^2*sinh(d*x + c) - 2*(a*d^2 + b*x)*cosh(d*x + c) + (a*d^3*x*Ei(
d*x) - a*d^3*x*Ei(-d*x))*cosh(c) + (a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*sin
h(c))/(d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3\right)\cosh\left(c+dx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)*cosh(c + d*x)/x**2, x)
```

Giac [B] time = 1.31986, size = 150, normalized size = 2.73

$$-\frac{ad^3x\text{Ei}\left(-dx\right)e^{(-c)}-ad^3x\text{Ei}\left(dx\right)e^c-bdx^2e^{(dx+c)}+bdx^2e^{(-dx-c)}+ad^2e^{(dx+c)}+ad^2e^{(-dx-c)}+bxe^{(dx+c)}+bxe^{(-dx-c)}}{2\,d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(a*d^3*x*Ei(-d*x)*e^(-c) - a*d^3*x*Ei(d*x)*e^c - b*d*x^2*e^(d*x + c) +
 b*d*x^2*e^(-d*x - c) + a*d^2*e^(d*x + c) + a*d^2*e^(-d*x - c) + b*x*e^(d*x
 + c) + b*x*e^(-d*x - c))/(d^2*x)
```

3.85 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}ad^2\cosh(c)\mathbf{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\mathbf{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+\frac{b\sinh(c+dx)}{d}$$

```
[Out] -(a*Cosh[c + d*x])/(2*x^2) + (a*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (b*Sinh[
c + d*x])/d - (a*d*Sinh[c + d*x])/(2*x) + (a*d^2*Sinh[c]*SinhIntegral[d*x])
/2
```

Rubi [A] time = 0.138576, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2\cosh(c)\mathbf{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\mathbf{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```


```
[Out] -(a*Cosh[c + d*x])/(2*x^2) + (a*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (b*Sinh[
c + d*x])/d - (a*d*Sinh[c + d*x])/(2*x) + (a*d^2*Sinh[c]*SinhIntegral[d*x])
/2
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)\cosh(c+dx)}{x^3}\,dx &= \int \left(b\cosh(c+dx)+\frac{a\cosh(c+dx)}{x^3}\right)dx \\
&= a\int \frac{\cosh(c+dx)}{x^3}\,dx + b\int \cosh(c+dx)\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}+\frac{1}{2}(ad)\int \frac{\sinh(c+dx)}{x^2}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}\left(ad^2\right)\int \frac{\cosh(c+dx)}{x}\,dx \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}\left(ad^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx + \\
&= -\frac{a\cosh(c+dx)}{2x^2}+\frac{1}{2}ad^2\cosh(c)\mathbf{Chi}(dx)+\frac{b\sinh(c+dx)}{d}-\frac{ad\sinh(c+dx)}{2x}+\frac{1}{2}ad^2\sin
\end{aligned}
$$

Mathematica [A] time = 0.141124, size = 86, normalized size = 1.25

$$\frac{1}{2}ad^2(\cosh(c)\mathbf{Chi}(dx)+\sinh(c)\mathbf{Shi}(dx))-\frac{a\cosh(dx)(dx\sinh(c)+\cosh(c))}{2x^2}-\frac{a\sinh(dx)(dx\cosh(c)+\sinh(c))}{2x^2}+\frac{b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

```
[Out] (b*Cosh[d*x]*Sinh[c])/d - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2) + (
b*Cosh[c]*Sinh[d*x])/d - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + (a
*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Maple [A] time = 0.056, size = 114, normalized size = 1.7

$$\frac{da\mathrm{e}^{-dx-c}}{4x}-\frac{a\mathrm{e}^{-dx-c}}{4x^2}-\frac{d^2a\mathrm{e}^{-c}\mathrm{Ei}\left(1,dx\right)}{4}-\frac{b\mathrm{e}^{-dx-c}}{2d}-\frac{a\mathrm{e}^{dx+c}}{4x^2}-\frac{da\mathrm{e}^{dx+c}}{4x}-\frac{d^2a\mathrm{e}^{c}\mathrm{Ei}\left(1,-dx\right)}{4}+\frac{b\mathrm{e}^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^3,x)
```

```
[Out] 1/4*d*a*exp(-d*x-c)/x-1/4*a*exp(-d*x-c)/x^2-1/4*d^2*a*exp(-c)*Ei(1,d*x)-1/2
*b/d*exp(-d*x-c)-1/4*a/x^2*exp(d*x+c)-1/4*d*a/x*exp(d*x+c)-1/4*d^2*a*exp(c)
*Ei(1,-d*x)+1/2*b/d*exp(d*x+c)
```

Maxima [A] time = 1.17684, size = 117, normalized size = 1.7

$$\frac{1}{4}\left(ade^{(-c)}\Gamma\left(-1,dx\right)+ade^{c}\Gamma\left(-1,-dx\right)-\frac{2\left(dxe^{c}-e^{c}\right)be^{(dx)}}{d^2}-\frac{2\left(dx+1\right)be^{(-dx-c)}}{d^2}\right)d+\frac{1}{2}\left(2\,bx-\frac{a}{x^2}\right)\cosh\left(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*(d*x*e^c - e^c
)*b*e^(d*x)/d^2 - 2*(d*x + 1)*b*e^(-d*x - c)/d^2)*d + 1/2*(2*b*x - a/x^2)*c
osh(d*x + c)
```

Fricas [A] time = 1.83341, size = 238, normalized size = 3.45

$$-\frac{2\,ad\cosh\left(dx+c\right) - \left(ad^{3}x^{2}\mathrm{Ei}\left(dx\right) + ad^{3}x^{2}\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right) + 2\left(ad^{2}x - 2\,bx^{2}\right)\sinh\left(dx+c\right) - \left(ad^{3}x^{2}\mathrm{Ei}\left(dx\right) - ad^{3}\right.}{4\,dx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*cosh(d*x + c) - (a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c
) + 2*(a*d^2*x - 2*b*x^2)*sinh(d*x + c) - (a*d^3*x^2*Ei(d*x) - a*d^3*x^2*Ei
(-d*x))*sinh(c))/(d*x^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.25569, size = 159, normalized size = 2.3

$$\frac{ad^{3}x^{2}\mathrm{Ei}\left(-dx\right)e^{(-c)} + ad^{3}x^{2}\mathrm{Ei}\left(dx\right)e^{c} - ad^{2}xe^{(dx+c)} + ad^{2}xe^{(-dx-c)} + 2\,bx^{2}e^{(dx+c)} - 2\,bx^{2}e^{(-dx-c)} - ade^{(dx+c)} - ade^{(-dx-c)}}{4\,dx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^3*x^2*Ei(-d*x)*e^(-c) + a*d^3*x^2*Ei(d*x)*e^c - a*d^2*x*e^(d*x + c
) + a*d^2*x*e^(-d*x - c) + 2*b*x^2*e^(d*x + c) - 2*b*x^2*e^(-d*x - c) - a*d
*e^(d*x + c) - a*d*e^(-d*x - c))/(d*x^2)
```

3.86 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$

Optimal. Leaf size=91

$$\frac{1}{6}ad^3\sinh(c)\mathbf{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathbf{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{a\cosh(c+dx)}{3x^3}+b\cosh(c)\mathbf{Chi}(dx$$

```
[Out] -(a*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*x])/(6*x) + b*Cosh[c]*CoshIn
tegral[d*x] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*
x^2) + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + b*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.215468, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3\sinh(c)\mathbf{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathbf{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{ad\sinh(c+dx)}{6x^2}-\frac{a\cosh(c+dx)}{3x^3}+b\cosh(c)\mathbf{Chi}(dx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(a*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d*x])/(6*x) + b*Cosh[c]*CoshIn
tegral[d*x] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*
x^2) + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + b*Sinh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)\cosh(c+dx)}{x^4}\,dx &= \int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x}\right)dx\\ &= a\int \frac{\cosh(c+dx)}{x^4}\,dx+b\int \frac{\cosh(c+dx)}{x}\,dx\\ &= -\frac{a\cosh(c+dx)}{3x^3}+\frac{1}{3}(ad)\int \frac{\sinh(c+dx)}{x^3}\,dx+(b\cosh(c))\int \frac{\cosh(dx)}{x}\,dx+(b\sinh(c\\ &= -\frac{a\cosh(c+dx)}{3x^3}+b\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}+b\sinh(c)\mathbf{Shi}(dx)+\frac{1}{6}\left(ad^2\right)\\ &= -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}+b\sinh(\\ &= -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\mathbf{Chi}(dx)-\frac{ad\sinh(c+dx)}{6x^2}+b\sinh(\\ &= -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\mathbf{Chi}(dx)+\frac{1}{6}ad^3\mathbf{Chi}(dx)\sinh(c)-\end{aligned}$$

Mathematica [A] time = 0.238487, size = 73, normalized size = 0.8

$$\frac{1}{6}\left(\mathbf{Chi}(dx)\left(ad^3\sinh(c)+6b\cosh(c)\right)+\mathbf{Shi}(dx)\left(ad^3\cosh(c)+6b\sinh(c)\right)-\frac{a\left(\left(d^2x^2+2\right)\cosh(c+dx)+dx\sinh(c\right.}{x^3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

```
[Out] (CoshIntegral[d*x]*(6*b*Cosh[c] + a*d^3*Sinh[c]) - (a*((2 + d^2*x^2)*Cosh[c
 + d*x] + d*x*Sinh[c + d*x]))/x^3 + (a*d^3*Cosh[c] + 6*b*Sinh[c])*SinhInteg
ral[d*x])/6
```

Maple [A] time = 0.066, size = 143, normalized size = 1.6

$$\frac{d^3ae^{-c}\mathrm{Ei}\,(1,dx)}{12}-\frac{be^{-c}\mathrm{Ei}\,(1,dx)}{2}-\frac{ad^2e^{-dx-c}}{12\,x}+\frac{dae^{-dx-c}}{12\,x^2}-\frac{ae^{-dx-c}}{6\,x^3}-\frac{ae^{dx+c}}{6\,x^3}-\frac{dae^{dx+c}}{12\,x^2}-\frac{ad^2e^{dx+c}}{12\,x}-\frac{d^3ae^{c}\mathrm{Ei}\,(1,-dx}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c)/x^4,x)
```

```
[Out] 1/12*d^3*a*exp(-c)*Ei(1,d*x)-1/2*b*exp(-c)*Ei(1,d*x)-1/12*d^2*a*exp(-d*x-c)
/x+1/12*d*a*exp(-d*x-c)/x^2-1/6*a*exp(-d*x-c)/x^3-1/6*a/x^3*exp(d*x+c)-1/12
*d*a/x^2*exp(d*x+c)-1/12*d^2*a/x*exp(d*x+c)-1/12*d^3*a*exp(c)*Ei(1,-d*x)-1/
2*b*exp(c)*Ei(1,-d*x)
```

Maxima [A] time = 1.22964, size = 128, normalized size = 1.41

$$\frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma\left(-2,dx\right)-d^2e^{c}\Gamma\left(-2,-dx\right)\right)a-\frac{2\,b\cosh\left(dx+c\right)\log\left(x^3\right)}{d}+\frac{3\left(\mathrm{Ei}\left(-dx\right)e^{(-c)}+\mathrm{Ei}\left(dx\right)e^{c}\right)b}{d}\right)d+\frac{1}{3}\left(b\log\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a - 2*b*cosh(d*x
 + c)*log(x^3)/d + 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/3*(b*log(x^
3) - a/x^3)*cosh(d*x + c)
```

Fricas [A] time = 1.72822, size = 277, normalized size = 3.04

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\left(ad^{2}x^{2}+2\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^{3}+6\,b\right)x^{3}\mathrm{Ei}\left(dx\right)-\left(ad^{3}-6\,b\right)x^{3}\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-\left(\left(ad^{3}+\right.\right.}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*(a*d^2*x^2 + 2*a)*cosh(d*x + c) - ((a*d^3
+ 6*b)*x^3*Ei(d*x) - (a*d^3 - 6*b)*x^3*Ei(-d*x))*cosh(c) - ((a*d^3 + 6*b)*x
^3*Ei(d*x) + (a*d^3 - 6*b)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.27178, size = 190, normalized size = 2.09

$$-\frac{ad^{3}x^{3}\mathrm{Ei}\left(-dx\right)e^{\left(-c\right)}-ad^{3}x^{3}\mathrm{Ei}\left(dx\right)e^{c}+ad^{2}x^{2}e^{\left(dx+c\right)}+ad^{2}x^{2}e^{\left(-dx-c\right)}-6\,bx^{3}\mathrm{Ei}\left(-dx\right)e^{\left(-c\right)}-6\,bx^{3}\mathrm{Ei}\left(dx\right)e^{c}+adxe^{\left(dx+c\right)}-}{12\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x
 + c) + a*d^2*x^2*e^(-d*x - c) - 6*b*x^3*Ei(-d*x)*e^(-c) - 6*b*x^3*Ei(d*x)*
e^c + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*
x - c))/x^3
```

3.87 $\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx$

Optimal. Leaf size=234

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh}{d^4}$$

```
[Out] (-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c
+ d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x])/
d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 + (48
*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[c +
d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])/d^5
+ (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2*x^7*S
inh[c + d*x])/d
```

Rubi [A] time = 0.39526, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5287, 3296, 2638, 2637}

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh}{d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```


```
[Out] (-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c
+ d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x])/
d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 + (48
*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[c +
d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])/d^5
+ (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2*x^7*S
inh[c + d*x])/d
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx=\int\left(a^2x\cosh(c+dx)+2abx^4\cosh(c+dx)+b^2x^7\cosh(c+dx)\right)dx$$
$$=a^2\int x\cosh(c+dx)\,dx+(2ab)\int x^4\cosh(c+dx)\,dx+b^2\int x^7\cosh(c+dx)\,dx$$
$$=\frac{a^2x\sinh(c+dx)}{d}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^7\sinh(c+dx)}{d}-\frac{a^2\int\sinh(c+dx)\,dx}{d}-$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{210b^2x^4\cosh(c+dx)}{d^4}$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{210b^2x^4\cosh(c+dx)}{d^4}$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}-\frac{8abx^3\cosh(c+dx)}{d^2}$$
$$=-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}-\frac{8abx^3\cosh(c+dx)}{d^2}$$
$$=-\frac{5040b^2\cosh(c+dx)}{d^8}-\frac{a^2\cosh(c+dx)}{d^2}-\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}$$

Mathematica [A] time = 0.291489, size = 139, normalized size = 0.59

$$\frac{d\left(a^2d^6x+2abd^2\left(d^4x^4+12d^2x^2+24\right)+b^2x\left(d^6x^6+42d^4x^4+840d^2x^2+5040\right)\right)\sinh(c+dx)-\left(a^2d^6+8abd^4x\left(d^2x^2+\right.\right.}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-((a^2*d^6 + 8*a*b*d^4*x*(6 + d^2*x^2) + 7*b^2*(720 + 360*d^2*x^2 + 30*d^4
*x^4 + d^6*x^6))*Cosh[c + d*x]) + d*(a^2*d^6*x + 2*a*b*d^2*(24 + 12*d^2*x^2
 + d^4*x^4) + b^2*x*(5040 + 840*d^2*x^2 + 42*d^4*x^4 + d^6*x^6))*Sinh[c + d
*x])/d^8
```

Maple [B] time = 0.008, size = 818, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d^2*(a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^6*b^2*((d*x+c)^7*sinh(d*x+
c)-7*(d*x+c)^6*cosh(d*x+c)+42*(d*x+c)^5*sinh(d*x+c)-210*(d*x+c)^4*cosh(d*x+
c)+840*(d*x+c)^3*sinh(d*x+c)-2520*(d*x+c)^2*cosh(d*x+c)+5040*(d*x+c)*sinh(d
*x+c)-5040*cosh(d*x+c))+7/d^6*b^2*c^6*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+12/
d^3*b*c^2*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-8/d
^3*b*c*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+
c)-6*cosh(d*x+c))-21/d^6*b^2*c^5*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+
```

```
c)+2*sinh(d*x+c))-8/d^3*b*c^3*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+35/d^6*b^
2*c^4*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-
6*cosh(d*x+c))-35/d^6*b^2*c^3*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c
)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+2/d^3*b*a
*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24
*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-7/d^6*b^2*c*((d*x+c)^6*sinh(d*x+c)-6*(
d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh(d*x+c)-120*(d*x+c)^3*cosh(d*x+c)+360
*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cosh(d*x+c)+720*sinh(d*x+c))+21/d^6*b^2*
c^2*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)
-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))-c*a^2*si
nh(d*x+c)-1/d^6*b^2*c^7*sinh(d*x+c)+2/d^3*b*c^4*a*sinh(d*x+c))
```

Maxima [A] time = 1.07065, size = 517, normalized size = 2.21

$$-\frac{1}{80}d\left(\frac{20\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)a^2e^{(dx)}}{d^3}+\frac{20\left(d^2x^2+2\,dx+2\right)a^2e^{(-dx-c)}}{d^3}+\frac{16\left(d^5x^5e^c-5\,d^4x^4e^c+20\,d^3x^3e^c-60\,d^2x\right.}{d^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*d*(20*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 20*(d^2*x^2
 + 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 16*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d
^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^(d*x)/d^6 + 16*(
d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^(-d*x
- c)/d^6 + 5*(d^8*x^8*e^c - 8*d^7*x^7*e^c + 56*d^6*x^6*e^c - 336*d^5*x^5*e^
c + 1680*d^4*x^4*e^c - 6720*d^3*x^3*e^c + 20160*d^2*x^2*e^c - 40320*d*x*e^c
 + 40320*e^c)*b^2*e^(d*x)/d^9 + 5*(d^8*x^8 + 8*d^7*x^7 + 56*d^6*x^6 + 336*d
^5*x^5 + 1680*d^4*x^4 + 6720*d^3*x^3 + 20160*d^2*x^2 + 40320*d*x + 40320)*b
^2*e^(-d*x - c)/d^9) + 1/40*(5*b^2*x^8 + 16*a*b*x^5 + 20*a^2*x^2)*cosh(d*x
+ c)
```

Fricas [A] time = 1.75902, size = 358, normalized size = 1.53

$$-\frac{\left(7\,b^2d^6x^6+8\,abd^6x^3+210\,b^2d^4x^4+a^2d^6+48\,abd^4x+2520\,b^2d^2x^2+5040\,b^2\right)\cosh\left(dx+c\right)-\left(b^2d^7x^7+2\,abd^7x^4+\right.}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -((7*b^2*d^6*x^6 + 8*a*b*d^6*x^3 + 210*b^2*d^4*x^4 + a^2*d^6 + 48*a*b*d^4*x
 + 2520*b^2*d^2*x^2 + 5040*b^2)*cosh(d*x + c) - (b^2*d^7*x^7 + 2*a*b*d^7*x^
4 + 42*b^2*d^5*x^5 + 24*a*b*d^5*x^2 + 840*b^2*d^3*x^3 + 48*a*b*d^3 + (a^2*d
^7 + 5040*b^2*d)*x)*sinh(d*x + c))/d^8
```

Sympy [A] time = 15.3885, size = 284, normalized size = 1.21

$$\begin{cases}\frac{a^2x\sinh(c+dx)}{d}-\frac{a^2\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{48abx\cosh(c+dx)}{d^4}+\frac{48ab\sinh(c+dx)}{d^5}+\frac{b^2}{} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^5}{5}+\frac{b^2x^8}{8}\right)\cosh(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**4*si
nh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)/d**2 + 24*a*b*x**2*sinh(c + d*x)/d
**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a*b*sinh(c + d*x)/d**5 + b**2*x**7*s
inh(c + d*x)/d - 7*b**2*x**6*cosh(c + d*x)/d**2 + 42*b**2*x**5*sinh(c + d*x
)/d**3 - 210*b**2*x**4*cosh(c + d*x)/d**4 + 840*b**2*x**3*sinh(c + d*x)/d**
5 - 2520*b**2*x**2*cosh(c + d*x)/d**6 + 5040*b**2*x*sinh(c + d*x)/d**7 - 50
40*b**2*cosh(c + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5 + b**2*
x**8/8)*cosh(c), True))
```

Giac [A] time = 1.23338, size = 409, normalized size = 1.75

$$\frac{\left(b^2d^7x^7 - 7\,b^2d^6x^6 + 2\,abd^7x^4 + 42\,b^2d^5x^5 - 8\,abd^6x^3 + a^2d^7x - 210\,b^2d^4x^4 + 24\,abd^5x^2 - a^2d^6 + 840\,b^2d^3x^3 - 48\,abd^4x \cdots\right.}{2\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^7*x^7 - 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 + 42*b^2*d^5*x^5 - 8*a*b*d
^6*x^3 + a^2*d^7*x - 210*b^2*d^4*x^4 + 24*a*b*d^5*x^2 - a^2*d^6 + 840*b^2*d
^3*x^3 - 48*a*b*d^4*x - 2520*b^2*d^2*x^2 + 48*a*b*d^3 + 5040*b^2*d*x - 5040
*b^2)*e^(d*x + c)/d^8 - 1/2*(b^2*d^7*x^7 + 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 +
42*b^2*d^5*x^5 + 8*a*b*d^6*x^3 + a^2*d^7*x + 210*b^2*d^4*x^4 + 24*a*b*d^5*x
^2 + a^2*d^6 + 840*b^2*d^3*x^3 + 48*a*b*d^4*x + 2520*b^2*d^2*x^2 + 48*a*b*d
^3 + 5040*b^2*d*x + 5040*b^2)*e^(-d*x - c)/d^8
```

3.88 $\int (a+bx^3)^2 \cosh(c+dx)\,dx$

Optimal. Leaf size=186

$$\frac{a^2\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh}{d^3}$$

```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c + d*
x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*x*S
inh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c + d
*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d
```

Rubi [A] time = 0.287106, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5277, 2637, 3296, 2638}

$$\frac{a^2\sinh(c+dx)}{d}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*Cosh[c + d*x],x]
```


```
[Out] (-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*Co
sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c + d*
x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*x*S
inh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c + d
*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d
```


Rule 5277

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \cosh(c+dx)\,dx = \int \left(a^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)\,dx$$
$$= a^2\int \cosh(c+dx)\,dx + (2ab)\int x^3\cosh(c+dx)\,dx + b^2\int x^6\cosh(c+dx)\,dx$$
$$= \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{b^2x^6\sinh(c+dx)}{d} - \frac{(6ab)\int x^2\sinh(c+dx)\,dx}{d}$$
$$= -\frac{6abx^2\cosh(c+dx)}{d^2} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{a^2\sinh(c+dx)}{d} + \frac{2abx^3\sinh(c+dx)}{d} + \frac{b}{}$$
$$= -\frac{6abx^2\cosh(c+dx)}{d^2} - \frac{6b^2x^5\cosh(c+dx)}{d^2} + \frac{a^2\sinh(c+dx)}{d} + \frac{12abx\sinh(c+dx)}{d^3} + \frac{2}{}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{6b^2x^5\cosh(c+dx)}{d^2}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+dx)}{d^4} - \frac{6b^2x^5\cosh(c+dx)}{d^2}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{720b^2x\cosh(c+dx)}{d^6} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+d}{d^4}$$
$$= -\frac{12ab\cosh(c+dx)}{d^4} - \frac{720b^2x\cosh(c+dx)}{d^6} - \frac{6abx^2\cosh(c+dx)}{d^2} - \frac{120b^2x^3\cosh(c+d}{d^4}$$

Mathematica [A] time = 0.219807, size = 111, normalized size = 0.6

$$\frac{\left(a^2d^6+2abd^4x\left(d^2x^2+6\right)+b^2\left(d^6x^6+30d^4x^4+360d^2x^2+720\right)\right)\sinh(c+dx)-6bd\left(ad^2\left(d^2x^2+2\right)+bx\left(d^4x^4+20d^2\right.\right.}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*Cosh[c + d*x],x]
```

```
[Out] (-6*b*d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a^2*d^6 + 2*a*b*d^4*x*(6 + d^2*x^2) + b^2*(720 + 360*d^2*x^2 + 30*d^
4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7
```

Maple [B] time = 0.009, size = 592, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c),x)
```

```
[Out] 1/d*(-6/d^6*b^2*c*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)
^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*
x+c))+15/d^6*b^2*c^2*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x
+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-20/d^6*b^2*c^3*((d
*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*
x+c))+2/d^3*b*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*si
nh(d*x+c)-6*cosh(d*x+c))+a^2*sinh(d*x+c)+1/d^6*b^2*((d*x+c)^6*sinh(d*x+c)-6
*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh(d*x+c)-120*(d*x+c)^3*cosh(d*x+c)+3
60*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cosh(d*x+c)+720*sinh(d*x+c))+1/d^6*b^2
*c^6*sinh(d*x+c)+6/d^3*b*c^2*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))-6/d^3*b*c*
a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2/d^3*b*c^3*a
*sinh(d*x+c)-6/d^6*b^2*c^5*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+15/d^6*b^2*c^4
```

```
*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c)))
```

Maxima [A] time = 1.04638, size = 328, normalized size = 1.76

$$\frac{a^2e^{(dx+c)}}{2d} - \frac{a^2e^{(-dx-c)}}{2d} + \frac{\left(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c\right)abe^{(dx)}}{d^4} - \frac{\left(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6\right)abe^{(-dx-c)}}{d^4} + \frac{\left(d^6x^6e^c - 6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^3*x^3*e^c - 3*d^2*x^2*e
^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4 - (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)
*a*b*e^(-d*x - c)/d^4 + 1/2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c -
 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7
 - 1/2*(d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*
d*x + 720)*b^2*e^(-d*x - c)/d^7
```

Fricas [A] time = 1.7411, size = 285, normalized size = 1.53

$$-\frac{6\left(b^2d^5x^5 + abd^5x^2 + 20\,b^2d^3x^3 + 2\,abd^3 + 120\,b^2dx\right)\cosh\left(dx + c\right) - \left(b^2d^6x^6 + 2\,abd^6x^3 + 30\,b^2d^4x^4 + a^2d^6 + 12\,ab\right.}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

```
[Out] -(6*(b^2*d^5*x^5 + a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 2*a*b*d^3 + 120*b^2*d*x)*
cosh(d*x + c) - (b^2*d^6*x^6 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + a^2*d^6 + 1
2*a*b*d^4*x + 360*b^2*d^2*x^2 + 720*b^2)*sinh(d*x + c))/d^7
```

Sympy [A] time = 9.36333, size = 226, normalized size = 1.22

$$\begin{cases} \frac{a^2\sinh\left(c+dx\right)}{d} + \frac{2abx^3\sinh\left(c+dx\right)}{d} - \frac{6abx^2\cosh\left(c+dx\right)}{d^2} + \frac{12abx\sinh\left(c+dx\right)}{d^3} - \frac{12ab\cosh\left(c+dx\right)}{d^4} + \frac{b^2x^6\sinh\left(c+dx\right)}{d} - \frac{6b^2x^5\cosh\left(c+dx\right)}{d^2} + \frac{30b}{} \\ \left(a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7}\right)\cosh\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c),x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*c
osh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4
 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*x)/d**2 + 30*b**2*x**
4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**4 + 360*b**2*x**2*sin
h(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*b**2*sinh(c + d*x)/d*
*7, Ne(d, 0)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*cosh(c), True))
```

Giac [A] time = 1.38075, size = 329, normalized size = 1.77

$$\frac{\left(b^2d^6x^6 - 6\,b^2d^5x^5 + 2\,abd^6x^3 + 30\,b^2d^4x^4 - 6\,abd^5x^2 + a^2d^6 - 120\,b^2d^3x^3 + 12\,abd^4x + 360\,b^2d^2x^2 - 12\,abd^3 - 720\,b^2\right.}{2\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^6*x^6 - 6*b^2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 - 6*a*b*d
^5*x^2 + a^2*d^6 - 120*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 - 12*a*
b*d^3 - 720*b^2*d*x + 720*b^2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 6*b^2*d
^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + 6*a*b*d^5*x^2 + a^2*d^6 + 120*b^2
*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 12*a*b*d^3 + 720*b^2*d*x + 720*
b^2)*e^(-d*x - c)/d^7
```

3.89 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} \, dx$

Optimal. Leaf size=160

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) + \frac{4ab \sinh(c+dx)}{d^3} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{20b^2x^3 \sinh}{d^3}$$

```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (4*a*b*x*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Co
sh[c + d*x])/d^4 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral
[d*x] + (4*a*b*Sinh[c + d*x])/d^3 + (120*b^2*x*Sinh[c + d*x])/d^5 + (2*a*b*
x^2*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (b^2*x^5*Sinh[c + d
*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.282343, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5287, 3303, 3298, 3301, 3296, 2637, 2638}

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) + \frac{4ab \sinh(c+dx)}{d^3} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{20b^2x^3 \sinh}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```


```
[Out] (-120*b^2*Cosh[c + d*x])/d^6 - (4*a*b*x*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Co
sh[c + d*x])/d^4 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral
[d*x] + (4*a*b*Sinh[c + d*x])/d^3 + (120*b^2*x*Sinh[c + d*x])/d^5 + (2*a*b*
x^2*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (b^2*x^5*Sinh[c + d
*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2\cosh(c+dx)}{x}\,dx=\int\left(\frac{a^2\cosh(c+dx)}{x}+2abx^2\cosh(c+dx)+b^2x^5\cosh(c+dx)\right)dx$$
$$=a^2\int\frac{\cosh(c+dx)}{x}\,dx+(2ab)\int x^2\cosh(c+dx)\,dx+b^2\int x^5\cosh(c+dx)\,dx$$
$$=\frac{2abx^2\sinh(c+dx)}{d}+\frac{b^2x^5\sinh(c+dx)}{d}-\frac{(4ab)\int x\sinh(c+dx)\,dx}{d}-\frac{\left(5b^2\right)\int x^4\sinh(c$$
$$=-\frac{4abx\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+a^2\cosh(c)\mathrm{Chi}(dx)+\frac{2abx^2\sinh(c+dx)}{d}+$$
$$=-\frac{4abx\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+a^2\cosh(c)\mathrm{Chi}(dx)+\frac{4ab\sinh(c+dx)}{d^3}+2$$
$$=-\frac{4abx\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+a^2\cosh(c)\mathrm{Chi}(dx)$$
$$=-\frac{4abx\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+a^2\cosh(c)\mathrm{Chi}(dx)$$
$$=-\frac{120b^2\cosh(c+dx)}{d^6}-\frac{4abx\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4}-\frac{5b^2x^4\cosh(c+dx)}{d^2}$$

Mathematica [A] time = 0.468257, size = 108, normalized size = 0.68

$$a^2\cosh(c)\mathrm{Chi}(dx)+a^2\sinh(c)\mathrm{Shi}(dx)+\frac{b\left(2ad^2\left(d^2x^2+2\right)+bx\left(d^4x^4+20d^2x^2+120\right)\right)\sinh(c+dx)}{d^5}-\frac{b\left(4ad^4x+5b\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

```
[Out] -((b*(4*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])/d^6) + a^
2*Cosh[c]*CoshIntegral[d*x] + (b*(2*a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2
*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Maple [B] time = 0.086, size = 335, normalized size = 2.1

$$-\frac{b^2e^{-dx-c}x^5}{2d}-\frac{5\,b^2e^{-dx-c}x^4}{2\,d^2}-10\,\frac{b^2e^{-dx-c}x^3}{d^3}-30\,\frac{b^2e^{-dx-c}x^2}{d^4}-60\,\frac{b^2e^{-dx-c}x}{d^5}-\frac{a^2e^{-c}\mathrm{Ei}\left(1,dx\right)}{2}-\frac{a^2e^{c}\mathrm{Ei}\left(1,-dx\right)}{2}-60\,\frac{e^{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x,x)
```

```
[Out] -1/2/d*b^2*exp(-d*x-c)*x^5-5/2/d^2*b^2*exp(-d*x-c)*x^4-10/d^3*b^2*exp(-d*x-
c)*x^3-30/d^4*b^2*exp(-d*x-c)*x^2-60/d^5*b^2*exp(-d*x-c)*x-1/2*a^2*exp(-c)*
Ei(1,d*x)-1/2*a^2*exp(c)*Ei(1,-d*x)-60/d^6*b^2*exp(d*x+c)-2/d^3*a*b*exp(-d*
x-c)+1/d*a*b*exp(d*x+c)*x^2-2/d^2*a*b*exp(d*x+c)*x+2/d^3*a*b*exp(d*x+c)-60/
d^6*b^2*exp(-d*x-c)-30/d^4*b^2*exp(d*x+c)*x^2+60/d^5*b^2*exp(d*x+c)*x+1/2/d
*b^2*exp(d*x+c)*x^5-5/2/d^2*b^2*exp(d*x+c)*x^4+10/d^3*b^2*exp(d*x+c)*x^3-1/
d*a*b*exp(-d*x-c)*x^2-2/d^2*a*b*exp(-d*x-c)*x
```

Maxima [A] time = 1.23268, size = 390, normalized size = 2.44

$$-\frac{1}{12}\left(4\,ab\left(\frac{\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)e^{(dx)}}{d^4}+\frac{\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)e^{(-dx-c)}}{d^4}\right)+b^2\left(\frac{\left(d^6x^6e^c-6\,d^5x^5e^c+30\,d\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4
 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + b^2*((d^6*x^6*e^c
- 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*
d*x*e^c + 720*e^c)*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^
3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*e^(-d*x - c)/d^7) + 4*a^2*cosh(d*x + c
)*log(x^3)/d - 6*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/6*(b^2*x^6 +
4*a*b*x^3 + 2*a^2*log(x^3))*cosh(d*x + c)
```

Fricas [A] time = 1.78628, size = 365, normalized size = 2.28

$$-\frac{2\left(5\,b^2d^4x^4+4\,abd^4x+60\,b^2d^2x^2+120\,b^2\right)\cosh\left(dx+c\right)-\left(a^2d^6\mathrm{Ei}\left(dx\right)+a^2d^6\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-2\left(b^2d^5x^5+2\,ab\right.}{2\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x + 60*b^2*d^2*x^2 + 120*b^2)*cosh(d*x +
 c) - (a^2*d^6*Ei(d*x) + a^2*d^6*Ei(-d*x))*cosh(c) - 2*(b^2*d^5*x^5 + 2*a*b
*d^5*x^2 + 20*b^2*d^3*x^3 + 4*a*b*d^3 + 120*b^2*d*x)*sinh(d*x + c) - (a^2*d
^6*Ei(d*x) - a^2*d^6*Ei(-d*x))*sinh(c))/d^6
```

Sympy [A] time = 11.71, size = 168, normalized size = 1.05

$$a^2\sinh(c)\,\mathrm{Shi}(dx)+a^2\cosh(c)\,\mathrm{Chi}(dx)+2ab\left(\begin{cases}\frac{x^2\sinh(c+dx)}{d}-\frac{2x\cosh(c+dx)}{d^2}+\frac{2\sinh(c+dx)}{d^3} & \text{for } d\neq 0\\ \frac{x^3\cosh(c)}{3} & \text{otherwise}\end{cases}\right)+b^2\left(\begin{cases}\frac{x^5\sin}{} \\ \frac{x^6\cos}{6}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x,x)
```

```
[Out] a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x**2*sinh(
c + d*x)/d - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**
3*cosh(c)/3, True)) + b**2*Piecewise((x**5*sinh(c + d*x)/d - 5*x**4*cosh(c
+ d*x)/d**2 + 20*x**3*sinh(c + d*x)/d**3 - 60*x**2*cosh(c + d*x)/d**4 + 120
*x*sinh(c + d*x)/d**5 - 120*cosh(c + d*x)/d**6, Ne(d, 0)), (x**6*cosh(c)/6,
 True))
```

Giac [B] time = 1.21348, size = 447, normalized size = 2.79

$$\frac{b^2d^5x^5e^{(dx+c)} - b^2d^5x^5e^{(-dx-c)} - 5\,b^2d^4x^4e^{(dx+c)} - 5\,b^2d^4x^4e^{(-dx-c)} + 2\,abd^5x^2e^{(dx+c)} - 2\,abd^5x^2e^{(-dx-c)} + a^2d^6\mathrm{Ei}\left(-dx\right)e^{(-c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^5*x^5*e^(d*x + c) - b^2*d^5*x^5*e^(-d*x - c) - 5*b^2*d^4*x^4*e^(
d*x + c) - 5*b^2*d^4*x^4*e^(-d*x - c) + 2*a*b*d^5*x^2*e^(d*x + c) - 2*a*b*d
^5*x^2*e^(-d*x - c) + a^2*d^6*Ei(-d*x)*e^(-c) + a^2*d^6*Ei(d*x)*e^c + 20*b^
2*d^3*x^3*e^(d*x + c) - 20*b^2*d^3*x^3*e^(-d*x - c) - 4*a*b*d^4*x*e^(d*x +
c) - 4*a*b*d^4*x*e^(-d*x - c) - 60*b^2*d^2*x^2*e^(d*x + c) - 60*b^2*d^2*x^2
*e^(-d*x - c) + 4*a*b*d^3*e^(d*x + c) - 4*a*b*d^3*e^(-d*x - c) + 120*b^2*d*
x*e^(d*x + c) - 120*b^2*d*x*e^(-d*x - c) - 120*b^2*e^(d*x + c) - 120*b^2*e^
(-d*x - c))/d^6
```

3.90 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$

Optimal. Leaf size=143

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c}{d^3}$$

```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.251996, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638, 2637}

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```


```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^3\right)^2 \cosh(c+dx)}{x^2}\,dx &= \int \left(\frac{a^2\cosh(c+dx)}{x^2}+2abx\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)dx\\
&= a^2\int \frac{\cosh(c+dx)}{x^2}\,dx+(2ab)\int x\cosh(c+dx)\,dx+b^2\int x^4\cosh(c+dx)\,dx\\
&= -\frac{a^2\cosh(c+dx)}{x}+\frac{2abx\sinh(c+dx)}{d}+\frac{b^2x^4\sinh(c+dx)}{d}-\frac{(2ab)\int \sinh(c+dx)\,dx}{d}-\\
&= -\frac{2ab\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{x}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{2abx\sinh(c+dx)}{d}+\frac{b^2x^4}{}\\
&= -\frac{2ab\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{x}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+a^2d\mathbf{Chi}(dx)\sinh(c)+\frac{2a}{}\\
&= -\frac{2ab\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{x}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+a^2c\\
&= -\frac{2ab\cosh(c+dx)}{d^2}-\frac{a^2\cosh(c+dx)}{x}-\frac{24b^2x\cosh(c+dx)}{d^4}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+a^2c
\end{aligned}
$$

Mathematica [A] time = 0.347878, size = 143, normalized size = 1.

$$a^2d\sinh(c)\mathbf{Chi}(dx)+a^2d\cosh(c)\mathbf{Shi}(dx)-\frac{a^2\cosh(c+dx)}{x}-\frac{2ab\cosh(c+dx)}{d^2}+\frac{2abx\sinh(c+dx)}{d}+\frac{12b^2x^2\sinh(c+}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

```
[Out] (-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*x
])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c] +
(24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c
 + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Maple [B] time = 0.102, size = 296, normalized size = 2.1

$$-\frac{b^2 e^{-dx-c} x^4}{2d} - 2\frac{b^2 e^{-dx-c} x^3}{d^2} - 6\frac{b^2 e^{-dx-c} x^2}{d^3} - 12\frac{b^2 e^{-dx-c} x}{d^4} + \frac{d a^2 e^{-c} \mathrm{Ei}(1, dx)}{2} - \frac{a b e^{-dx-c}}{d^2} - \frac{a^2 e^{-dx-c}}{2x} - 12\frac{b^2 e^{-dx-c}}{d^5} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^2,x)
```

```
[Out] -1/2/d*b^2*exp(-d*x-c)*x^4-2/d^2*b^2*exp(-d*x-c)*x^3-6/d^3*b^2*exp(-d*x-c)*
x^2-12/d^4*b^2*exp(-d*x-c)*x+1/2*d*a^2*exp(-c)*Ei(1,d*x)-a/d^2*b*exp(-d*x-c
)-1/2*a^2*exp(-d*x-c)/x-12/d^5*b^2*exp(-d*x-c)-a/d*b*exp(-d*x-c)*x-1/2*a^2/
x*exp(d*x+c)-1/2*d*a^2*exp(c)*Ei(1,-d*x)+12/d^5*b^2*exp(d*x+c)+a/d*b*exp(d*
x+c)*x-a/d^2*b*exp(d*x+c)+6/d^3*b^2*exp(d*x+c)*x^2-12/d^4*b^2*exp(d*x+c)*x+
1/2/d*b^2*exp(d*x+c)*x^4-2/d^2*b^2*exp(d*x+c)*x^3
```

Maxima [A] time = 1.19747, size = 317, normalized size = 2.22

$$-\frac{1}{10}\left(5\,a^2 \mathrm{Ei}\left(-dx\right) e^{(-c)} - 5\,a^2 \mathrm{Ei}\left(dx\right) e^{c} + \frac{5\left(d^2x^2e^c - 2\,dxe^c + 2\,e^c\right) abe^{(dx)}}{d^3} + \frac{5\left(d^2x^2 + 2\,dx + 2\right) abe^{(-dx-c)}}{d^3} + \frac{\left(d^5x^5e^c - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

```
[Out] -1/10*(5*a^2*Ei(-d*x)*e^(-c) - 5*a^2*Ei(d*x)*e^c + 5*(d^2*x^2*e^c - 2*d*x*e
^c + 2*e^c)*a*b*e^(d*x)/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3
+ (d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*
e^c - 120*e^c)*b^2*e^(d*x)/d^6 + (d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2
*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6)*d + 1/5*(b^2*x^5 + 5*a*b*x^2 -
5*a^2/x)*cosh(d*x + c)
```

Fricas [A] time = 1.77474, size = 358, normalized size = 2.5

$$-\frac{2\left(4\,b^2d^3x^4 + a^2d^5 + 2\,abd^3x + 24\,b^2dx^2\right)\cosh\left(dx + c\right) - \left(a^2d^6x\mathrm{Ei}\left(dx\right) - a^2d^6x\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right) - 2\left(b^2d^4x^5 + 2\,ab\right.}{2\,d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(4*b^2*d^3*x^4 + a^2*d^5 + 2*a*b*d^3*x + 24*b^2*d*x^2)*cosh(d*x + c
) - (a^2*d^6*x*Ei(d*x) - a^2*d^6*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^4*x^5 + 2*a
*b*d^4*x^2 + 12*b^2*d^2*x^3 + 24*b^2*x)*sinh(d*x + c) - (a^2*d^6*x*Ei(d*x)
+ a^2*d^6*x*Ei(-d*x))*sinh(c))/(d^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^2 \cosh\left(c + dx\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**2,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**2, x)
```

Giac [B] time = 1.26092, size = 416, normalized size = 2.91

$$\frac{b^2d^4x^5e^{(dx+c)} - b^2d^4x^5e^{(-dx-c)} - a^2d^6x\text{Ei}\left(-dx\right)e^{(-c)} + a^2d^6x\text{Ei}\left(dx\right)e^{c} - 4\,b^2d^3x^4e^{(dx+c)} - 4\,b^2d^3x^4e^{(-dx-c)} + 2\,abd^4x^2e^{(dx+c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^4*x^5*e^(d*x + c) - b^2*d^4*x^5*e^(-d*x - c) - a^2*d^6*x*Ei(-d*x
)*e^(-c) + a^2*d^6*x*Ei(d*x)*e^c - 4*b^2*d^3*x^4*e^(d*x + c) - 4*b^2*d^3*x^
4*e^(-d*x - c) + 2*a*b*d^4*x^2*e^(d*x + c) - 2*a*b*d^4*x^2*e^(-d*x - c) - a
^2*d^5*e^(d*x + c) + 12*b^2*d^2*x^3*e^(d*x + c) - a^2*d^5*e^(-d*x - c) - 12
*b^2*d^2*x^3*e^(-d*x - c) - 2*a*b*d^3*x*e^(d*x + c) - 2*a*b*d^3*x*e^(-d*x -
 c) - 24*b^2*d*x^2*e^(d*x + c) - 24*b^2*d*x^2*e^(-d*x - c) + 24*b^2*x*e^(d*
x + c) - 24*b^2*x*e^(-d*x - c))/(d^5*x)
```

3.91 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} \, dx$

Optimal. Leaf size=141

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx) - \frac{a^2\cosh(c+dx)}{2x^2} - \frac{a^2d\sinh(c+dx)}{2x} + \frac{2ab\sinh(c+dx)}{d} - \frac{3b^2x^2\cos}{d}$$

```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/(2*x^2) - (3*b^2*x^2*Cosh[
c + d*x])/d^2 + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (2*a*b*Sinh[c + d*x
])/d - (a^2*d*Sinh[c + d*x])/(2*x) + (6*b^2*x*Sinh[c + d*x])/d^3 + (b^2*x^3
*Sinh[c + d*x])/d + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] time = 0.232806, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5287, 2637, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx) - \frac{a^2\cosh(c+dx)}{2x^2} - \frac{a^2d\sinh(c+dx)}{2x} + \frac{2ab\sinh(c+dx)}{d} - \frac{3b^2x^2\cos}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```


```
[Out] (-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/(2*x^2) - (3*b^2*x^2*Cosh[
c + d*x])/d^2 + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (2*a*b*Sinh[c + d*x
])/d - (a^2*d*Sinh[c + d*x])/(2*x) + (6*b^2*x*Sinh[c + d*x])/d^3 + (b^2*x^3
*Sinh[c + d*x])/d + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```


Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \int\left(2ab\cosh(c+dx)+\frac{a^2\cosh(c+dx)}{x^3}+b^2x^3\cosh(c+dx)\right)dx$$

$$= a^2\int\frac{\cosh(c+dx)}{x^3}\,dx+(2ab)\int\cosh(c+dx)\,dx+b^2\int x^3\cosh(c+dx)\,dx$$

$$= -\frac{a^2\cosh(c+dx)}{2x^2}+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{(3b^2)\int x^2\sinh(c+dx)\,dx}{d}$$

$$= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{2x}+\frac{b^2x^3 s}{}$$

$$= -\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{2ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{2x}+\frac{6b^2x s}{}$$

$$= -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{1}{2}a^2d^2\cosh(c)\mathrm{Chi}(dx)+$$

Mathematica [A] time = 0.347951, size = 136, normalized size = 0.96

$$\frac{1}{2}\left(a^2d^2\cosh(c)\mathrm{Chi}(dx)+a^2d^2\sinh(c)\mathrm{Shi}(dx)-\frac{a^2\cosh(c+dx)}{x^2}-\frac{a^2d\sinh(c+dx)}{x}+\frac{4ab\sinh(c+dx)}{d}-\frac{6b^2x^2\cosh(c}{d^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

```
[Out] ((-12*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/x^2 - (6*b^2*x^2*Cosh[c
+ d*x])/d^2 + a^2*d^2*Cosh[c]*CoshIntegral[d*x] + (4*a*b*Sinh[c + d*x])/d -
 (a^2*d*Sinh[c + d*x])/x + (12*b^2*x*Sinh[c + d*x])/d^3 + (2*b^2*x^3*Sinh[c
 + d*x])/d + a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Maple [A] time = 0.115, size = 265, normalized size = 1.9

$$-\frac{b^2e^{-dx-c}x^3}{2d}-\frac{3b^2e^{-dx-c}x^2}{2d^2}-3\frac{b^2e^{-dx-c}x}{d^3}-\frac{d^2a^2e^{-c}\mathrm{Ei}(1,dx)}{4}-\frac{abe^{-dx-c}}{d}+\frac{da^2e^{-dx-c}}{4x}-\frac{a^2e^{-dx-c}}{4x^2}-3\frac{b^2e^{-dx-c}}{d^4}-\frac{d^2a^2e^{c}\mathrm{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^3,x)
```

```
[Out] -1/2/d*b^2*exp(-d*x-c)*x^3-3/2/d^2*b^2*exp(-d*x-c)*x^2-3/d^3*b^2*exp(-d*x-c
)*x-1/4*d^2*a^2*exp(-c)*Ei(1,d*x)-a*b/d*exp(-d*x-c)+1/4*d*a^2*exp(-d*x-c)/x
-1/4*a^2*exp(-d*x-c)/x^2-3/d^4*b^2*exp(-d*x-c)-1/4*d^2*a^2*exp(c)*Ei(1,-d*x
)-3/d^4*b^2*exp(d*x+c)+a*b/d*exp(d*x+c)-3/2/d^2*b^2*exp(d*x+c)*x^2+3/d^3*b^
2*exp(d*x+c)*x-1/4*a^2/x^2*exp(d*x+c)-1/4*d*a^2/x*exp(d*x+c)+1/2/d*b^2*exp(
d*x+c)*x^3
```

Maxima [A] time = 1.20039, size = 274, normalized size = 1.94

$$\frac{1}{8}\left(2\,a^2de^{(-c)}\Gamma\left(-1,dx\right)+2\,a^2de^c\Gamma\left(-1,-dx\right)-\frac{8\,(dxe^c-e^c)abe^{(dx)}}{d^2}-\frac{8\,(dx+1)abe^{(-dx-c)}}{d^2}-\frac{\left(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2\right.}{d^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(2*a^2*d*e^(-c)*gamma(-1, d*x) + 2*a^2*d*e^c*gamma(-1, -d*x) - 8*(d*x*e
^c - e^c)*a*b*e^(d*x)/d^2 - 8*(d*x + 1)*a*b*e^(-d*x - c)/d^2 - (d^4*x^4*e^c
 - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 -
(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d +
1/4*(b^2*x^4 + 8*a*b*x - 2*a^2/x^2)*cosh(d*x + c)
```

Fricas [A] time = 1.75178, size = 351, normalized size = 2.49

$$-\frac{2\left(6\,b^2d^2x^4+a^2d^4+12\,b^2x^2\right)\cosh\left(dx+c\right)-\left(a^2d^6x^2\mathrm{Ei}\left(dx\right)+a^2d^6x^2\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-2\left(2\,b^2d^3x^5-a^2d^5x+4\,ab\right.}{4\,d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*b^2*d^2*x^4 + a^2*d^4 + 12*b^2*x^2)*cosh(d*x + c) - (a^2*d^6*x^2
*Ei(d*x) + a^2*d^6*x^2*Ei(-d*x))*cosh(c) - 2*(2*b^2*d^3*x^5 - a^2*d^5*x + 4
*a*b*d^3*x^2 + 12*b^2*d*x^3)*sinh(d*x + c) - (a^2*d^6*x^2*Ei(d*x) - a^2*d^6
*x^2*Ei(-d*x))*sinh(c))/(d^4*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3\right)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**3,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**3, x)
```

Giac [B] time = 1.33646, size = 378, normalized size = 2.68

$$a^2d^6x^2\mathrm{Ei}\left(-dx\right)e^{(-c)} + a^2d^6x^2\mathrm{Ei}\left(dx\right)e^{c} + 2\,b^2d^3x^5e^{(dx+c)} - 2\,b^2d^3x^5e^{(-dx-c)} - a^2d^5xe^{(dx+c)} - 6\,b^2d^2x^4e^{(dx+c)} + a^2d^5xe^{(-dx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*d^6*x^2*Ei(-d*x)*e^(-c) + a^2*d^6*x^2*Ei(d*x)*e^c + 2*b^2*d^3*x^5*
e^(d*x + c) - 2*b^2*d^3*x^5*e^(-d*x - c) - a^2*d^5*x*e^(d*x + c) - 6*b^2*d^
2*x^4*e^(d*x + c) + a^2*d^5*x*e^(-d*x - c) - 6*b^2*d^2*x^4*e^(-d*x - c) + 4
*a*b*d^3*x^2*e^(d*x + c) - 4*a*b*d^3*x^2*e^(-d*x - c) - a^2*d^4*e^(d*x + c)
 + 12*b^2*d*x^3*e^(d*x + c) - a^2*d^4*e^(-d*x - c) - 12*b^2*d*x^3*e^(-d*x -
 c) - 12*b^2*x^2*e^(d*x + c) - 12*b^2*x^2*e^(-d*x - c))/(d^4*x^2)
```

3.92 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx$

Optimal. Leaf size=150

$$\frac{1}{6}a^2d^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2d\sinh(c+dx)}{6x^2}-\frac{a^2\cosh(c+dx)}{3x^3}+2ab\cosh$$

```
[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (a^2*d^2*Cosh[c + d*x])/(6*x) - (2*b^2*x*Cos
h[c + d*x])/d^2 + 2*a*b*Cosh[c]*CoshIntegral[d*x] + (a^2*d^3*CoshIntegral[d
*x]*Sinh[c])/6 + (2*b^2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/(6*x^2)
+ (b^2*x^2*Sinh[c + d*x])/d + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + 2*a*b
*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] time = 0.278846, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2637}

$$\frac{1}{6}a^2d^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2d\sinh(c+dx)}{6x^2}-\frac{a^2\cosh(c+dx)}{3x^3}+2ab\cosh$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] -(a^2*Cosh[c + d*x])/(3*x^3) - (a^2*d^2*Cosh[c + d*x])/(6*x) - (2*b^2*x*Cos
h[c + d*x])/d^2 + 2*a*b*Cosh[c]*CoshIntegral[d*x] + (a^2*d^3*CoshIntegral[d
*x]*Sinh[c])/6 + (2*b^2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/(6*x^2)
+ (b^2*x^2*Sinh[c + d*x])/d + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + 2*a*b
*Sinh[c]*SinhIntegral[d*x]
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2\cosh(c+dx)}{x^4}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x}+b^2x^2\cosh(c+dx)\right)dx$$

$$= a^2\int \frac{\cosh(c+dx)}{x^4}\,dx + (2ab)\int \frac{\cosh(c+dx)}{x}\,dx + b^2\int x^2\cosh(c+dx)\,dx$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}+\frac{b^2x^2\sinh(c+dx)}{d}-\frac{\left(2b^2\right)\int x\sinh(c+dx)\,dx}{d}+\frac{1}{3}\left(a^2d\right)\int \frac{\sinh(c}{x^3}$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)-\frac{a^2d\sinh(c+dx)}{6x^2}+\frac{b^2x^2}{}$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)+\frac{2b}{}$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)+\frac{2b}{}$$

$$= -\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{2b^2x\cosh(c+dx)}{d^2}+2ab\cosh(c)\text{Chi}(dx)+\frac{1}{6}a$$

Mathematica [A] time = 0.569738, size = 135, normalized size = 0.9

$$\frac{1}{6}\left(-\frac{a^2d^2\cosh(c+dx)}{x}-\frac{a^2d\sinh(c+dx)}{x^2}-\frac{2a^2\cosh(c+dx)}{x^3}+a\text{Chi}(dx)\left(ad^3\sinh(c)+12b\cosh(c)\right)+a\text{Shi}(dx)\left(ad^3\text{c}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

```
[Out] ((-2*a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/x - (12*b^2*x*Cosh[c
+ d*x])/d^2 + a*CoshIntegral[d*x]*(12*b*Cosh[c] + a*d^3*Sinh[c]) + (12*b^2*
Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/x^2 + (6*b^2*x^2*Sinh[c + d*x])/
d + a*(a*d^3*Cosh[c] + 12*b*Sinh[c])*SinhIntegral[d*x])/6
```

Maple [A] time = 0.128, size = 261, normalized size = 1.7

$$-\frac{b^2e^{-dx-c}x^2}{2d}-\frac{b^2e^{-dx-c}x}{d^2}+\frac{d^3a^2e^{-c}\text{Ei}(1,dx)}{12}-\frac{a^2d^2e^{-dx-c}}{12x}+\frac{da^2e^{-dx-c}}{12x^2}-\frac{a^2e^{-dx-c}}{6x^3}-abe^{-c}\text{Ei}(1,dx)-\frac{b^2e^{-dx-c}}{d^3}-\frac{d^3a^2e^{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^4,x)
```

```
[Out] -1/2/d*b^2*exp(-d*x-c)*x^2-1/d^2*b^2*exp(-d*x-c)*x+1/12*d^3*a^2*exp(-c)*Ei(
1,d*x)-1/12*d^2*a^2*exp(-d*x-c)/x+1/12*d*a^2*exp(-d*x-c)/x^2-1/6*a^2*exp(-d
*x-c)/x^3-a*b*exp(-c)*Ei(1,d*x)-1/d^3*b^2*exp(-d*x-c)-1/12*d^3*a^2*exp(c)*E
i(1,-d*x)+1/d^3*b^2*exp(d*x+c)-1/6*a^2/x^3*exp(d*x+c)-1/12*d*a^2/x^2*exp(d*
x+c)-1/12*d^2*a^2/x*exp(d*x+c)-a*b*exp(c)*Ei(1,-d*x)+1/2/d*b^2*exp(d*x+c)*x
^2-1/d^2*b^2*exp(d*x+c)*x
```

Maxima [A] time = 1.25149, size = 254, normalized size = 1.69

$$\frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma\left(-2,dx\right)-d^2e^c\Gamma\left(-2,-dx\right)\right)a^2-b^2\left(\frac{\left(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c\right)e^{(dx)}}{d^4}+\frac{\left(d^3x^3+3\,d^2x^2+6\,dx+6\right)e^{(-}}{d^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a^2 - b^2*((d^3*
x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2
*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) - 4*a*b*cosh(d*x + c)*log(x^3)/d + 6*(E
i(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + 1/3*(b^2*x^3 + 2*a*b*log(x^3) - a^
2/x^3)*cosh(d*x + c)
```

Fricas [A] time = 1.75958, size = 412, normalized size = 2.75

$$-\frac{2\left(a^2d^5x^2+12\,b^2dx^4+2\,a^2d^3\right)\cosh\left(dx+c\right)-\left(\left(a^2d^6+12\,abd^3\right)x^3\mathrm{Ei}\left(dx\right)-\left(a^2d^6-12\,abd^3\right)x^3\mathrm{Ei}\left(-dx\right)\right)\cosh\left(c\right)-}{12\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(a^2*d^5*x^2 + 12*b^2*d*x^4 + 2*a^2*d^3)*cosh(d*x + c) - ((a^2*d^6
 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*cosh(c) -
 2*(6*b^2*d^2*x^5 - a^2*d^4*x + 12*b^2*x^3)*sinh(d*x + c) - ((a^2*d^6 + 12*
a*b*d^3)*x^3*Ei(d*x) + (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))/(d^3*x
^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3\right)^2\cosh\left(c+dx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**4,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**4, x)
```

Giac [A] time = 1.33523, size = 377, normalized size = 2.51

$$-\frac{a^2d^6x^3\mathrm{Ei}(-dx)e^{(-c)} - a^2d^6x^3\mathrm{Ei}(dx)e^c + a^2d^5x^2e^{(dx+c)} - 6b^2d^2x^5e^{(dx+c)} + a^2d^5x^2e^{(-dx-c)} + 6b^2d^2x^5e^{(-dx-c)} - 12abd^3x^3\mathrm{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a^2*d^6*x^3*Ei(-d*x)*e^(-c) - a^2*d^6*x^3*Ei(d*x)*e^c + a^2*d^5*x^2*
e^(d*x + c) - 6*b^2*d^2*x^5*e^(d*x + c) + a^2*d^5*x^2*e^(-d*x - c) + 6*b^2*
d^2*x^5*e^(-d*x - c) - 12*a*b*d^3*x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^3*x^3*Ei(d
*x)*e^c + a^2*d^4*x*e^(d*x + c) + 12*b^2*d*x^4*e^(d*x + c) - a^2*d^4*x*e^(-
d*x - c) + 12*b^2*d*x^4*e^(-d*x - c) + 2*a^2*d^3*e^(d*x + c) - 12*b^2*x^3*e
^(d*x + c) + 2*a^2*d^3*e^(-d*x - c) + 12*b^2*x^3*e^(-d*x - c))/(d^3*x^3)
```

3.93 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$

Optimal. Leaf size=167

$$\frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2c}{}$$

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(4*x^4) - (a^2*d^2*Cosh[c
+ d*x])/(24*x^2) - (2*a*b*Cosh[c + d*x])/x + (a^2*d^4*Cosh[c]*CoshIntegral[
d*x])/24 + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(12*x^
3) - (a^2*d^3*Sinh[c + d*x])/(24*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*d*Cos
h[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rubi [A] time = 0.30798, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{24}a^2d^4\cosh(c)\mathrm{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\mathrm{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d\sinh(c+dx)}{12x^3}-\frac{a^2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] -((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(4*x^4) - (a^2*d^2*Cosh[c
+ d*x])/(24*x^2) - (2*a*b*Cosh[c + d*x])/x + (a^2*d^4*Cosh[c]*CoshIntegral[
d*x])/24 + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(12*x^
3) - (a^2*d^3*Sinh[c + d*x])/(24*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*d*Cos
h[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Rule 5287

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2\cosh(c+dx)}{x^5}\,dx = \int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^2}+b^2x\cosh(c+dx)\right)dx$$
$$= a^2\int \frac{\cosh(c+dx)}{x^5}\,dx + (2ab)\int \frac{\cosh(c+dx)}{x^2}\,dx + b^2\int x\cosh(c+dx)\,dx$$
$$= -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{x} + \frac{b^2x\sinh(c+dx)}{d} - \frac{b^2\int \sinh(c+dx)\,dx}{d} + \frac{1}{4}\left(a\right.$$
$$= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{2ab\cosh(c+dx)}{x} - \frac{a^2d\sinh(c+dx)}{12x^3} + \frac{b^2x\sinh(}{d}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{2ab\cosh(c+dx)}{x} + 2abd\mathrm{Ch}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{2ab\cosh(c+dx)}{x} + 2abd\mathrm{Ch}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{2ab\cosh(c+dx)}{x} + 2abd\mathrm{Ch}$$
$$= -\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} - \frac{2ab\cosh(c+dx)}{x} + \frac{1}{24}a^2d^4$$

Mathematica [A] time = 0.539467, size = 150, normalized size = 0.9

$$\frac{1}{24}\left(-\frac{a^2d^2\cosh(c+dx)}{x^2} - \frac{a^2d^3\sinh(c+dx)}{x} - \frac{2a^2d\sinh(c+dx)}{x^3} - \frac{6a^2\cosh(c+dx)}{x^4} + ad\mathrm{Chi}(dx)\left(ad^3\cosh(c)+48b\,\mathrm{si}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

```
[Out] ((-24*b^2*Cosh[c + d*x])/d^2 - (6*a^2*Cosh[c + d*x])/x^4 - (a^2*d^2*Cosh[c
+ d*x])/x^2 - (48*a*b*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*(a*d^3*Cosh[
c] + 48*b*Sinh[c]) - (2*a^2*d*Sinh[c + d*x])/x^3 - (a^2*d^3*Sinh[c + d*x])/
x + (24*b^2*x*Sinh[c + d*x])/d + a*d*(48*b*Cosh[c] + a*d^3*Sinh[c])*SinhInt
egral[d*x])/24
```

Maple [A] time = 0.174, size = 292, normalized size = 1.8

$$-\frac{b^2e^{-dx-c}x}{2d} - \frac{d^4a^2e^{-c}\mathrm{Ei}(1,dx)}{48} + \frac{d^3a^2e^{-dx-c}}{48x} - \frac{b^2e^{-dx-c}}{2d^2} - \frac{a^2d^2e^{-dx-c}}{48x^2} + \frac{da^2e^{-dx-c}}{24x^3} - \frac{a^2e^{-dx-c}}{8x^4} - \frac{abe^{-dx-c}}{x} + dabe^{-c}\mathrm{Ei}(1,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*cosh(d*x+c)/x^5,x)
```

```
[Out] -1/2*b^2/d*exp(-d*x-c)*x-1/48*d^4*a^2*exp(-c)*Ei(1,d*x)+1/48*d^3*a^2*exp(-d
*x-c)/x-1/2*b^2/d^2*exp(-d*x-c)-1/48*d^2*a^2*exp(-d*x-c)/x^2+1/24*d*a^2*exp
(-d*x-c)/x^3-1/8*a^2*exp(-d*x-c)/x^4-a*b*exp(-d*x-c)/x+d*a*b*exp(-c)*Ei(1,d
*x)-1/48*d^4*a^2*exp(c)*Ei(1,-d*x)-1/2*b^2/d^2*exp(d*x+c)-1/48*d^2*a^2/x^2*
exp(d*x+c)-1/48*d^3*a^2/x*exp(d*x+c)-1/8*a^2/x^4*exp(d*x+c)-1/24*d*a^2/x^3*
exp(d*x+c)-d*a*b*exp(c)*Ei(1,-d*x)+1/2*b^2/d*exp(d*x+c)*x-a*b/x*exp(d*x+c)
```

Maxima [A] time = 1.24745, size = 208, normalized size = 1.25

$$\frac{1}{8}\left(a^2d^3e^{(-c)}\Gamma\left(-3,dx\right)+a^2d^3e^c\Gamma\left(-3,-dx\right)-8\,ab\text{Ei}\left(-dx\right)e^{(-c)}+8\,ab\text{Ei}\left(dx\right)e^c-\frac{2\left(d^2x^2e^c-2\,dxe^c+2\,e^c\right)b^2e^{(dx)}}{d^3}-\frac{2(\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*d^3*e^(-c)*gamma(-3, d*x) + a^2*d^3*e^c*gamma(-3, -d*x) - 8*a*b*Ei
(-d*x)*e^(-c) + 8*a*b*Ei(d*x)*e^c - 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b^2
*e^(d*x)/d^3 - 2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3)*d + 1/4*(2*b^2
*x^2 - (8*a*b*x^3 + a^2)/x^4)*cosh(d*x + c)
```

Fricas [A] time = 1.80379, size = 435, normalized size = 2.6

$$-\frac{2\left(a^2d^4x^2+48\,abd^2x^3+24\,b^2x^4+6\,a^2d^2\right)\cosh\left(dx+c\right)-\left(\left(a^2d^6+48\,abd^3\right)x^4\text{Ei}\left(dx\right)+\left(a^2d^6-48\,abd^3\right)x^4\text{Ei}\left(-dx\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*(a^2*d^4*x^2 + 48*a*b*d^2*x^3 + 24*b^2*x^4 + 6*a^2*d^2)*cosh(d*x +
 c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) + (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-
d*x))*cosh(c) + 2*(a^2*d^5*x^3 - 24*b^2*d*x^5 + 2*a^2*d^3*x)*sinh(d*x + c)
- ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) - (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-d*x)
)*sinh(c))/(d^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^2\cosh\left(c+dx\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**3)**2*cosh(c + d*x)/x**5, x)
```

Giac [B] time = 1.33292, size = 425, normalized size = 2.54

$$\frac{a^2d^6x^4\mathrm{Ei}\left(-dx\right)e^{(-c)} + a^2d^6x^4\mathrm{Ei}\left(dx\right)e^c - a^2d^5x^3e^{(dx+c)} + a^2d^5x^3e^{(-dx-c)} - 48\,abd^3x^4\mathrm{Ei}\left(-dx\right)e^{(-c)} + 48\,abd^3x^4\mathrm{Ei}\left(dx\right)e^c - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

```
[Out] 1/48*(a^2*d^6*x^4*Ei(-d*x)*e^(-c) + a^2*d^6*x^4*Ei(d*x)*e^c - a^2*d^5*x^3*e
^(d*x + c) + a^2*d^5*x^3*e^(-d*x - c) - 48*a*b*d^3*x^4*Ei(-d*x)*e^(-c) + 48
*a*b*d^3*x^4*Ei(d*x)*e^c - a^2*d^4*x^2*e^(d*x + c) + 24*b^2*d*x^5*e^(d*x +
c) - a^2*d^4*x^2*e^(-d*x - c) - 24*b^2*d*x^5*e^(-d*x - c) - 48*a*b*d^2*x^3*
e^(d*x + c) - 48*a*b*d^2*x^3*e^(-d*x - c) - 2*a^2*d^3*x*e^(d*x + c) - 24*b^
2*x^4*e^(d*x + c) + 2*a^2*d^3*x*e^(-d*x - c) - 24*b^2*x^4*e^(-d*x - c) - 6*
a^2*d^2*e^(d*x + c) - 6*a^2*d^2*e^(-d*x - c))/(d^2*x^4)
```

3.94 $\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=373

$$\frac{(-1)^{2/3}a^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}}+a^{2/3}\cosh$$

```
[Out] -(Cosh[c + d*x]/(b*d^2)) + ((-1)^(2/3)*a^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)
) - ((-1)^(1/3)*a^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b^(5/3)) + (a^(2/3)*Cosh[c
- (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
 + (x*Sinh[c + d*x])/(b*d) - ((-1)^(2/3)*a^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/
3)) + (a^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```

Rubi [A] time = 0.937188, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3296, 2638, 3303, 3298, 3301}

$$\frac{(-1)^{2/3}a^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}-\frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}}+a^{2/3}\cosh$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```



```
[Out] -(Cosh[c + d*x]/(b*d^2)) + ((-1)^(2/3)*a^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/3)
) - ((-1)^(1/3)*a^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b^(5/3)) + (a^(2/3)*Cosh[c
- (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
 + (x*Sinh[c + d*x])/(b*d) - ((-1)^(2/3)*a^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/
3)) + (a^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(5/3))
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx = \int \left(\frac{x\cosh(c+dx)}{b} - \frac{ax\cosh(c+dx)}{b\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int x\cosh(c+dx)\,dx}{b} - \frac{a\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{b}$$

$$= \frac{x\sinh(c+dx)}{bd} - \frac{a\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b} - \frac{\int \sin\ldots}{}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} + \frac{a^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\right)\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b^{4/3}} + \frac{\left((-1)^{2/\ldots}\right.}{}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd} + \frac{\left(a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}a^{2/3}\cosh\left(c\ldots\right.\right.}{}$$

$$= -\frac{\cosh(c+dx)}{bd^2} + \frac{(-1)^{2/3}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\ldots}{\sqrt[3]{b}}\right.}{3b\ldots}$$

Mathematica [C] time = 0.462271, size = 213, normalized size = 0.57

$$-\frac{ad^2\text{RootSum}\left[\#1^3b+a\&,\frac{-\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\sinh(\#1d+c)\text{Shi}(d(x-\#1))-\cosh(\#1d+c)\text{Shi}(d(x-\#1))}{\#1}\&\right]+ad\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(a*d^2*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x -
 #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + a*d^2*RootSum[a +
 b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x -
```

```
#1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*
#1]*SinhIntegral[d*(x - #1)])/#1 & ] + 6*b*(Cosh[c + d*x] - d*x*Sinh[c + d*
x]))/(6*b^2*d^2)
```

Maple [C] time = 0.087, size = 925, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b*x-1/2/d^2*exp(-d*x-c)/b-1/6/d^2/b^2*sum((6*_R1^2*b*c^2
-_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(
1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*
c^4/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-1/d^2*c^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c
),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*sum((
3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x
-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*exp(d
*x+c)*x-1/2/d^2/b*exp(d*x+c)-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1
*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1
=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*c^4/b*sum(1/(_R1
^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(
1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/d^2*c
^2/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*sum((3*_R1^2*b*c-3*_R
1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13585, size = 2627, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
```

```
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(
(sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(
a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((s
qrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
 - c) + (a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x +
 c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(2
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*d*x*sinh(d*x + c
) - 12*cosh(d*x + c))/(b*d^2*cosh(d*x + c)^2 - b*d^2*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx+c)}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")

[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a), x)
```

3.95 $\int \frac{x^3 \cosh(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=358

$$\frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\cosh\left(c\right.}{}$$

```
[Out] ((-1)^(1/3)*a^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*Cos
h[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*Cosh
Integral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) + Sinh[c + d*x]/(b*d) - ((
-1)^(1/3)*a^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^
(2/3)*a^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rubi [A] time = 0.648427, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 2637, 5281, 3303, 3298, 3301}

$$\frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\cosh\left(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] ((-1)^(1/3)*a^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*Cos
h[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*Cosh
Integral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) + Sinh[c + d*x]/(b*d) - ((
-1)^(1/3)*a^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) - ((-1)^
(2/3)*a^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx &= \int \left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b\left(a+bx^3\right)}\right)dx\\
&= \frac{\int \cosh(c+dx)\,dx}{b}-\frac{a\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{b}\\
&= \frac{\sinh(c+dx)}{bd}-\frac{a\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{b}\\
&= \frac{\sinh(c+dx)}{bd}+\frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b}+\frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3b}+\frac{\sqrt[3]{a}\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3b}\\
&= \frac{\sinh(c+dx)}{bd}+\frac{\left(\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3b}+\frac{\left(\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}\ldots}{\sqrt[3]{b}}\ldots\right)}{-\sqrt[3]{a}-(-1)^{2/3}\ldots}}{3b}\\
&= \frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}\ldots}{\sqrt[3]{b}}\ldots\right)}{3b^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.259845, size = 198, normalized size = 0.55

$$
-\frac{ad\,\mathrm{RootSum}\left[\#1^3b+a\&,\frac{-\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\sinh(\#1d+c)\mathrm{Shi}(d(x-\#1))-\cosh(\#1d+c)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right]+ad\ldots}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -(a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #
1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + a*d*RootSum[a + b
```

```
*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)])/#1^2 & ] - 6*b*Sinh[c + d*x])/(6*b^2*d)
```

Maple [C] time = 0.036, size = 671, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_
R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1
,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d*c^2
/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/2/d/b*exp(d*x+c)-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3
)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*
c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*E
i(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d
*c^2/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*sum(_R1^2/(_R1^2-2*_R1*c+
c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20258, size = 2591, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c
)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(
(sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x + 1
```

```
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(
a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c
)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((s
qrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
 - c) + (a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x +
 c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*sinh(d*x + c))/(
b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh (c + dx)}{a + bx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh (dx + c)}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a), x)
```

3.96 $\int \frac{x^2 \cosh(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=283

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\cdots}{\sqrt[3]{\cdots}}\right.}{3b}$$

```
[Out] (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(3*b) + (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIn
tegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b) + (Cosh[c - (a^(1/3)
*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b) - (Sinh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(3*b) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b) + (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rubi [A] time = 0.455137, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}\cdots}{\sqrt[3]{\cdots}}\right.}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*
d)/b^(1/3) - d*x])/(3*b) + (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIn
tegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b) + (Cosh[c - (a^(1/3)
*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b) - (Sinh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(3*b) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*b) + (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx = \int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$=\frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}$$

$$=\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{2/3}}+\frac{\cosh\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{}}\right.}{}$$

$$=\frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c\right.}{}$$

Mathematica [C] time = 0.225213, size = 170, normalized size = 0.6

$$\text{RootSum}\left[\#1^3b+a\&,-\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\sinh(\#1d+c)\text{Shi}(d(x-\#1))-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInteg
ral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] + RootSum[a + b*#1^3 & , Cosh[c
 + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]
 + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*
(x - #1)] & ])/(6*b)
```

Maple [C] time = 0.032, size = 423, normalized size = 1.5

$$-\frac{1}{6b}\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-bc^3\right)}\frac{\_R1^2e^{-\_R1}\text{Ei}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}-\frac{c^2}{6b}\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/6/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*c^2/b*sum(1/(_R1^2-2*_R1*c+c^
2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))+1/3*c/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=R
```

```
ootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*sum(_R1^2/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))-1/6*c^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c
),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3*c/b*sum(_R1/(_R
1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(dx^2e^{(2c)} + xe^{(2c)}\right)e^{(dx)} - \left(dx^2 - x\right)e^{(-dx)}}{2\left(bd^2x^3e^c + ad^2e^c\right)} + \frac{1}{2}\int \frac{\left(2bx^3e^c - 3adxe^c - ae^c\right)e^{(dx)}}{b^2d^2x^6 + 2abd^2x^3 + a^2d^2}\,dx + \frac{1}{2}\int \frac{\left(2bx^3 + 3adx - a\right)e^{(-dx)}}{b^2d^2x^6e^c + 2abd^2x^3e^c + a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b*d^2*x^3
*e^c + a*d^2*e^c) + 1/2*integrate((2*b*x^3*e^c - 3*a*d*x*e^c - a*e^c)*e^(d*
x)/(b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2), x) + 1/2*integrate((2*b*x^3 + 3
*a*d*x - a)*e^(-d*x)/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c), x
)
```

Fricas [B] time = 1.95249, size = 1322, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c
) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + Ei(d*x + (a*d^
3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c)
 - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))
/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\cosh(c + dx)}{a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x**2*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a), x)
```

3.97 $\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$

Optimal. Leaf size=345

$$-\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\cdots}{3\sqrt[3]{a}\cdots}$$

```
[Out] -((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(3*a^(1/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Sinh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(3*a^(1/3)*b^(2/3)) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x
])/(3*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.411566, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5293, 3303, 3298, 3301}

$$-\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\cdots}{3\sqrt[3]{a}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] -((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(3*a^(1/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Sinh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(3*a^(1/3)*b^(2/3)) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x
])/(3*a^(1/3)*b^(2/3))
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\left((-1)^{2/3}\right.}{}$$

$$= -\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-d\right.}{3\sqrt[3]{a}b^{2/3}}$$

Mathematica [C] time = 0.210721, size = 180, normalized size = 0.52

$$\frac{\operatorname{RootSum}\left[\#1^3 b+a\&,\frac{-\sinh(\#1d+c)\operatorname{Chi}(d(x-\#1))+\cosh(\#1d+c)\operatorname{Chi}(d(x-\#1))+\sinh(\#1d+c)\operatorname{Shi}(d(x-\#1))-\cosh(\#1d+c)\operatorname{Shi}(d(x-\#1))}{\#1}\&\right]+\operatorname{RootS}}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
 Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + RootSum[a + b*#1^3 & , (
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c +
 d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInteg
ral[d*(x - #1)])/#1 & ])/(6*b)
```

Maple [C] time = 0.027, size = 280, normalized size = 0.8

$$-\frac{d}{6b}\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Z bc^2+ad^3-bc^3\right)}\frac{\_R1\,e^{-\_R1}\operatorname{Ei}(1,dx-\_R1+c)}{\_R1^2-2\_R1c+c^2}+\frac{cd}{6b}\sum_{\_R1=\operatorname{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Z bc^2+ad^3-bc^3\right)}\frac{e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d*c/b*sum(1/(_R1^2-2*_R1*c+c^
2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))-1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=R
ootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d*c/b*sum(1/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13832, size = 1773, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) - (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(2/3)*(s
qrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cosh(dx+c)}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a), x)
```

3.98 $\int \frac{\cosh(c+dx)}{a+bx^3} \, dx$

Optimal. Leaf size=345

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/}}$$

```
[Out] -((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Sinh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x
])/(3*a^(2/3)*b^(1/3))
```

Rubi [A] time = 0.401321, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5281, 3303, 3298, 3301}

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^3),x]
```

```
[Out] -((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegra
l[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Sinh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x
])/(3*a^(2/3)*b^(1/3))
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right) dx$$

$$= -\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}}$$

$$= -\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}d}{\sqrt[3]{b}}-idx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{2/3}} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{\ldots}}{\sqrt[3]{b}}\right.}{\ldots}$$

$$= -\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right.}{3a^{2/3}\sqrt[3]{b}}$$

Mathematica [C] time = 0.135678, size = 180, normalized size = 0.52

$$\frac{\mathrm{RootSum}\left[\#1^3 b+a\&,\frac{-\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\sinh(\#1d+c)\mathrm{Shi}(d(x-\#1))-\cosh(\#1d+c)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right]+\mathrm{RootS}\ldots}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^3),x]
```

```
[Out] (RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] +
 Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + RootSum[a + b*#1^3 & ,
 (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c
 + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)])/#1^2 & ])/(6*b)
```

Maple [C] time = 0.022, size = 143, normalized size = 0.4

$$-\frac{d^2}{6\,b}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Z\,bc^2+ad^3-bc^3\right)}\frac{\mathrm{e}^{-\_R1}\mathrm{Ei}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1\,c+c^2}-\frac{d^2}{6\,b}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Z\,bc^2+ad^3-bc^3\right)}\frac{\mathrm{e}^{\_R1}\mathrm{E}\ldots}{\_R\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^3+a),x)
```

```
[Out] -1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^
```

```
2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04197, size = 1770, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")

[Out] -1/12*((a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) + (-a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(1/3)*(s
qrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*(sq
rt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3
))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))
*sinh(-c + (a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (c + dx)}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**3+a),x)

[Out] Integral(cosh(c + d*x)/(a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (dx + c)}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^3 + a), x)
```

3.99 $\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=303

$$-\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\ldots\right.}{3a}$$

```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*C
oshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a)
 - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(3*a)
```

Rubi [A] time = 0.523232, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5293, 3303, 3298, 3301}

$$-\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\ldots\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```


```
[Out] (Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*C
oshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/
3) + d*x])/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/3)*a^
(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a)
 - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(3*a)
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx^2\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a} - \frac{b\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{b\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{\cosh(c)\int \frac{\cosh(dx)}{x}\,dx}{a} + \frac{\sinh(c}{}$$

$$= \frac{\cosh(c)\mathbf{Chi}(dx)}{a} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{\cosh(c+dx}{(-1)^{2/3}\sqrt[3]{a}+}}{3a}$$

$$= \frac{\cosh(c)\mathbf{Chi}(dx)}{a} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a} - \frac{\left(\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a} - \frac{\left(\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}}{}\right.\right.}{}$$

$$= \frac{\cosh(c)\mathbf{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathbf{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathbf{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{3a}$$

Mathematica [C] time = 0.267678, size = 186, normalized size = 0.61

$$-\frac{\text{RootSum}\left[\#1^3b+a\&,-\sinh(\#1d+c)\mathbf{Chi}(d(x-\#1))+\cosh(\#1d+c)\mathbf{Chi}(d(x-\#1))+\sinh(\#1d+c)\mathbf{Shi}(d(x-\#1))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```

```
[Out] -(-6*Cosh[c]*CoshIntegral[d*x] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*Cosh
Integral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d
*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ]
 + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshInt
egral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
+ Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] - 6*Sinh[c]*SinhIntegral[d*x]
)/(6*a)
```

Maple [C] time = 0.036, size = 138, normalized size = 0.5

$$-\frac{e^{-c}\text{Ei}(1,dx)}{2a} + \frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-bc^3\right)} e^{-\_R1}\text{Ei}(1,dx-\_R1+c)}{6a} - \frac{e^{c}\text{Ei}(1,-dx)}{2a} + \frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^3+a),x)
```

```
[Out] -1/2/a*exp(-c)*Ei(1,d*x)+1/6/a*sum(exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a*exp(c)*Ei(1,-d*x)+1/6/a*sum(e
xp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (dx + c)}{\left(bx^3 + a\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Fricas [B] time = 2.04368, size = 1412, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)
*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(
-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) +
c) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) +
Ei(-d*x))*cosh(c) + Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) +
Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
 - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - E
i(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d*x) - Ei(-d*
x))*sinh(c) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/a
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^3+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

3.100 $\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=381

$$\frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cosh\left(c\right.\cdots}{\cdots}$$

```
[Out] -(Cosh[c + d*x]/(a*x)) + ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3))
- ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral
[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) +
 (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - ((-1)^
(2/3)*b^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3
)*b^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))
```

Rubi [A] time = 0.600099, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5293, 3297, 3303, 3298, 3301}

$$\frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cosh\left(c\right.\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] -(Cosh[c + d*x]/(a*x)) + ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3))
- ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral
[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) +
 (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - ((-1)^
(2/3)*b^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3
)*b^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)
*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{a} - \frac{b\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{ax} - \frac{b\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{d\int \frac{\sinh}{\ldots}}{\ldots}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{b^{2/3}\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}b^{2/3}\right)\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{4/3}} + \frac{\left((-1)^{2/3}b^{2/3}\right)\int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x}\,d\ldots}{3a^{4/3}}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{d\,\mathrm{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a} + \frac{\left(b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}}$$

$$= -\frac{\cosh(c+dx)}{ax} + \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\ldots}{3a^{4/3}}$$

Mathematica [C] time = 0.406599, size = 215, normalized size = 0.56

$$-\frac{x\mathrm{RootSum}\left[\#1^3 b+a\&,\frac{-\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\sinh(\#1d+c)\mathrm{Shi}(d(x-\#1))-\cosh(\#1d+c)\mathrm{Shi}(d(x-\#1))}{\#1}\&\right]+x\mathrm{Ro}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

```
[Out] -(6*Cosh[c + d*x] + x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d
*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + x*R
ootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegr
al[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + S
```

```
inh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] - 6*d*x*CoshIntegral[d*x]*Si
nh[c] - 6*d*x*Cosh[c]*SinhIntegral[d*x])/(6*a*x)
```

Maple [C] time = 0.047, size = 187, normalized size = 0.5

$$-\frac{e^{-dx-c}}{2ax}+\frac{de^{-c}\mathrm{Ei}(1,dx)}{2a}+\frac{d}{6a}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-bc^3\right)}\frac{e^{-\_R1}\mathrm{Ei}(1,dx-\_R1+c)}{\_R1-c}-\frac{e^{dx+c}}{2ax}-\frac{de^{c}\mathrm{Ei}(1,-}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^3+a),x)
```

```
[Out] -1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/6*d/a*sum(1/(_R1-c)*exp(-_
R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1
/2/a/x*exp(d*x+c)-1/2*d/a*exp(c)*Ei(1,-d*x)+1/6*d/a*sum(1/(_R1-c)*exp(_R1)*
Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18902, size = 2921, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*d^2*cosh(d*x + c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(
d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3
/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh
(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^
3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^
3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sin
h(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d
^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c
)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)
) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(2/3)*Ei(d*x +
(a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*
x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1)
```

```
+ c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b
*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*s
inh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b
*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x +
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
 - c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*
b*x + b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*
sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x*cosh(d*x + c)^2 - b*
x*sinh(d*x + c)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-
a*d^3/b)^(1/3)) + 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(
2/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 6*(a*d^3*x*Ei(d
*x) - a*d^3*x*Ei(-d*x))*cosh(c) - 6*(a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*si
nh(c))/(a^2*d^2*x*cosh(d*x + c)^2 - a^2*d^2*x*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^3+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^2), x)
```

3.101 $\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal. Leaf size=410

$$\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{b^{2/3}\cosh(\dots}{\dots}$$

```
[Out] -Cosh[c + d*x]/(2*a*x^2) + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) + ((-1)^(1
/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - (d*Sinh[c + d*x])/(2*a*x) + (d^2
*Sinh[c]*SinhIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Sinh[c + ((-1)^(1/3
)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
3*a^(5/3)) - (b^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Sinh[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^
(5/3))
```

Rubi [A] time = 0.639085, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{b^{2/3}\cosh(\dots}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```



```
[Out] -Cosh[c + d*x]/(2*a*x^2) + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) + ((-1)^(1
/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - (d*Sinh[c + d*x])/(2*a*x) + (d^2
*Sinh[c]*SinhIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Sinh[c + ((-1)^(1/3
)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
3*a^(5/3)) - (b^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Sinh[c - ((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^
(5/3))
```


Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^3\right)}\,dx = \int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$= \frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{a} - \frac{b\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{a} + \frac{d\int \frac{\sinh(c+dx)}{x^2}}{2a}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{3a^{5/3}} + \frac{b\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{3a^{5/3}}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d\sinh(c+dx)}{2ax} + \frac{\left(d^2\cosh(c)\right)\int \frac{\cosh(dx)}{x}\,dx}{2a} + \frac{\left(b\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}}{3a^{5/3}}$$

$$= -\frac{\cosh(c+dx)}{2ax^2} + \frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} + \frac{\sqrt[3]{-1}b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3}}{}$$

Mathematica [C] time = 0.37738, size = 237, normalized size = 0.58

$$-\frac{x^2\mathrm{RootSum}\left[\#1^3b+a\&,\frac{-\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\sinh(\#1d+c)\mathrm{Shi}(d(x-\#1))-\cosh(\#1d+c)\mathrm{Shi}(d(x-\#1))}{\#1^2}\&\right]+x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

```
[Out] -(3*Cosh[c + d*x] - 3*d^2*x^2*Cosh[c]*CoshIntegral[d*x] + x^2*RootSum[a + b
*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)])/#1^2 & ] + x^2*RootSum[a + b*#1^3 & , (Cosh[c +
 d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] +
 Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x
 - #1)])/#1^2 & ] + 3*d*x*Sinh[c + d*x] - 3*d^2*x^2*Sinh[c]*SinhIntegral[d*
x])/(6*a*x^2)
```

Maple [C] time = 0.054, size = 240, normalized size = 0.6

$$\frac{d\mathrm{e}^{-dx-c}}{4\,ax}-\frac{\mathrm{e}^{-dx-c}}{4\,ax^{2}}-\frac{d^{2}\mathrm{e}^{-c}\mathrm{Ei}\left(1,dx\right)}{4\,a}+\frac{d^{2}}{6\,a}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^{3}-3\_Z^{2}bc+3\_Z\,bc^{2}+ad^{3}-bc^{3}\right)}\frac{\mathrm{e}^{-\_R1}\mathrm{Ei}\left(1,dx-\_R1+c\right)}{\_R1^{2}-2\_R1\,c+c^{2}}-\frac{d\mathrm{e}^{dx+c}}{4\,ax}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^3/(b*x^3+a),x)
```

```
[Out] 1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2-1/4*d^2/a*exp(-c)*Ei(1,d*x)+1/6
*d^2/a*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/4*d/a/x*exp(d*x+c)-1/4/a/x^2*exp(d*
x+c)-1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/6*d^2/a*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R
1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13039, size = 3071, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*d^2*x*sinh(d*x + c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*
cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*
x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(1/3)*((sqrt(-3
)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2
```

```
)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)
^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x^2
*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^
3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*s
inh(d*x + c)^2)*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/
b)^(1/3)) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sq
rt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(1/3)*((
sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x
 + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-
a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2
- b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*si
nh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x^2*cosh(d*x + c)^2 - b*
x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c +
(-a*d^3/b)^(1/3)) - 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(a*d^
3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 6*a*d*cos
h(d*x + c) - 3*(a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c) - 3*(a*d^3*
x^2*Ei(d*x) - a*d^3*x^2*Ei(-d*x))*sinh(c))/(a^2*d*x^2*cosh(d*x + c)^2 - a^2
*d*x^2*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**3/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{\left(bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^3), x)
```

3.102 $\int \frac{x^3\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=718

result too large to display

```
[Out] -(x*Cosh[c + d*x])/(3*b*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(
1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(
2/3)*b^(4/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIn
tegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(2/3)*b^(4/3)) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2
/3)*b^(4/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)
*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) - ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1
/3)*b^(5/3)) + ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
 - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + ((-
1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3
)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*Sinh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(9*a^(2/3)*b^(4/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (Sinh[c - (a^(1/3)*d)/b^(1
/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)^(
1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^
(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Sinh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^(2/3)*b^(4/3))
```

Rubi [A] time = 1.08119, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5291, 5281, 3303, 3298, 3301, 5292}

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```


```
[Out] -(x*Cosh[c + d*x])/(3*b*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(
1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(
2/3)*b^(4/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIn
tegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(2/3)*b^(4/3)) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2
/3)*b^(4/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)
*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) - ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1
/3)*b^(5/3)) + ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
 - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + ((-
1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3
)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*Sinh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(9*a^(2/3)*b^(4/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (Sinh[c - (a^(1/3)*d)/b^(1
/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)^(
```

```
1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^
(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Sinh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]
)/(9*a^(2/3)*b^(4/3))
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^2}\,dx = -\frac{x\cosh(c+dx)}{3b(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{3b} + \frac{d\int \frac{x\sinh(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b} + \frac{d\int\Big(}{}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}}{9\sqrt[3]{a}b^{4/3}}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9\sqrt[3]{a}b^{4/3}}$$

$$= -\frac{x\cosh(c+dx)}{3b(a+bx^3)} - \frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{9a^{2/3}}$$

Mathematica [C] time = 0.335862, size = 363, normalized size = 0.51

$$-\mathrm{RootSum}\left[\#1^3 b + a\&, \frac{\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1)) - \#1d\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1)) - \cosh(\#1d+c)\mathrm{Chi}(d(x-\#1)) + \#1d\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1)) - \mathrm{si}}{\#1^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*x*Cosh[c + d*x])/(a + b*x^3) - RootSum[a + b*#1^3 & , (-(Cosh[c + d*
#1]*CoshIntegral[d*(x - #1)]) + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d*#1]*SinhIntegral[d*(x -
 #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d
*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3
& , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sin
h[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ])/(18*b
^2)
```

Maple [C] time = 0.15, size = 877, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)*x-1/18/d/a/b^2*sum((3*_R1^2*b*c^2-
_R1*a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))+1/18/d*c^3/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*E
```

```
i(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*
c^2/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+
c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c/a/b^2*sum(
(2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R
1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/
6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)*x+1/18/d/a/b^2*sum((3*_R1^2*b*c^2-_R1*
a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c
+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))-1/18/d*c^3/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-
d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^2/
a/b*sum((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*c/a/b^2*sum((2*_
R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei
(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.40792, size = 5017, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
 a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
```

```
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
 a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3
))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c
)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-
a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b*x^3 +
sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) +
 a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b*x^3 + sqrt(-
3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*si
nh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^
2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)
*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a
*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b*x^3 + a)
*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 +
 a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3
))*sinh(-c + (a*d^3/b)^(1/3)))/((a*b^2*d*x^3 + a^2*b*d)*cosh(d*x + c)^2 - (
a*b^2*d*x^3 + a^2*b*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx+c)}{\left(bx^3+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

3.103 $\int \frac{x^2 \cosh(c+dx)}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=373

$$\frac{d \sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{\sqrt[3]{-1}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}d \sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(-x\right.}{9a^{2/3}b^{4/3}}$$

```
[Out] -Cosh[c + d*x]/(3*b*(a + b*x^3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*
x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshI
ntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*
d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^
(2/3)*b^(4/3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Sin
hIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + (d*C
osh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^
(2/3)*b^(4/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sin
hIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.601165, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5289, 5280, 3303, 3298, 3301}

$$\frac{d \sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}-\frac{\sqrt[3]{-1}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}+\frac{(-1)^{2/3}d \sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right) \text{Chi}\left(-x\right.}{9a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] -Cosh[c + d*x]/(3*b*(a + b*x^3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*
x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshI
ntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*
d)/b^(1/3)])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^
(2/3)*b^(4/3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Sin
hIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + (d*C
osh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^
(2/3)*b^(4/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sin
hIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

```
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx = -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} + \frac{d\int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,dx}{9a^{2/3}b}$$

$$= -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{2/3}b} - \frac{\left(id\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sin\left(\frac{(-1)^{5/6}\ldots}{\sqrt[3]{b}}\ldots\right)}{-\sqrt[3]{a}-(-1)\ldots}}{9a^{2/3}b}$$

$$= -\frac{\cosh(c+dx)}{3b\left(a+bx^3\right)} + \frac{d\,\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}d\,\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\ldots}{\ldots}\right)}{9a^{2/3}b^{4/3}}$$

Mathematica [C] time = 0.159526, size = 203, normalized size = 0.54

$$-d\,\text{RootSum}\left[\#1^3b+a\&,\frac{-\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\sinh(\#1d+c)\text{Shi}(d(x-\#1))-\cosh(\#1d+c)\text{Shi}(d(x-\#1))}{\#1^2}\&\right]+d\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*Cosh[c + d*x])/(a + b*x^3) - d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh
[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1
)])/#1^2 & ] + d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x -
 #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegr
```

```
al[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/(18*b^2
)
```

Maple [C] time = 0.044, size = 594, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)-1/18/a/b^2*sum((2*_R1^2*b*c-3*_R1*
b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),
_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*c^2/a/b*sum((_R1
-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*c/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)+1/18/a/b^2*sum((2*_R1
^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1
,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*c^2
/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c/a/b*sum((_R1^2-_R1*c-_R1-
c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.22021, size = 3200, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*((a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*
d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 +
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(
1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
- 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(1/3)*((b*x
```

```
^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 +
 a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1/3)*((b*x^3 +
a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3
))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c
)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d
^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x
 + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 -
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*
d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 -
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/
3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*
(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3)
- 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*((
b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3
/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*((b*x^3 + a)*cosh
(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-
c + (a*d^3/b)^(1/3)) + 12*a*cosh(d*x + c))/((a*b^2*x^3 + a^2*b)*cosh(d*x +
c)^2 - (a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx+c)}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

3.104 $\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=695

$$-\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ch}}{9a^{4/3}b^{2}}$$

```
[Out] Cosh[c + d*x]/(3*a*b*x) - Cosh[c + d*x]/(3*b*x*(a + b*x^3)) - ((-1)^(2/3)*C
osh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*
a^(4/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a*b) - (d*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b)
 - (d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b) + (d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a*b) + ((-1)^(
2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3
)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d
*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/
3))
```

Rubi [A] time = 1.32363, antiderivative size = 695, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292}

$$-\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ch}}{9a^{4/3}b^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x) - Cosh[c + d*x]/(3*b*x*(a + b*x^3)) - ((-1)^(2/3)*C
osh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)
/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*
a^(4/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a*b) - (d*CoshIntegral[((-1)^(1/3)*
a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b)
 - (d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b) + (d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a*b) + ((-1)^(
2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^(2/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3
)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a*b) - (Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/3)) - (d
```

```
*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*
d)/b^(1/3) + d*x])/(9*a*b) + ((-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^(2/
3))
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx = -\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{3b} + \frac{d\int \frac{\sinh(c+dx)}{x\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax} - \frac{bx^2\sinh(c+dx)}{a\left(a+bx^3\right)}\right)dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{3a} - \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{3ab} - \frac{d\int \frac{x^2\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\sinh(c+dx)}{x}\,dx}{3ab}$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)} + \frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right.}{3a}$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} + \frac{d\,\text{Chi}(dx)\sinh(c)}{3ab} + \frac{d\cosh(c)\text{Shi}(dx)}{3ab} - \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}}{\ldots}$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\cosh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\ldots\right)}{\sqrt[3]{a}+\ldots}}{9ab^{2/3}}$$

$$= \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.188285, size = 387, normalized size = 0.56

$$\frac{\left(a+bx^3\right)\text{RootSum}\left[\#1^3b+a\&,\frac{-\sinh(\#1d+c)\text{Chi}(d(x-\#1))-\#1d\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1d\cosh(\#1d+c)\text{Chi}(d(x-\ldots}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] (6*b*x^2*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[
c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)
] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sin
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ] - (a + b*x^3)*RootSum[a + b
*#1^3 & , (-(Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) - CoshIntegral[d*(x -
 #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d
*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#
1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ]
)/(18*a*b*(a + b*x^3))
```

Maple [C] time = 0.037, size = 395, normalized size = 0.6

$$\frac{d^3e^{-dx-c}x^2}{6a\left(bd^3x^3+ad^3\right)} - \frac{d}{18ab}\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-bc^3\right)}\frac{\left(\_R1^2-\_R1c+\_R1+c\right)e^{-\_R1}\text{Ei}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2} + \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x^2/a/(b*d^3*x^3+a*d^3)-1/18*d/a/b*sum((_R1^2-_R1*c+_R1
+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d*c/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/6*d^3*exp(d*x+c)*x^2/a/(b*d^3*x^3+a*d^3)+1/18*d/a/b*sum((_R1^2-_R1*
c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*d*c/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c
+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39469, size = 4891, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b*d^2*x^2*cosh(d*x + c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x +
c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^
3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt
(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (2*(a*b*d^3*x^3 +
a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*
d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 -
(b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) -
 c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3
)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a
*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x
+ c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1
/3)*(sqrt(-3) - 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a
*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b +
sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*
x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((a*b*d^3*x^3 + a^2*d^3
)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(2
/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei
(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*b*d^3*x^3 + a^
2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (a*d^3/b
```

```
)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2)
)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (2*(a*b*d^3*x^3 +
a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d
^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (
b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*si
nh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b)
)*cosh(d*x + c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c
)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1
/3)*(sqrt(-3) + 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a
*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + s
qrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x
^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*s
inh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*co
sh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3
)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 +
a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^
(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((a
*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x +
c)^2 - (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*
sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) +
2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d
*x + c)^2 + (a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a
*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))
/((a^2*b^2*d^2*x^3 + a^3*b*d^2)*cosh(d*x + c)^2 - (a^2*b^2*d^2*x^3 + a^3*b*
d^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

3.105 $\int \frac{\cosh(c+dx)}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=739

result too large to display

```
[Out] Cosh[c + d*x]/(3*a*b*x^2) - Cosh[c + d*x]/(3*b*x^2*(a + b*x^3)) - (2*(-1)^(
1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*Cosh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d
*x])/(9*a^(5/3)*b^(1/3)) + (2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (d*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^
(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*d*CoshIntegral[
-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(9*a^(5/3)*b^(1/3)) - ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rubi [A] time = 1.32306, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5279, 5293, 3297, 3303, 3298, 3301, 5281, 5292}

$$\frac{d \sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}+\frac{(-1)^{2/3}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}}-\frac{\sqrt[3]{-1}d \sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}}{9a^{4/3}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x^2) - Cosh[c + d*x]/(3*b*x^2*(a + b*x^3)) - (2*(-1)^(
1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*Cosh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d
*x])/(9*a^(5/3)*b^(1/3)) + (2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a
^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) + (d*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^
(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(
1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*d*CoshIntegral[
-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3)*b^(1/3)) + (d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3)
```

```
+ d*x])/(9*a^(5/3)*b^(1/3)) - ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(4/3)*b^
(2/3)) + (2*(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3))
```

Rule 5279

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
```

```
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx = -\frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\int \frac{\cosh(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{3b} + \frac{d\int \frac{\sinh(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\int \left(\frac{\cosh(c+dx)}{ax^3} - \frac{b\cosh(c+dx)}{a\left(a+bx^3\right)}\right) dx}{3b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax^2} - \frac{bx\sinh(c+dx)}{a\left(a+bx^3\right)}\right) dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} + \frac{2\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{3a} - \frac{2\int \frac{\cosh(c+dx)}{x^3}\,dx}{3ab} - \frac{d\int \frac{x\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int \frac{\sinh(c+dx)}{x^2}\,dx}{3ab}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{3abx} + \frac{2\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)} - \ldots\right.}{3a}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{9a^{5/3}} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{9a^{5/3}} - \frac{2\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}\,d\ldots}{9a^{5/3}}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} + \frac{d^2\cosh(c)\text{Chi}(dx)}{3ab} + \frac{d^2\sinh(c)\text{Shi}(dx)}{3ab} - \frac{\left(d^2\cosh(c)\right)\int \ldots}{3ab}$$

$$= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2\left(a+bx^3\right)} - \frac{2\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2(-1)^{2/3}\cosh\ldots}{\ldots}$$

Mathematica [C] time = 0.193522, size = 387, normalized size = 0.52

$$\frac{\left(a+bx^3\right)\text{RootSum}\left[\#1^3 b+a\&, \frac{-2\sinh(\#1d+c)\text{Chi}(d(x-\#1))-\#1d\sinh(\#1d+c)\text{Chi}(d(x-\#1))+2\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1d\cosh(\#1d+c)\text{C}\ldots}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

```
[Out] (6*b*x*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x
 - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
 d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (a + b*x^3)*RootS
um[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshInteg
ral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
- 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral
[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c +
 d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #
1)]*#1)/#1^2 & ])/(18*a*b*(a + b*x^3))
```

Maple [C] time = 0.026, size = 226, normalized size = 0.3

$$\frac{d^3 e^{-dx-c} x}{6a\left(bd^3x^3+ad^3\right)}-\frac{d^2}{18ab}\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-bc^3\right)}\frac{(\_R1-c+2)\,e^{-\_R1}\mathrm{Ei}\left(1,dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}+\frac{d^3 e^{dx+c} x}{6a\left(bd^3x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x/a/(b*d^3*x^3+a*d^3)-1/18*d^2/a/b*sum((_R1-c+2)/(_R1^2
-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))+1/6*d^3*exp(d*x+c)*x/a/(b*d^3*x^3+a*d^3)+1/18*d^2/a/b*su
m((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3497, size = 5042, normalized size = 6.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 +
 a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)
^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((
-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^
3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b*x^3
 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a
) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cos
h(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^3 + sqr
t(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)
*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*c
osh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*
x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
- 1) - c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x
+ c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)
^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(
-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt
(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2
/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(-a*d^3
```

```
/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-
d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)*((b
*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3
)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*
d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(
-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*s
inh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cos
h(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x
- 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1) + c) + ((-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x +
c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*(
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x
^3 + a) + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3
 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x +
c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3
)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) +
a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) +
2*(-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b
*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((
-a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)
 + 2*(-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x +
 c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/
b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) + 2*(a
*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*
Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/((a^2*b*d*x^3 + a^3*d
)*cosh(d*x + c)^2 - (a^2*b*d*x^3 + a^3*d)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**3+a)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

3.106 $\int \frac{\cosh(c+dx)}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=697

result too large to display

```
[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*
CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])
/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a
^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) +
(Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3
)*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a
^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) - ((-1)
^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2)
```

Rubi [A] time = 1.47159, antiderivative size = 697, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5280}

$$-\frac{d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{-1}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{5/3}\sqrt[3]{b}}-\frac{(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\right.}{9a^{5/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*
CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])
/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a
^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) +
(Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(5/3
)*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a
^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2) - ((-1)
^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
```

```
a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2)
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx = -\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{\int \frac{\cosh(c+dx)}{x^4\left(a+bx^3\right)}\,dx}{b} + \frac{d\int \frac{\sinh(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^4} - \frac{b\cosh(c+dx)}{a^2x} + \frac{b^2x^2\cosh(c+dx)}{a^2\left(a+bx^3\right)}\right)dx}{b} + \frac{d\int \left(\frac{\sinh(c+dx)}{ax^3} - \frac{b\sinh(c+dx)}{a\left(a+bx^3\right)}\right) d}{3b}$$

$$= -\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\int \frac{\cosh(c+dx)}{x}\,dx}{a^2} - \frac{\int \frac{\cosh(c+dx)}{x^4}\,dx}{ab} - \frac{b\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx}{a^2} - \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{3a} + \frac{d\int}{}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{6abx^2} - \frac{b\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\cosh(c+dx)}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\text{co}}{3b^{2/3}(-}\right.}{a^2}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{d^2\cosh(c+dx)}{6abx} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a^2} - \frac{\sqrt[3]{b}\int \frac{\text{cos}}{\sqrt[3]{}}}{3}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} + \frac{\sinh(c)\mathbf{Shi}(dx)}{a^2} - \frac{d^3\int \frac{\sinh(c+dx)}{x}\,dx}{6ab} + \frac{\left(d^3\cos\right.}{}$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \cos$$

$$= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)} + \frac{\cosh(c)\mathbf{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^2} - \cos$$

Mathematica [C] time = 8.61079, size = 5530, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.072, size = 338, normalized size = 0.5

$$\frac{e^{-dx-c}d^3}{6a\left((dx+c)^3b-3(dx+c)^2bc+3(dx+c)bc^2+ad^3-bc^3\right)} - \frac{e^{-c}\text{Ei}(1,dx)}{2a^2} + \frac{1}{18a^2b}\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3\_Z^2bc+3\_Zbc^2+ad^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x/(b*x^3+a)^2,x)
```

```
[Out] 1/6*exp(-d*x-c)*d^3/a/((d*x+c)^3*b-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*
c^3)-1/2/a^2*exp(-c)*Ei(1,d*x)+1/18/a^2/b*sum((-a*d^3+3*_R1^2*b-6*_R1*b*c+3
*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*exp(d*x+c)*d^3/a/((d*x+c)^3*b-3*(d*x+c
```

```
)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*c^3)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/18/a^2/b*s
um((a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d
*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.32726, size = 4420, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*((6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*
d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 +
 sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (6*(b*x^3 + a
)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^
3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 +
a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^
2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x
^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x
+ c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1
/3)*(sqrt(-3) - 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*si
nh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d
*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x +
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1) + c) + 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2
+ (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)
^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(3*(b*x^3 +
 a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b*x
^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^
(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x
^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) +
a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*E
i(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) + 1) + c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c
)^2 - (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
- (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^
3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) -
(6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^
(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-
3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (6*(b*x^3 + a)*cosh(
d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqr
```

```
t(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)
*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3
*(b*x^3 + a)*sinh(d*x + c)^2 + (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^
2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*
d^3/b)^(1/3)) - 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x +
 c)^2 + (a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x
 + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*a*cosh(
d*x + c) - 18*((b*x^3 + a)*Ei(d*x) + (b*x^3 + a)*Ei(-d*x))*cosh(c) - 18*((b
*x^3 + a)*Ei(d*x) - (b*x^3 + a)*Ei(-d*x))*sinh(c))/((a^2*b*x^3 + a^3)*cosh(
d*x + c)^2 - (a^2*b*x^3 + a^3)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{\left(bx^3+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)^2*x), x)
```

3.107 $\int \frac{x^5 \cosh(c+dx)}{\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=784

result too large to display

```
[Out] -(x^3*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[c + d*x]/(6*b^2*(a + b*x^3)
) - ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + ((-1)^(1/3)*d^
2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(
1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*
CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*
a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) +
 (2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c +
d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(2/3)*
b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*Co
sh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/
(27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3
))
```

Rubi [A] time = 1.6262, antiderivative size = 784, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5291, 5289, 5280, 3303, 3298, 3301, 5290, 5293}

$$\frac{2d\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{-1}d\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{2/3}b^{7/3}}+\frac{2(-1)^{2/3}d\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{C}}{27a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -(x^3*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[c + d*x]/(6*b^2*(a + b*x^3)
) - ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + ((-1)^(1/3)*d^
2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(
1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*
CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*
a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) +
 (2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c +
d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(2/3)*
b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhInt
```

```
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*Co
sh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/
(27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3
))
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
```

```
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^5\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{\int \frac{x^2\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{2b} + \frac{d\int \frac{x^3\sinh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{dx\sinh(c+dx)}{18b^2\left(a+bx^3\right)} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{18b^2} + \frac{d\int \frac{\sinh(c+dx)}{a+bx^3}\,dx}{6b^2} +$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{dx\sinh(c+dx)}{18b^2\left(a+bx^3\right)} + \frac{d\int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\right.}\right.}{18b^2}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{dx\sinh(c+dx)}{18b^2\left(a+bx^3\right)} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2} - \frac{d\int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,d}{54a^{2/3}b^2}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{dx\sinh(c+dx)}{18b^2\left(a+bx^3\right)} - \frac{\left(d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\right)\int \frac{\sinh\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{54a^{2/3}b^2}$$

$$= -\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{6b^2\left(a+bx^3\right)} - \frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54\sqrt[3]{a}b^{8/3}} + \sqrt[3]{-1}$$

Mathematica [C] time = 0.595173, size = 397, normalized size = 0.51

$$d\text{RootSum}\left[\#1^3b+a\&,\frac{4\sinh(\#1d+c)\text{Chi}(d(x-\#1))-\#1d\sinh(\#1d+c)\text{Chi}(d(x-\#1))-4\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1d\cosh(\#1d+c)\text{Chi}(d(x-\#1))-}{\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (d*RootSum[a + b*#1^3 & , (-4*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 4*C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c + d*#1]*SinhIntegral[d*(x
 - #1)] - 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*Cosh
Integral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[
d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (4*Cosh[c + d*#1]*CoshI
ntegral[d*(x - #1)] + 4*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c
+ d*#1]*SinhIntegral[d*(x - #1)] + 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)
] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1
)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sin
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*(3*(a + 2*b*x^3)*C
```

```
osh[c + d*x] + d*x*(a + b*x^3)*Sinh[c + d*x]))/(a + b*x^3)^2)/(108*b^3)
```

Maple [C] time = 0.52, size = 2448, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/108/d^3/a^2/b^3*sum((10*_R1^2*a*b*c^2*d^3-_R1^2*b^2*c^5-_R1*a^2*d^6-10*_R
1*a*b*c^3*d^3+2*_R1*b^2*c^6-4*a^2*c*d^6+5*a*b*c^4*d^3-b^2*c^7-10*_R1^2*a*b*
c*d^3-20*_R1^2*b^2*c^4+20*_R1*a*b*c^2*d^3+34*_R1*b^2*c^5+4*a^2*d^6+10*a*b*c
^3*d^3-14*b^2*c^6-10*_R1*a*b*c*d^3-20*_R1*b^2*c^4-10*a*b*c^2*d^3+10*b^2*c^5
)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*
c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^
3+a^2*d^6)*x^4+1/36*d^7*exp(-d*x-c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^
6)*x+1/108/d^3*c^5/a^2/b*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*
c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*
d^3-b*c^3))-5/108/d^3*c^4/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*
_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1
,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*
c^3/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*
c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-
_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-
1/6*d^6*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+5/54/d^3*c^2/
a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^
5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^
3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/12*d^6*exp(-d*x-c)*a/b^2/(b^2*d^6*x^6+2*a*b*d
^6*x^3+a^2*d^6)-5/108/d^3*c/a^2/b^3*sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*
_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*d^3
-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^5-
2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp
(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)+1/108/d^3*c^5/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c
^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3
-b*c^3))-5/108/d^3*c^4/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1
^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*
x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^3
/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+1
0*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)
*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/5
4/d^3*c^2/a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^
2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*
d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/12*d^6*exp(d*x+c)*a/b^2/(b^2*d^6*x^
6+2*a*b*d^6*x^3+a^2*d^6)-1/6*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^
2*d^6)*x^3+1/108/d^3/a^2/b^3*sum((10*_R1^2*a*b*c^2*d^3-_R1^2*b^2*c^5-_R1*a^
2*d^6-10*_R1*a*b*c^3*d^3+2*_R1*b^2*c^6-4*a^2*c*d^6+5*a*b*c^4*d^3-b^2*c^7+10
*_R1^2*a*b*c*d^3+20*_R1^2*b^2*c^4-20*_R1*a*b*c^2*d^3-34*_R1*b^2*c^5-4*a^2*d
^6-10*a*b*c^3*d^3+14*b^2*c^6-10*_R1*a*b*c*d^3-20*_R1*b^2*c^4-10*a*b*c^2*d^3
+10*b^2*c^5)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*
b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-5/108/d^3*c/a^2/b^3*sum((4*_R1^2*a*b*
c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b
^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a
*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(
```

```
_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3
*_Z*b*c^2+a*d^3-b*c^3))-1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^
2*d^6)*x^4-1/36*d^7*exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 3.09944, size = 6886, normalized size = 8.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")

[Out] 1/216*(((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2
*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 +
 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
(b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
 + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^
3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - s
qrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((-
a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6
 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2)
)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh
(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3
)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2
*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*
a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^
2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*d^3/b)^(2/3)
```

```
*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d
*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt
(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 +
2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (
b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
+ c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 +
 sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((-a*d^3/b)^(2/3)*((b^2*x^
6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2
 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(
d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^
2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sq
rt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2
*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6
+ 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*
Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)
*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x
 + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^
(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 36*(2*a*b*x^3 + a^2)*cosh(d*x + c) - 12
*(a*b*d*x^4 + a^2*d*x)*sinh(d*x + c))/((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2
)*cosh(d*x + c)^2 - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \cosh(dx+c)}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

3.108 $\int \frac{x^4 \cosh(c+dx)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=1105

result too large to display

```
[Out] Cosh[c + d*x]/(9*a*b^2*x) - (x^2*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[
c + d*x]/(9*b^2*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*
b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) + ((-1)^(
1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x])/(54*a^(2/3)*b^(7/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + (d^2*Cosh[c - (a^(1/3
)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(2/3)*b^(7/3))
 - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]
)/(27*a*b^2) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a*b^2) - (d*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(2
7*a*b^2) - (d*Sinh[c + d*x])/(18*b^2*(a + b*x^3)) + (d*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(2
7*a*b^2) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(1/3)
*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) - (Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3))
 + (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) + ((-1)^(1/3)*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Sinh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(5
4*a^(2/3)*b^(7/3))
```

Rubi [A] time = 1.85304, antiderivative size = 1105, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5288, 5281}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```


```
[Out] Cosh[c + d*x]/(9*a*b^2*x) - (x^2*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[
c + d*x]/(9*b^2*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*
b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) + ((-1)^(
1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^
(1/3)*d)/b^(1/3)) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
```

```
)) - d*x])/(54*a^(2/3)*b^(7/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + (d^2*Cosh[c - (a^(1/3
)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(2/3)*b^(7/3))
 - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]
)/(27*a*b^2) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a*b^2) - (d*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(2
7*a*b^2) - (d*Sinh[c + d*x])/(18*b^2*(a + b*x^3)) + (d*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(2
7*a*b^2) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(1/3)
*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) - (Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3))
 + (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a*b^2) + ((-1)^(1/3)*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x])/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Sinh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(5
4*a^(2/3)*b^(7/3))
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5288

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^4 \cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} + \frac{\int \frac{x\cosh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{3b} + \frac{d\int \frac{x^2\sinh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)} - \frac{\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{9b^2} + \frac{d\int \frac{\sinh(c+dx)}{x\left(a+bx^3\right)}\,dx}{9b^2} + \cdots$$

$$= -\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)} - \frac{\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{bx\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2} + \cdots$$

$$= -\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)} - \frac{\int \frac{\cosh(c+dx)}{x^2}\,dx}{9ab^2} + \frac{\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{9ab} + \cdots$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2\left(a+bx^3\right)} + \frac{\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \cdots\right)}{\cdots}$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{\sqrt[3]{-1}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\cdots\right)}{54a^{2/3}b^{7/3}}\cdots$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{\sqrt[3]{-1}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\cdots\right)}{54a^{2/3}b^{7/3}}\cdots$$

$$= \frac{\cosh(c+dx)}{9ab^2x} - \frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2} - \frac{\cosh(c+dx)}{9b^2x\left(a+bx^3\right)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\cdots\right)}{27a^{4/3}b^{5/3}}\cdots$$

Mathematica [C] time = 0.524039, size = 675, normalized size = 0.61

$$\text{RootSum}\left[\#1^3b+a\&, \frac{-2\#1^2bd\sinh(\#1d+c)\text{Chi}(d(x-\#1))+2\#1^2bd\cosh(\#1d+c)\text{Chi}(d(x-\#1))+2\#1^2bd\sinh(\#1d+c)\text{Shi}(d(x-\#1))-2\#1^2bd\cosh(\#1d\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - a*
d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - a*d^2*Cosh[c + d*#1]*SinhInte
gral[d*(x - #1)] + a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*b*Cosh
[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[
c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*Sinh[c
+ d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(
x - #1)]*#1^2 - 2*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 2*b*d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c +
 d*#1]*CoshIntegral[d*(x - #1)]) - a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c +
d*#1] - a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)] - 2*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1
 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1 - 2*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
```

```
2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 2*b*d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1^2 + 2*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*
b*Cosh[d*x]*(b*x^2*(-a + 2*b*x^3)*Cosh[c] - a*d*(a + b*x^3)*Sinh[c]))/(a +
b*x^3)^2 + (6*b*(-(a*d*(a + b*x^3)*Cosh[c]) + b*x^2*(-a + 2*b*x^3)*Sinh[c])
*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^3)
```

Maple [C] time = 0.143, size = 1927, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/27/d^2*c^3/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1
*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108/d^2*c^4/a^2/b*su
m((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_
R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/27/d^2*c/a^2/
b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12
*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(
_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3
*_Z*b*c^2+a*d^3-b*c^3))-1/18/d^2*c^2/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_
R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2
*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d^2/a^2/b^3*sum((4*_R1^2*a*b*c*d^3-_
R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-
2*_R1^2*a*b*d^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*
d^3-10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2
*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c
^2+a*d^3-b*c^3))+1/18*d^6*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)
*x^5-1/36*d^6*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^
7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+1/36*d^7*exp(-d*x-c
)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)+1/108/d^2/a^2/b^3*sum((4*_R1^2*
a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d
^3-b^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-
10*a*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^
4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d^6*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^
3+a^2*d^6)*x^5-1/36*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^
2-1/36*d^7*exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/27/d^2*c/
a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^
5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^
3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/27/d^2*c^3/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2
-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)
*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))-1/18/d^2*c^2/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b
*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))-1/108/d^2*c^4/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R
1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))-1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*
d^6)*x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.4755, size = 10106, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(
2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^
2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6
+ 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqr
t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*Ei(d*x
 - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(-a*d^3/
b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*((a*b^2*d^3
*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3
+ sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*E
i(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sq
rt(-3) + 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x
+ c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*
(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b
^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3
)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*
d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^
3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)
^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d
*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 +
 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2
*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*
((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x
 + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) - 1) + c) - 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
```

```
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*(
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^
6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3)
)*cosh(c + (-a*d^3/b)^(1/3)) + 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d
*x + c)^2 + 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((
a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*c
osh(-c + (a*d^3/b)^(1/3)) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x +
c)^2 - 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^
6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
- sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (a*d^3/b)^(1
/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 +
 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh
(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x
 + c)^2 + 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^
3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (-a*d^3/
b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*
x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*
b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))
*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
sinh(d*x + c)^2 - 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) + (
a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^
2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 +
2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + s
qrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b
^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2)
 + (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x
^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(2*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
- (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x +
c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*
x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(2*(a*b^2*d^3*x^6 + 2*
a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
- (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c
)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x
+ (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(2*a*b^2*d^2*x^5 - a^2*b
*d^2*x^2)*cosh(d*x + c) + 12*(a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c))/((a^2
```

```
*b^4*d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*cosh(d*x + c)^2 - (a^2*b^4*
d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh (dx + c)}{(bx^3 + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

3.109 $\int \frac{x^3\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=776

result too large to display

```
[Out] Cosh[c + d*x]/(18*a*b^2*x^2) - (x*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh
[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5
/3)*b^(4/3)) - (d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + ((-1)^(2/3)*Cosh[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
 - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a*b^2) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(
5/3)*b^(4/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(54*a*b^2) + (d*Sinh[c + d*x])/(18*a*b^2*x) - (d*Sinh[c +
d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^
(4/3)) + (d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]
*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[
c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2
) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(54*a*b^2)
```

Rubi [A] time = 2.65879, antiderivative size = 776, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5291, 5279, 5293, 3297, 3303, 3298, 3301, 5281, 5292, 5290}

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{27a^{5/3}b^{4/3}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Ch}}{27a^{5/3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] Cosh[c + d*x]/(18*a*b^2*x^2) - (x*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh
[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5
/3)*b^(4/3)) - (d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + ((-1)^(2/3)*Cosh[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
 - d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a*b^2) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(
5/3)*b^(4/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(54*a*b^2) + (d*Sinh[c + d*x])/(18*a*b^2*x) - (d*Sinh[c +
d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^
(4/3)) + (d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a*b^2) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]
```

```
*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[
c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a*b^2
) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x])/(54*a*b^2)
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5279

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rubi steps

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}+\frac{\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx}{6b}+\frac{d\int \frac{x\sinh(c+dx)}{(a+bx^3)^2}\,dx}{6b}$$

$$= -\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x(a+bx^3)}-\frac{\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx}{9b^2}+\frac{d^2\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx}{18b^2}$$

$$= -\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x(a+bx^3)}-\frac{\int \left(\frac{\cosh(c+dx)}{ax^3}-\frac{b\cosh(c+dx)}{a(a+bx^3)}\right)dx}{9b^2}+\frac{d^2\int}{}$$

$$= -\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x(a+bx^3)}-\frac{\int \frac{\cosh(c+dx)}{x^3}\,dx}{9ab^2}+\frac{\int \frac{\cosh(c+dx)}{a+bx^3}\,dx}{9ab}+\frac{d^2}{}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}-\frac{d\sinh(c+dx)}{18b^2x(a+bx^3)}+\frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{}{3a}\right.}{}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{18ab^2}+\frac{d\sinh(c+dx)}{18ab^2x}-\frac{d}{18}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{18ab^2}+\frac{d\sinh(c+dx)}{18ab^2x}-\frac{d}{18}$$

$$= \frac{\cosh(c+dx)}{18ab^2x^2}-\frac{x\cosh(c+dx)}{6b(a+bx^3)^2}-\frac{\cosh(c+dx)}{18b^2x^2(a+bx^3)}-\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right.}{27a^{5/3}b^{4/3}}$$

Mathematica [C] time = 0.636095, size = 429, normalized size = 0.55

$$-\frac{\text{RootSum}\left[\#1^3b+a\&,\frac{-\#1^2d^2\sinh(\#1d+c)\text{Chi}(d(x-\#1))+\#1^2d^2\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1^2d^2\sinh(\#1d+c)\text{Shi}(d(x-\#1))-\#1^2d^2\cosh(\#1d+c)\text{Shi}}{\#}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -(RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 2*Co
shIntegral[d*(x - #1)]*Sinh[c + d*#1] + 2*Cosh[c + d*#1]*SinhIntegral[d*(x
- #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d^2*Cosh[c + d*#1]*Cos
hIntegral[d*(x - #1)]*#1^2 - d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1
^2 - d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c + d*#1]*
SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 & , (-2*Cosh[c
 + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#
1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhInteg
ral[d*(x - #1)] + d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + d^2*Co
shIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + d^2*Cosh[c + d*#1]*SinhIntegra
l[d*(x - #1)]*#1^2 + d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2
 & ] - (6*b*x*((-2*a + b*x^3)*Cosh[c + d*x] + d*x*(a + b*x^3)*Sinh[c + d*x]
))/(a + b*x^3)^2)/(108*a*b^2)
```

Maple [C] time = 0.112, size = 1456, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/108/d/a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*
d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^
3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d*c^3/a^2/b*sum((_R1^2-2*_R1*c+c^
2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36/d*c/a^2/b^2*sum((_R1^2*b*c^2-
_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^
3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/36/d*c^2/a^2/b^2*sum((_R1^2
*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R
1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_
Z*b*c^2+a*d^3-b*c^3))-1/36*d^7*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2
*d^6)*x^5-1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/
36*d^6*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/18*d^6*exp(-
d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/36/d*c^2/a^2/b^2*sum((_R1^
2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_
R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))+1/36/d*c/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*
c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^
2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b
*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d/a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R
1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3
+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d
*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*ex
p(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5+1/36*d^7*exp(d*x+c)/b/(b
^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/108/d*c^3/a^2/b*sum((_R1^2-2*_R1*c+
c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^6*exp(d*x+c)/a/(b^2*d^6*x
^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/18*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*
x^3+a^2*d^6)*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.63863, size = 6494, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*(
(b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*
cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*
b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
+ 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +
 a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*
b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 +
2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x +
c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(
1/3)*(sqrt(-3) + 1) - c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cos
h(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2
+ (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a
*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 +
 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x +
 c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 +
a^2*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*co
sh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(-d*x +
 (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + ((a*b^2*d^3*x^6 + 2*a^2*b*d
^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*c
osh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(d*x +
 (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^
3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*
Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)
```

```
^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/
b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3
)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*b^2*d
^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^
2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^
2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 -
 (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))
*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3
*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2
*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(-d*x +
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1) + c) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a
*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 2*(-a*d^3/b)^(1
/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/
b)^(1/3)) - ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 2*(a*d^3/b)^(
1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^3 + a^2*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b
)^(1/3)) - 6*(a*b^2*d*x^4 - 2*a^2*b*d*x)*cosh(d*x + c) - 6*(a*b^2*d^2*x^5 +
 a^2*b*d^2*x^2)*sinh(d*x + c))/((a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*
d)*cosh(d*x + c)^2 - (a^2*b^4*d*x^6 + 2*a^3*b^3*d*x^3 + a^4*b^2*d)*sinh(d*x
 + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

3.110 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} dx$

Optimal. Leaf size=781

result too large to display

```
[Out] -Cosh[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*
a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(4/3)*b^(5/3)
) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d
*x])/(54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh
[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3
)*b^(4/3)) + (d*Sinh[c + d*x])/(18*a*b^2*x^2) - (d*Sinh[c + d*x])/(18*b^2*x
^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Si
nhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - ((
-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/3)
*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x
])/(54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/
3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3))
```

Rubi [A] time = 1.4493, antiderivative size = 781, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5289, 5278, 5292, 3297, 3303, 3298, 3301, 5280, 5293}

$$\frac{(-1)^{2/3} d^2 \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}}-\frac{\sqrt[3]{-1}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}+\frac{d^2\cosh\left(c-\frac{\sqrt[3]{a}\ldots}{\sqrt[3]{b}\ldots}\right.}{54a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -Cosh[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*
a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(4/3)*b^(5/3)
) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d
*x])/(54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh
[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3
)*b^(4/3)) + (d*Sinh[c + d*x])/(18*a*b^2*x^2) - (d*Sinh[c + d*x])/(18*b^2*x
^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Si
nhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(5/3)*b^(4/3)) - ((
-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/3)
```

```
*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x
])/(54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(5/3)*b^(4/
3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(4/3)*b^(5/3))
```

Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5278

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Si
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Sinh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x^2\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{d\int\frac{\sinh(c+dx)}{\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\int\frac{\sinh(c+dx)}{x^3\left(a+bx^3\right)}\,dx}{9b^2}+\frac{d^2\int\frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)}\,dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\int\left(\frac{\sinh(c+dx)}{ax^3}-\frac{b\sinh(c+dx)}{a\left(a+bx^3\right)}\right)dx}{9b^2}+\frac{d^2\int\left(\frac{\cosh(c+dx)}{ax^2}-\frac{bx\cosh}{a(a+}\right.}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}-\frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\int\frac{\sinh(c+dx)}{x^3}\,dx}{9ab^2}+\frac{d\int\frac{\sinh(c+dx)}{a+bx^3}\,dx}{9ab}+\frac{d^2\int\frac{\cosh(c+dx)}{x^2}\,dx}{18ab^2}-$$

$$= -\frac{d^2\cosh(c+dx)}{18ab^2x}-\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{d\sinh(c+dx)}{18ab^2x^2}-\frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)}+\frac{d\int\left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}\right.}{}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{d\sinh(c+dx)}{18ab^2x^2}-\frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)}-\frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{b}x}\,dx}{27a^{5/3}b}-\frac{d\int\frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x}\,dx}{27a^{5/3}b}-$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{d^3\mathbf{Chi}(dx)\sinh(c)}{18ab^2}+\frac{d\sinh(c+dx)}{18ab^2x^2}-\frac{d\sinh(c+dx)}{18b^2x^2\left(a+bx^3\right)}+\frac{d^3\cosh(c)\mathbf{Shi}(dx)}{18ab^2}$$

$$= -\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}+\frac{(-1)^{2/3}d^2\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{54a^{4/3}b^{5/3}}-\frac{\sqrt[3]{-1}d^2\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{54a^{4/3}}$$

Mathematica [C] time = 0.421524, size = 423, normalized size = 0.54

$$-\frac{d\mathrm{RootSum}\left[\#1^3b+a\&,\frac{-2\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1))-\#1d\sinh(\#1d+c)\mathrm{Chi}(d(x-\#1))+2\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1))+\#1d\cosh(\#1d+c)\mathrm{Chi}(d(x-\#1))+}{\#1^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -(d*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x
```

```
 - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*Cosh
Integral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[
d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*Cosh
Integral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c
 + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1
)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #
1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Si
nh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*Cosh[d*x]*(-3*a*C
osh[c] + d*x*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 - (6*b*(d*x*(a + b*x^3)*Co
sh[c] - 3*a*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^2)
```

Maple [C] time = 0.086, size = 994, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] -1/36*d^7*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/36*d^7*ex
p(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(-d*x-c)/b/(b
^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-
2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*
c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/a^2/b*sum((_R1^2-2*_R1*c+c^2+
6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3
*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/a^2/b^2*sum((_R1^2*b*c-2*_R1*
b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c
+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))+1/36*d^7*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4+1/
36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(d*x+
c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/a^2/b^2*sum((_R1^2*b*c^2-_R1
*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8
*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/a^2/b*sum((_R1^2-2*_R1
*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/a^2/b^2*sum((_R1^2*b*c
-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-
2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53715, size = 6869, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*(36*a^2*cosh(d*x + c) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*
d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*
a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 +
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d
^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) +
 ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x
^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6
 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b
*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x
^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^
2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-
3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^
2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*co
sh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*co
sh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6
 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x
+ c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 -
 (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*
cosh(c + (-a*d^3/b)^(1/3)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2
)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(a*d^3
/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(
1/3)) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6
+ 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
 (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*
x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
 sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d
^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a
^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b^2
*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c
)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*si
nh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(
b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3
```

```
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - ((
-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*
x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^
6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2
))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)
*(sqrt(-3) - 1) + c) + 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cos
h(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^
(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
 a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/
3)) + 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^
2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b^2*x^6 + 2
*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)
^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(a*b*d*x^4 +
 a^2*d*x)*sinh(d*x + c))/((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*cosh(d*x +
c)^2 - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

3.111 $\int \frac{x\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=1147

result too large to display

```
[Out] -Cosh[c + d*x]/(18*a*b^2*x^4) + (2*Cosh[c + d*x])/(9*a^2*b*x) - Cosh[c + d*
x]/(6*b*x*(a + b*x^3)^2) + Cosh[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-1)
^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^
(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(1/3)*d^2*Cosh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(54*a^(5/3)*b^(4/3)) + (2*(-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(27*a^(7/3
)*b^(2/3)) - ((-1)^(2/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(5/3)*b^(4/3)) - (2
*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27
*a^(7/3)*b^(2/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3
)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*CoshIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^2*b) - (2*d*CoshIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(27*a^2*b) - (2*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*
x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^2*b) + (d*Sinh[c + d*x])
/(18*a*b^2*x^3) - (d*Sinh[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*Cosh[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x])/(27*a^2*b) + (2*(-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)
) - ((-1)^(1/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) -
(2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
27*a^(7/3)*b^(2/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
2*b) + (2*(-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) - ((-1)^(2/3)*d
^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3))
```

Rubi [A] time = 3.19773, antiderivative size = 1147, normalized size of antiderivative = 1., number of steps used = 89, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5290, 5281}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```


```
[Out] -Cosh[c + d*x]/(18*a*b^2*x^4) + (2*Cosh[c + d*x])/(9*a^2*b*x) - Cosh[c + d*
x]/(6*b*x*(a + b*x^3)^2) + Cosh[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-1)
^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^
(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)) + ((-1)^(1/3)*d^2*Cosh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(54*a^(5/3)*b^(4/3)) + (2*(-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(27*a^(7/3
```

```
)*b^(2/3)) - ((-1)^(2/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(5/3)*b^(4/3)) - (2
*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27
*a^(7/3)*b^(2/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3
)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*CoshIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*a^2*b) - (2*d*CoshIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(27*a^2*b) - (2*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*
x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^2*b) + (d*Sinh[c + d*x])
/(18*a*b^2*x^3) - (d*Sinh[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*Cosh[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x])/(27*a^2*b) + (2*(-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(7/3)*b^(2/3)
) - ((-1)^(1/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^2*b) -
(2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
27*a^(7/3)*b^(2/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1
/3)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
2*b) + (2*(-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(27*a^(7/3)*b^(2/3)) - ((-1)^(2/3)*d
^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3
)*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3))
```

Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\int \frac{x\cosh(c+dx)}{\left(a+bx^3\right)^3}\,dx = -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} - \frac{\int \frac{\cosh(c+dx)}{x^2\left(a+bx^3\right)^2}\,dx}{6b} + \frac{d\int \frac{\sinh(c+dx)}{x\left(a+bx^3\right)^2}\,dx}{6b}$$

$$= -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2\int \frac{\cosh(c+dx)}{x^5\left(a+bx^3\right)}\,dx}{9b^2} - \frac{d\int \frac{\sinh(c+dx)}{x^4\left(a+bx^3\right)}\,dx}{18b^2}$$

$$= -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2\int \left(\frac{\cosh(c+dx)}{ax^5} - \frac{b\cosh(c+dx)}{a^2x^2} + \frac{b^2x\cosh(c+dx)}{a^2(\ldots}\right.}{9b^2}$$

$$= -\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d\sinh(c+dx)}{18b^2x^3\left(a+bx^3\right)} + \frac{2\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx}{9a^2} + \frac{2\int \frac{\cosh(c+dx)}{x^5}\,d}{9ab^2}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} - \frac{d^2\cosh(c+dx)}{36ab^2x^2} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2a}{}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{d^2\cosh(c+dx)}{108ab^2x^2} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{2a}{}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{d\sinh(c+dx)}{18ab^2x^3} + \frac{d^3}{}$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} + \frac{d^4\cosh(c)\mathbf{Chi}(dx)}{36ab^2} -$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{d^4\cosh(c)\mathbf{Chi}(dx)}{108ab^2} -$$

$$= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2\cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2} + \frac{\cosh(c+dx)}{18b^2x^4\left(a+bx^3\right)} - \frac{2(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{}}{}\right.}{27}$$

Mathematica [C] time = 0.457546, size = 669, normalized size = 0.58

$$\text{RootSum}\left[\#1^3 b + a\&, \frac{-4\#1^2 bd\sinh(\#1d+c)\text{Chi}(d(x-\#1))+4\#1^2 bd\cosh(\#1d+c)\text{Chi}(d(x-\#1))+4\#1^2 bd\sinh(\#1d+c)\text{Shi}(d(x-\#1))-4\#1^2 bd\cosh(\#1d\ldots}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] (RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) +
 a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 4*b*C
osh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)]*Si
nh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshIntegral[
d*(x - #1)]*#1^2 - 4*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 - 4*b
*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*#1]*Sinh
```

```
Integral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (a*d^2*Cosh[c
+ d*#1]*CoshIntegral[d*(x - #1)] + a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c +
d*#1] + a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + a*d^2*Sinh[c + d*#1
]*SinhIntegral[d*(x - #1)] - 4*b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1
 - 4*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*Sinh
Integral[d*(x - #1)]*#1 - 4*b*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
4*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 4*b*d*CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1]*#1^2 + 4*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)
]*#1^2 + 4*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*
b*Cosh[d*x]*(b*x^2*(7*a + 4*b*x^3)*Cosh[c] + a*d*(a + b*x^3)*Sinh[c]))/(a +
 b*x^3)^2 + (6*b*(a*d*(a + b*x^3)*Cosh[c] + b*x^2*(7*a + 4*b*x^3)*Sinh[c])*
Sinh[d*x])/(a + b*x^3)^2)/(108*a^2*b^2)
```

Maple [C] time = 0.063, size = 810, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/9*d^6*exp(-d*x-c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5-1/36*d^7*
exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*exp(-d*x-c)/
a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2-1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x
^6+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+
b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_
R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1
/108*d*c/a^2/b*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp
(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)+1/9*d^6*exp(d*x+c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5+1/36*d^7
*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*exp(d*x+c)/a
/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6
+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*
c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)
*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/1
08*d*c/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_
R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.0848, size = 10103, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/216*((8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 8*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*(a*d^3/b)^(
2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^
2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6
+ 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a
^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqr
t(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*Ei(d*x
 - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) +
 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 -
 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + 4*(-a*d^3/
b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3
+ a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*
x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((a*b^2*d^3
*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3
+ sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)^2))*E
i(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sq
rt(-3) + 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x
+ c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - 4*
(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b
^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3
)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b
^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*
d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^
3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x + c)
^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d
*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 +
 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2
*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqr
t(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*
((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh(d*x
 + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b
)^(1/3)*(sqrt(-3) - 1) + c) - 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d
^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*
x + c)^2 - 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*(
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^
6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3)
)*cosh(c + (-a*d^3/b)^(1/3)) + 2*(4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*
d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d
*x + c)^2 + 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*cosh(d*x + c
)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((
a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6
 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*c
osh(-c + (a*d^3/b)^(1/3)) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x +
c)^2 - 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^
6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b
- sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (a*d^3/b)^(1
/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 +
 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3
*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*sinh
(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/
```

```
b)^(1/3)*(sqrt(-3) + 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x
 + c)^2 + 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^
3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^
2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (-a*d^3/
b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*
x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*
b*d^3*x^3 + a^3*d^3 + sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))
*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*
sinh(d*x + c)^2 - 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(
-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x
^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2) - (
a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^
2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 +
2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3
*d^3))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(
1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^
3)*sinh(d*x + c)^2 + 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + s
qrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b
^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2)
 - (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*
(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*cosh(d*x + c)^2 - (a*b^2*d^3*x
^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - sqrt(-3)*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3
+ a^3*d^3))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1)
)*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(4*(a*b^2*d^3*x^6 + 2*a
^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^
3 + a^3*d^3)*sinh(d*x + c)^2 - 4*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
+ (-a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x +
c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(-d*
x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(4*(a*b^2*d^3*x^6 + 2*
a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x
^3 + a^3*d^3)*sinh(d*x + c)^2 + 4*(a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 +
 a^2*b)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sinh(d*x + c)^2)
+ (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c
)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2))*Ei(d*x
+ (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) - 12*(4*a*b^2*d^2*x^5 + 7*a^2
*b*d^2*x^2)*cosh(d*x + c) - 12*(a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c))/((a
^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*cosh(d*x + c)^2 - (a^3*b^3*
d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh (dx + c)}{\left(bx^3 + a\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
    not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```